Electronic noise:
the first two decades

Most of the basic knowledge in the field of electronic noise was gained in the 20-year period following World War I. A considerable amount of the vacuum-tube data obtained was later translated into the semiconductor language

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You could hear a pin drop. (English saying)

You could hear the grass grow. (German version)

Such harmony is in immortal souls;
But whilst this muddy vestment of decay
Doth grossly close it in we cannot hear it.
(Merchant of Venice)

Fifty-two years ago the classical paper on noise in amplifiers was written by Dr. Walther Schottky.[31] The high-vacuum thermionic amplifier could then be called about six years old. Its development had taken place along nearly parallel lines in several countries, including Germany, mostly under rules of strict secrecy. It seems now almost incredible that out of the Germany of those years, faced with military defeat and economic collapse, could come a scientific paper of the quality and technical importance of this paper of Schottky's.

The amplifiers developed at the Siemens-Halske Works no doubt had the same kind of faults as those produced at other laboratories—poor welds, mechanical resonances, unstable cathodes, inadequate pumping, etc. These faults could distort the signals applied to the amplifiers and, since thermionic amplifiers were then being installed in commercial and military telephone systems, the faults became a technical liability. At this time I was employed in the Engineering Department of the Western Electric Company, the Engineering and Supply Division for the Bell System. I was assigned to study some of the many projects on vacuum-tube research, came early in touch with Schottky's work, and have some memories of the work that went on. With this as my background, the Editor of IEEE SPECTRUM asked me to write this article on the study of amplifier noise as I saw it develop during about the first two decades of its progress into a rather broad scientific field.

'Wärmeeffekt' and 'Schroteffekt'

In the 1918 paper, Dr. Schottky evidently assumes that the grosser current fluctuations produced by faulty tube structures such as those just enumerated have been, or can be, eliminated, and he is left with two sources of noise that are of a much more fundamental nature. One he calls the "Wärmeeffekt," in English now commonly named "thermal noise." This is a fluctuating voltage generated by electric current flowing through a resistance in the input circuit of an amplifier, not in the amplifier itself. The motion of charge is a spontaneous and random flow of the electric charge in the conductor in response to the heat motion of its molecules. The voltage between the ends of the conductor varies and is impressed upon the input to the amplifier as a fluctuating noise. This flow of energy between molecules and electric current involves not the charge of the electron but rather the rate of flow of power between charge and momentum. It involves the Boltzmann constant k times the absolute temperature T of the system, and a power flow of at least 10^{-17} watt to be audible in a telephone. Schottky believed any other noise source would be much stronger than this.

And here, for the sake of history, we may digress a bit. In estimating the total of noise that is going to be contributed by the "Schroteffekt," the integration of a certain expression is needed that has come to be called the Schottky equation. Schottky performed this integration

* John Bertrand Johnson (F) died at the age of 83 on November 27, 1970, the day he completed work on this manuscript. Dr. Johnson's obituary appears on page 107 of the January issue of IEEE SPECTRUM.

and got the result $2\pi/r^2$, where r is a damping factor of the circuit, $r = R/L\omega$.

My recollection is that because of some postal delay the 1918 paper did not get to the United States until about 1920. On reading it, I became suspicious of the integration, but in the then-available tables of integration could find no solution for the Schottky equation. I asked my friend, Dr. L. A. MacColl, mathematician, for assistance. He suggested splitting the Schottky expression into four complex factors, integrating each separately and then recombining them for the final result, $2/r$. When, after much labor on my part, this was done, MacColl again looked at the equation and said this was a case for the method of poles and residues and, without putting pencil to paper, read off the correct result. This was impressive, but evidently the method had not yet penetrated down to physicists and engineers, and the more cumbersome method was left in. The method of residues was evidently used later by Fry[5] and by Hull and Williams,[11] but before them several other methods had also been suggested. We correctors, however, chose to abide by Schottky's word that the thermal-effect noise is much smaller than the shot noise, and recognition of the technical importance of the thermal noise was delayed by about a decade. This probably did not matter much, because it was a busy decade spent on other phases of the project.

