Dispersion Relations

Dispersion relations describe the relationship between the energy and the momentum of a particle. They are expressed by the functional relationships $E(p)$ and $\omega(k)$. Since $E = \hbar\omega$ and $p = \hbar k$ the two expressions are equivalent.

First consider free particles.

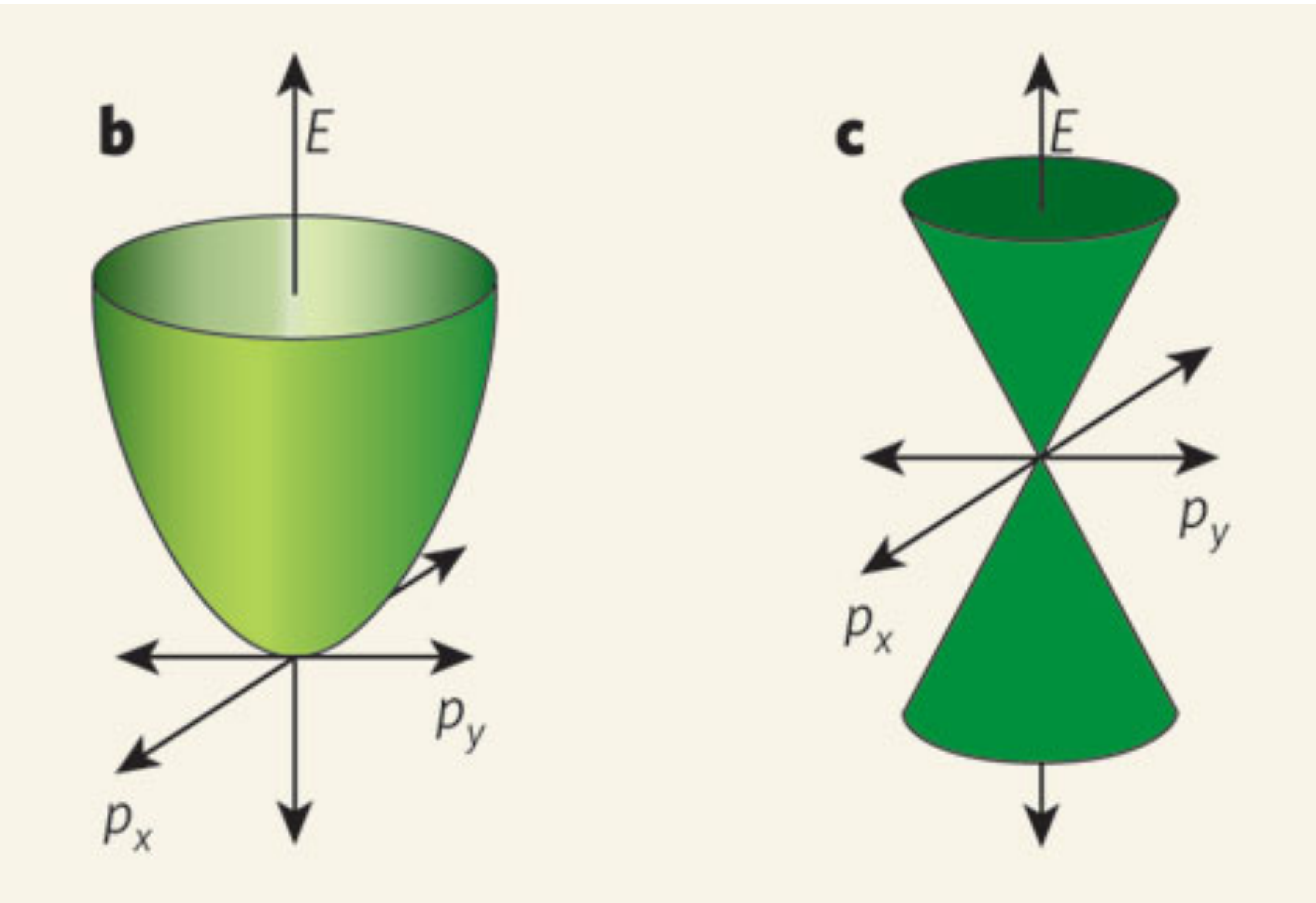
For non-relativistic particles with mass, the relationship between energy and momentum is $E = p^2/2m$, or equivalently $\omega = \hbar^2 k^2/2m$. So, the dispersion relation is parabolic.

