Physics 227 Exam 2

Rutherford said that if you really understand something you should be able to explain it to your grandmother.

For each of the topics on the next two pages, write clear, concise, physical descriptions that demonstrate you really understand the important qualitative aspects of quantum mechanics. You should be able to do this in a few sentences to a paragraph for each topic.

Explain the physics for each topic in your own words. You do not have to write a perfect essay on each topic, but do write enough to convince me that you really do understand the topic. Make sure to include any important pictures, graphs, and equations.

Write down or draw at least four important things for each topic.

Read **QED: The Strange Story of Light and Matter** by Richard Feynman and then write a one-half to one page summary for each of the four chapters.

Read **Probability and Uncertainty---the Quantum Mechanical View of Nature** by Richard Feynman and then write a one-half to one page summary.

	A
1	The time-dependent Schrodinger equation (TDSE)
2	The time-independent Schrodinger equation (TISE)
3	Converting the TDSE into the TISE by separating variables
4	Solving the TISE by finding the stationary states
5	Solving the TDSE by expanding the initial state in terms of the stationary states
6	The wavefunction
7	The probability density
8	The quantum mechanical expectation value
9	The quantum mechanical uncertainty
10	The energy levels of the square well
11	Sketch the potential for the square well and the first four energy eigenfunctions
12	Sketch the first four probability distributions for the square well
13	The energy levels of the simple harmonic oscillator (SHO)
14	Sketch the potential for the SHO and the first four energy eigenfunctions
15	Sketch the first four probability distributions for the SHO
16	The zero-point energy and the zero-point motion of the harmonic oscillator
17	The transmission and reflection coefficients for a potential step up
18	The transmission and reflection coefficients for a potential step down
19	The transmission and reflection coefficients for a potential well (down)
20	The transmission and reflection coefficients for a potential barrier (up)
21	The position-momentum Heisenberg uncertainty relation
22	The eigenstates of energy and momentum for the one-dimensional free particle
23	The dispersion relation for free electrons in empty space
24	The phase velocity and the group velocity for free electrons in empty space
25	The spreading of a Gaussian wave packet---past, present, and future
26	Charles Hermite, the Hermite equation, and the Hermite polynomials
27	Adrien-Marie Legendre, the Legendre equation, and the Legendre polynomials
28	Edmond Laguerre, the Laguerre equation, and the Laguerre polynomials
29	The spherical harmonics

	A
1	The geometric meaning of the divergence operator
2	The geometric meaning of the curl operator
3	The geometric meaning of the gradient operator
4	The geometric meaning of the Laplacian operator
5	The form of the divergence operator in Cartesian, spherical, and cylindrical coordinates
6	The form of the gradient operator in Cartesian, spherical, and cylindrical coordinates
7	The form of the curl operator in Cartesian, spherical, and cylindrical coordinates
8	The form of the Laplacian operator in Cartesian, spherical, and cylindrical coordinates
9	Maxwell's equation for the divergence of E (M1)
10	Maxwell's equation for the curl of E (M2)
11	Maxwell's equation for the divergence of B (M3)
12	Maxwell's equation for the curl of B (M4)
13	Gauss's theorem (GT)
14	Stoke's theorem (ST)
15	The fundamental theorem of calculus (FTC)
16	Convert M1 into its integral form using GT, ST, or FTC
17	Convert M2 into its integral form using GT, ST, or FTC
18	Convert M3 into its integral form using GT, ST, or FTC
19	Convert M4 into its integral form using GT, ST, or FTC
20	The electric field
21	The scalar potential and the electric field
22	The electric potential inside and outside of a spherical charge distribution
23	The electric field inside and outside of a spherical charge distribution
24	The magnetic field
25	The vector potential and the magnetic field
26	The vector potential due to a current carrying wire
27	The magnetic field due to a current carrying wire
28	The energy density in E and the energy density in B
29	The electric current density

Problem 1. Consider a square well that extends from 0 to L.