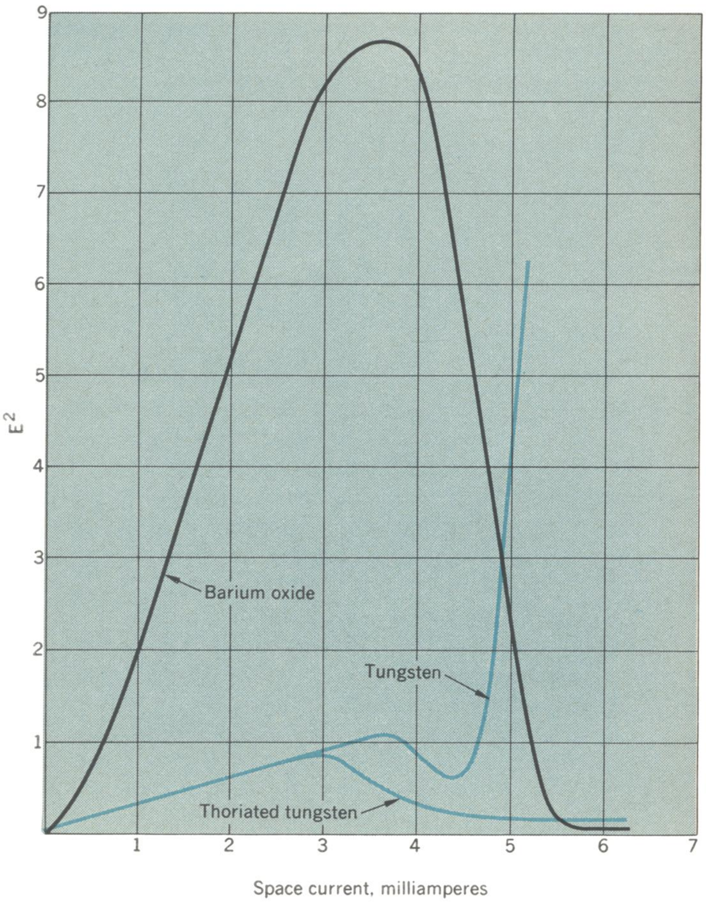
In the case of the "thermal noise," as we shall call it, the electric charge is in effect held in long bags with walls relatively impervious to electrons at low temperature. The mass transport of charge along the bag, or wires, under the influence of the heat motion, sets up the potential differences that generate the fluctuating output of the amplifier.

When now one end of the conductor, the "cathode" of the tube, is heated to incandescence, electrons can be emitted from the cathode surface to travel across the vacuum toward the anode. The electrons are emitted at random times, independent of each other, and they travel at different velocities, depending on initial velocity and voltage distribution for electron passage. In the case of a small electron emission, a small nearly steady flow of current results, with a superimposed smaller alternating current whose amplitude can be calculated from statistical theory. This small current flowing through the amplifier generates the "Schroteffekt," or shot effect, in the amplifier.

The first experimental work on identifying and measuring the shot effect was done in Schottky's laboratory and published by C. A. Hartmann in 1921. This seemed like a well-designed set of tests, but was a little ahead of its time in the new art. After corrections, it left little doubt of the existence of the shot effect.

The next step came with the publication of three papers in 1925. T. C. Fry[5] covered parts of the theory that he wanted put on a firmer mathematical basis. Through Fry, the work of Hull and Williams[11] at General Electric and Johnson[14] at Western Electric–Bell Laboratories became known to the participants, which may have given added impetus to the efforts. At GE, the first application of Hull's screen-grid tube in the amplifier increased the accuracy of the GE work to such a point that the value of the charge of the electron found by the shot effect came out close to that of the oil-drop method. The work of Johnson at lower frequencies revealed the existence of the "flicker effect," which could be many times greater than the shot effect, as well as the effect of space charge in reducing the magnitude of both shot effect and flicker effect by large factors (also recognized by Hull and Williams).