For light, $E = pc$ and $\omega = ck$. So, the dispersion relation is linear.

Now consider interacting particles.

For interacting particles, the relationship between the energy and the momentum can be richer. We are interested in the dispersion relation for photons inside a material and the dispersion relation for surface plasmons.

The dispersion relation for photons inside a material is $\omega = (c/n)k$.



Parabolic Dispersion

particles with rest mass

electrons

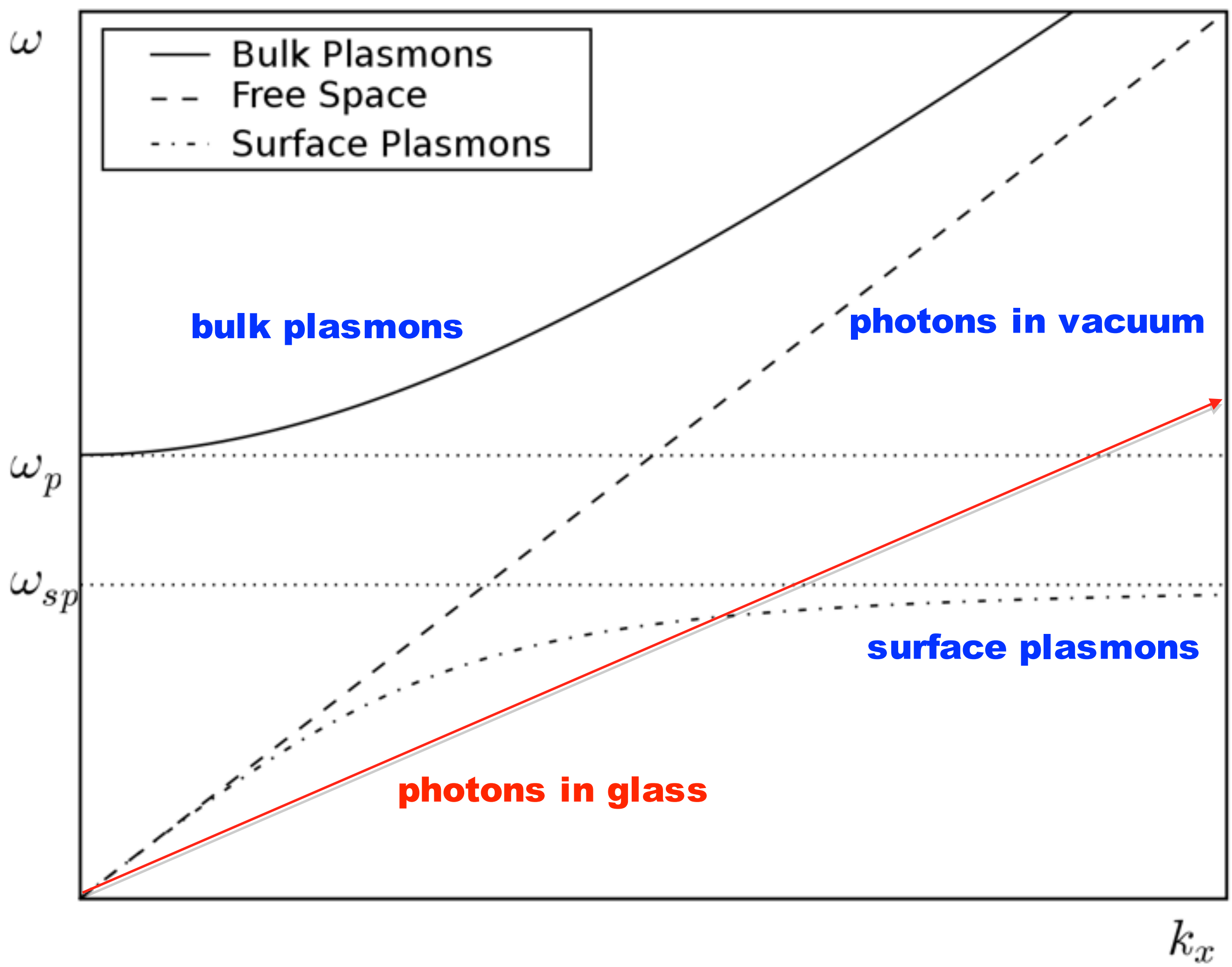
proton

neutrons

Linear Dispersion

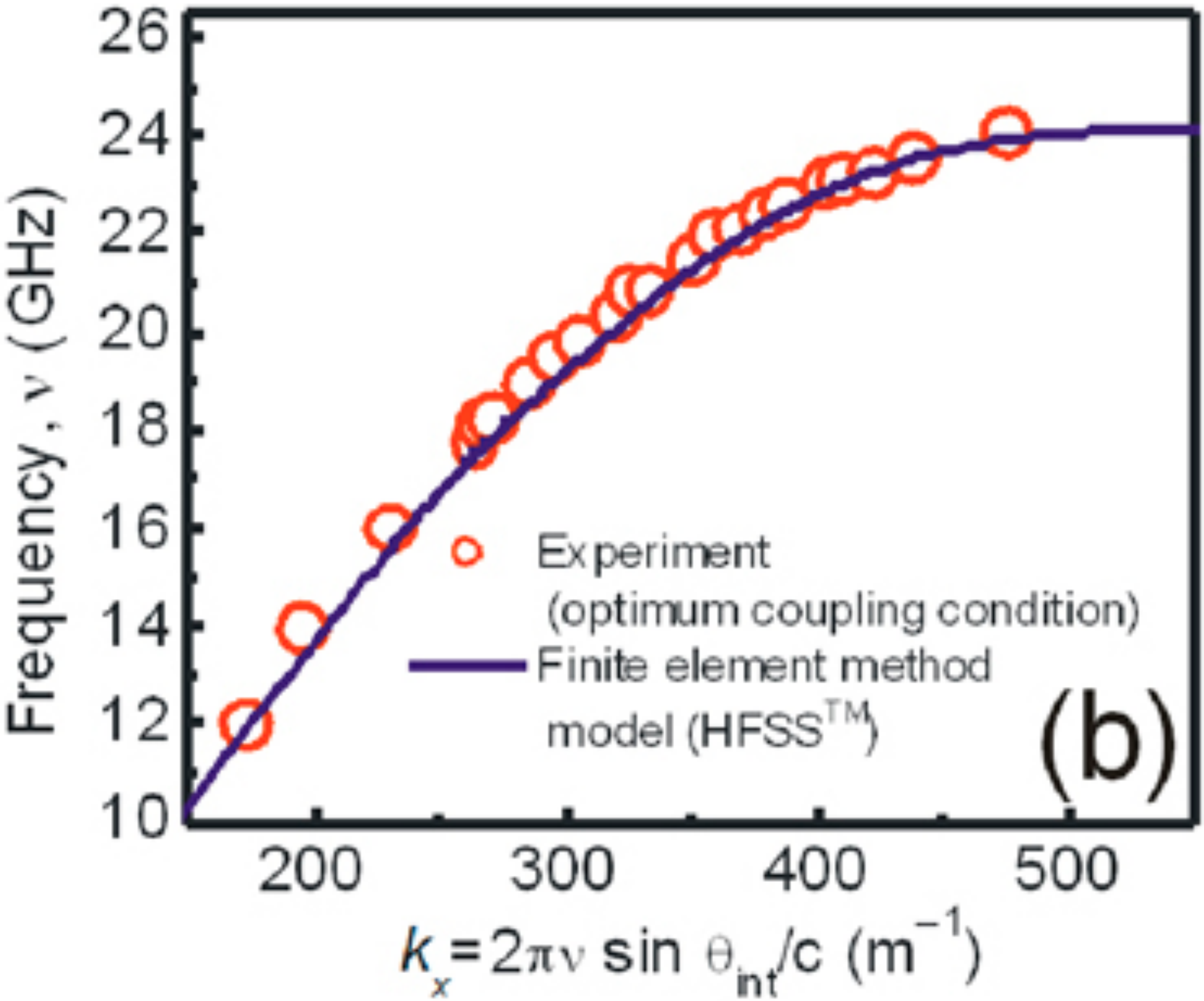
particles with zero rest mass