(a) Write down the general solution for the wave function inside the well.

(b) Determine the specific solutions inside the well for the ground state and for the first excited state by applying the boundary conditions at $x = 0$ and at $x = L$.

Now consider a 50:50 superposition of the ground state and the first excited state.

(c) Write down the normalized state vector for this superposition state using Dirac notation.

(d) Write down the normalized wave function for this superposition state using wave functions in position-space.

(e) Explain the time-dependence of the ground state wavefunction in the complex plane.

(f) Explain the time-dependence of the first excited state wavefunction in the complex plane.

(g) Explain the time-dependence of the 50:50 superposition state wavefunction in the complex plane.

(h) Explain the time-dependence of the 50:50 superposition state probability density in the complex plane.

(i) What is the oscillation frequency of the 50:50 superposition state probability density?

(j) Calculate the time-dependent expectation value of the position $< x(t) >$ for the 50:50 superposition state.

(k) Calculate the time-dependent expectation value of the momentum $< p(t) >$ for the 50:50 superposition state.

(l) Show that your answers for parts c through k above agree with the applet at

$$http://falstad.com/qm1d$$

Problem 2. Reflection and the transmission through a potential barrier

Consider an incident wave of unit amplitude that is incident from minus infinity.
In region 1 (from minus infinity to $x = 0$) the potential is $V(x) = 0$.
In region 2 (from $x = 0$ to a) the potential is $V(x) = V_0$.
In region 3 (from $x = a$ to infinity) the potential is $V(x) = 0$.
The energy E of the particle is greater than V_0.

(a) Write down the general solution in region 1.

(b) Write down the general solution in region 2.

(c) Write down the general solution in region 3.

(d) Match the boundary conditions on the wavefunctions at $x = 0$.

(e) Match the boundary conditions on the derivatives of the wavefunctions at $x = 0$.

(f) Match the boundary conditions on the wavefunctions at $x = a$.

(g) Match the boundary conditions on the derivatives of the wavefunctions at $x = a$.

(h) Solve the resulting equations for the reflection and transmission amplitudes r and t.

(i) Convert your amplitudes r and t into the associated probabilities R and T.

(j) Explain why the wave is partially reflected and partially transmitted.

(k) Show that your reflection and transmission probabilities R and T agree with the applet at

$$http://phet.colorado.edu/simulations/$$

Problem 3. Consider all the beautiful hydrogen atoms...

(a) Sketch the energy levels of the hydrogen atom for $n = 1, 2, 3, 4$ and 5 versus n and l. Label the l levels spectroscopically and by their l value. Indicate the m degeneracy of each l level and add up the total degeneracy for each n level. Remember that the degeneracy of the n^{th} level is n^2.

(b) Make a composite sketch showing the $l = 0, 1, 2,$ and 3 effective potentials for the hydrogen atom. Add the locations of the $n = 1, 2, 3,$ and 4 energy levels to your effective potential sketch, and explain how the energy levels in the different wells are lined up.

(c) Sketch just the $l = 0$ effective potential. Add the locations of the $n = 1, 2, 3,$ and 4 energy levels to your effective potential sketch. Sketch the radial wavefunctions for each of these energy levels and label each wavefunction with the appropriate R_{nl} designation. Sketch the corresponding radial probability distribution functions.

(d) Sketch just the $l = 1$ effective potential. Add the locations of the $n = 1, 2, 3,$ and 4 energy levels to your effective potential sketch. Sketch the radial wavefunctions for each of these energy levels and label each wavefunction with the appropriate R_{nl} designation. Sketch the corresponding radial probability distribution functions.

(e) Sketch the spherical harmonics for $l = 0$, 1, and 2. Be sure to show all of the allowed values of m. Explain the general rule for how many values of m there are for each value of l. Explain how the spherical harmonics vary with ϕ.

Problem 4. Coming soon !!! An interesting EM problem.