FIGURE 1. The effect of space charge on fluctuation noise. Three tubes have filaments composed of tungsten, thoriated tungsten, and barium oxide. $\overline{E^2}$ is the mean-square noise voltage across the output measuring device expressed in arbitrary units. The variation in space current was obtained by changing the cathode temperature, the plate voltage remaining constant. (Copyright 1934, The American Telephone and Telegraph Co.; reprinted by permission)



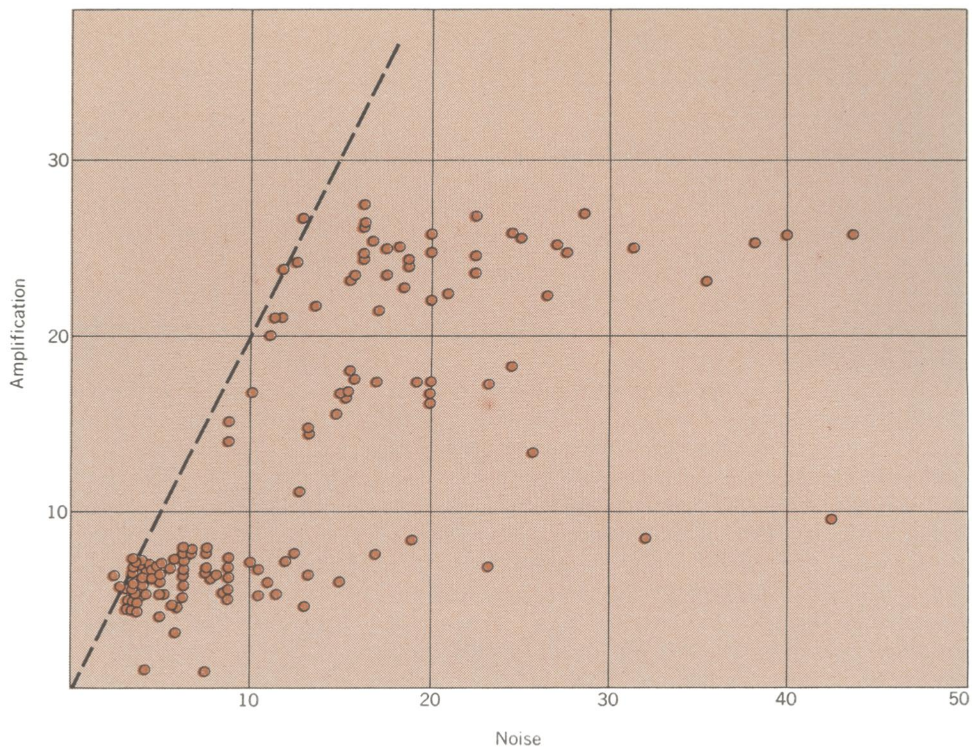


FIGURE 2. Amplification as a function of noise in three-electrode tubes; noise in arbitrary units; each point represents a tube. (Physical Review, 1925, reprinted by permission)

Each of these phenomenons will be discussed in connection with Fig. 1, which is reproduced from the 1934 paper by Pearson.[27]

By the early 1930s, the shot effect had been fairly well established for thermionic diodes, simple amplifiers, and photoelectric tubes.

A typical event that took place during the shot-effect work will be described here. We were visited by Sir J. J. Thomson, and the shot effect was demonstrated to him. Our explanation of it may not have been satisfactory, for as he left the room, the discoverer of the electron, with a forbearing smile and a gentle shake of the head, muttered, "Oh, no, no, no!"

Toward the end of the shot-noise work, a rough exploratory test was made. About 100 triode tubes of various kinds were picked out at random and tested for gain and noise in a circuit of fixed voltage, frequency range, etc. A resistance of 500 kΩ was connected across the input of the tube under test, with the output of the tube resistance–capacitance coupled to the amplifier. For each tube, the observed noise was plotted against the separately measured amplification of the tube, as in Fig. 13 of the 1925 article, here reproduced as Fig. 2. There is one point for each tube and these points are scattered over the right-hand side of the diagram. On the left, the point distribution stops abruptly along a straight sloping line. This suggests that along this line the noise pulses that the amplifier responds to have been amplified by the tube under test by its gain factor, from incoming pulses of more nearly constant value. Could this be the thermal effect predicted by Schottky?