photons

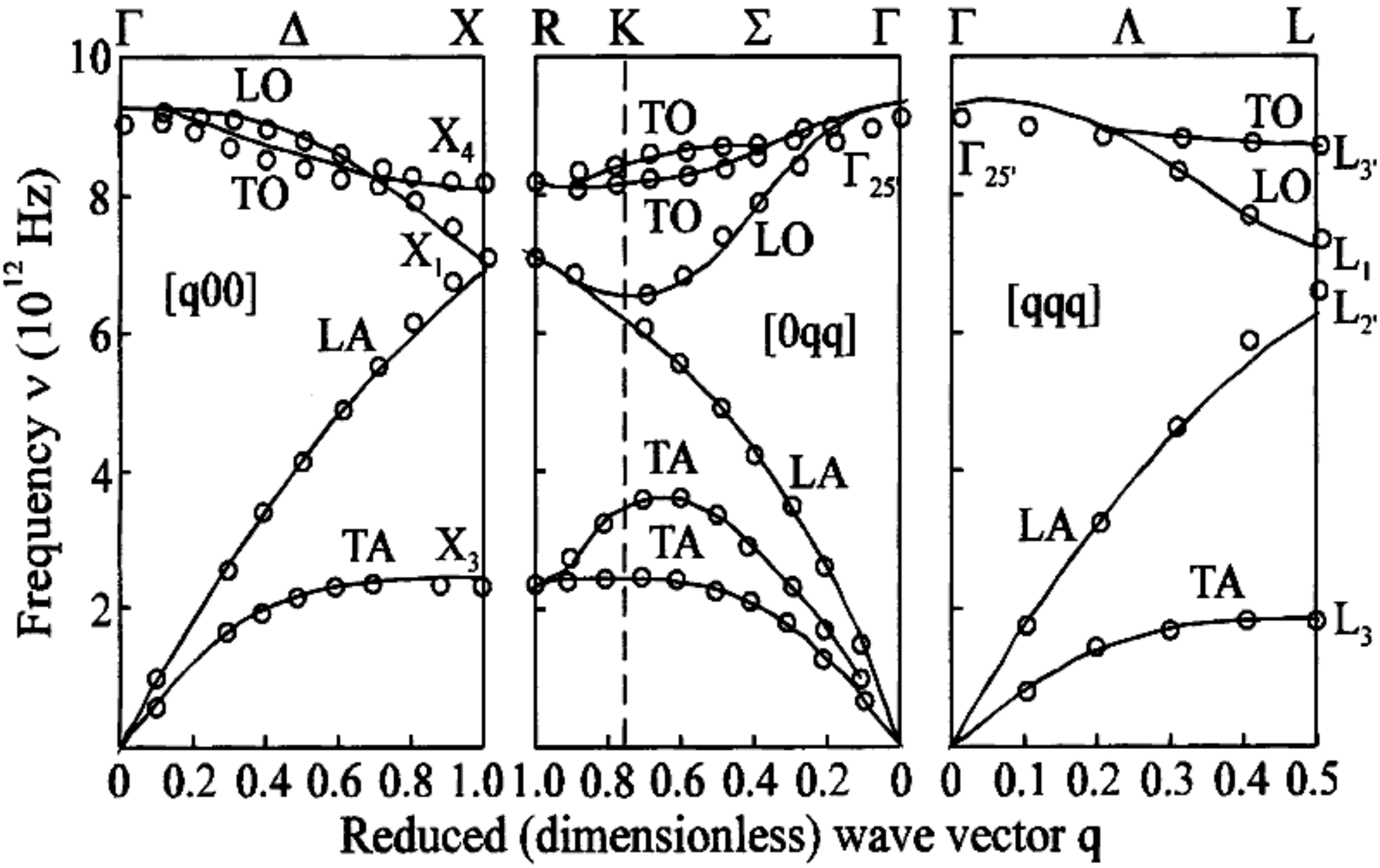


ω
Bulk Plasmons
Free Space
Surface Plasmons
bulk plasmons
photons in vacuum
ω_p
ω_{sp}
surface plasmons
photons in glass
k_x

Surface Plasmons



Dispersion Relation for phonons in germanium



The frequency depends on the direction of propagation