A few simple tests, such as varying the electrical value of the input resistor, its temperature, its size, its material, soon answered the question in the affirmative. The results were discussed with Dr. H. Nyquist, who in a matter of a month or so came up with the famous formula for the effect, based essentially on the thermodynamics of a telephone line, and covering almost all one needs to know about the thermal noise.

The two effects: A and B (or T and S?)

We have, then, two different sources of electrical noise obeying statistical laws. Both have the properties in common that the noise can be described as a power dissipated by the noise source at a point of the amplifier circuit, and that for frequencies above certain values the noise power is constant up to very high frequencies. For thermal noise this constant power extends also to low values, while for shot noise there are many exceptions and variations.

T: the thermal effect. By the Nyquist[23] formulation, the thermal effect may be expressed as a voltage applied by the source to the input point of the amplifier at the high-impedance grid-leak resistor:

$$\text{Thermal formulation } \overline{V_T^2} = 4kTR \tag{1}$$

Here $\overline{V_T^2}$ is the mean-square noise fluctuation per unit bandwidth as measured by a thermocouple voltmeter; R is the resistance of the input circuit; T is temperature in degrees Kelvin; k is Boltzmann's constant, 1.38×10^{-23} joules/degree K. This can also be written

$$W_T = 4kT \text{ watts per unit resistance} \tag{2}$$

per unit bandwidth. The total for any case is then obtained by linear integration over the resistance and bandwidth range.

There is not much more to be done with this formulation except to consider the slope resistance of the tube, which will be done later.

S: the shot effect. The Schottky formulation for the shot effect per unit bandwidth may similarly be written

$$\overline{J_S^2} = 2ei \tag{3}$$

or

$$W_S = 2eiR_1 \tag{4}$$

where the charge on the electron $e = 1.602 \times 10^{-19}$ coulomb; i = dc space current, in amperes, flowing in space from cathode to anode (negative); R_1 = total resistance between cathode and anode, including that internal to the tube (function of frequency); and W_S = power per cycle dissipated in R_1.

This formulation was found by the early workers to hold under some carefully controlled conditions, including choice of cathode materials, freedom from space-charge effects, and choice of frequency band. When these conditions were judiciously selected, the experiments yielded, for instance, very nearly the correct value for the charge on the electron, as was shown in the tests of the 1920s. More complicated effects were also observed; they were subjected to a rather concentrated theoretical attack in the 1930s and will briefly be described in the following paragraphs.

1. *The flicker effect.* With some cathodes there is superimposed on the pure shot effect a fluctuation in current that is much greater than the shot current itself. This is illustrated in Fig. 1. The linear portion of the curve, obtained from tubes having filaments of tungsten and thoriated tungsten, gives the values the pure shot noise should have. The noise data were recorded as the temperature of the cathode was raised, the plate voltage of the diode being supplied by a fixed battery through a constant resistance. A measure of the cathode temperature is given by the indicated total current, in milliamperes. In the barium oxide tube, the noise increased more rapidly and reached a maximum value approximately ten times that of pure shot noise. The reason for this excess noise was surmised by Johnson to be fluctuations in the work function of the cathode surface due to particle migration, and was discussed at length by Schottky, who called it "Fackelneffekt."

2. *Space-charge depression.* Still in Fig. 1, after passing through a maximum, the noise in all three of the tubes decreases toward values eventually far below the theoretical shot value, at first thought to be effectively zero. This is an important feature, for it is in this low noise range that thermionic devices can be used as amplifiers. Schottky ascribes this noise depression to the smoothing effect of a dense space-charge layer near the cathode—between cathode and grid in a triode, for instance—and he works out a plausible theory for it.

3. *Frequency and flicker effect.* With fixed operating conditions, except for the natural frequency of a narrow-band circuit that the device works into, the noise output depends on this frequency. Normally the noise varies with this frequency f as

$$\overline{J_S^2} = f^{-n} \tag{5}$$

where n may lie in the range 1.2–0.9, depending on the material and condition of the cathode. For very pure materials, this increase in noise may be unobservable at frequencies above a few thousand hertz. Oxide cathodes, and perhaps all cathodes, show the effect down to very low frequencies, such as perhaps one cycle per month, where the noise has merged with the natural drift of the device.

The f^{-n} law has been discussed theoretically by Schottky and others.

4. *Ionic effects.* Ions may be generated from gas in the device, or from the electrodes, either by photoelectric or collision processes. The ion current would normally be small and make only a small addition to the dc electron current. But if, say, a heavy positive ion becomes trapped in the negative potential well that is created by the electron space charge, then a large pulse of electrons may be released through the potential minimum to make a noise pulse. This effect was described by Johnson[14] and studied by Ballantine[1] and others. The sharp rise of the noise at high currents as depicted for the tungsten tube in Fig. 1 is a result of ions emitted from its filament.

5. *Thermal noise in plate current.* A curious situation developed in about 1930. Llewellyn[21] suggested that the internal resistance R_0 of the thermionic device is really in parallel with the external resistance R_1, the parallel combination taken as the thermal noise source of the output circuit. Llewellyn suggested that this slope or differential resistance should be considered at the cathode temperature in combining it with the external resistance at room temperature. The result seemed to give reasonable agreement with observations.

6. *The half-temperature rule.* In making more careful measurements, however, Pearson[28] concluded that the temperature of the slope resistance should be half of the absolute temperature of the cathode in order to get agreement with Eq. (1) or (2). There seems at first to be no physical basis for this peculiar situation, but further experiments seemed to agree. Some found it hard to believe that there could be such a coupling between a stream of electrons and their source (the cathode). The most careful calculation of the effect, based on certain assumptions, was made by Rack,[29] who found that over a considerable part of the mid-temperature range the value of the temperature should be taken as $0.644T$ instead of $0.500T$. The most plausible explanation of the effect is probably presented by Schottky,[33] who arrived at about 0.500 for the factor, but his presentation has to do with a certain rectification of the noise signal in the output circuit of the device and is not easy to repeat here.

Another facet of the $\frac{1}{2}T$ rule is that for very small currents the noise can be derived from either the Schottky equation ei or the Nyquist equation kT. This is in the region where the current to the anode is too small to set up appreciable space charge, because of too low a cathode temperature. This seems to have been first noticed by F. C. Williams,[42] but was also discussed by Schottky and others. The temperature must again be taken as $\frac{1}{2}T$, but tubes are probably not often used in these regions, except possibly for logarithmic response.

Rating of tubes

We have, then, two fundamental sources of noise in an electronic circuit: thermal noise, which can be calculated from the input parameters; and shot noise, which is modified by various device parameters and can, in some cases, be calculated, or can be measured for individual devices. In the device, the effects of these sources are added into a noise-power spectrum. In a diode, this is fairly simple, but in a grid-controlled tube it is more complicated since each electrode must be considered.

Up to 1940, the period of this review, the method proposed by Johnson for grid-controlled tubes was followed closely by others. This technique involved short-circuiting the input, setting the other parameters at some operating condition, and measuring the noise at the output of the device in this condition. This was then considered the noise figure introduced by the device itself, and it could be expressed in terms of a resistance at the input that would give the same amount of thermal noise. This would normally be a few hundred to a few thousand ohms.

Several measurements of this kind will be referred to but no details will be given here because methods may have changed and, moreover, because most of the tubes tested are now obsolete.

Tests on a few U.S. tube types were made by Pearson,[27] whereas Moullín and Ellis[22] reported tests on some British tubes. Spenke[38] studied some German tubes, on which he presented extended and careful discussions. Probably the most extensive and detailed discussion and measurements on U.S.-made tubes for our period were reported by Thompson, North, and Harris.[41]

I would like to acknowledge some debts: for the short early days of my participation, three friends, long departed: Hendrick van der Bijl, Oliver Buckley, and Harold Arnold, for technical guidance and management support; for aid in the preparation of this manuscript, the staffs of Bell Telephone Laboratories and of Thomas A. Edison Industries; for love, cooperation, and understanding: in the early times, Clara, and in the latter days, Ruth.

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The following list of references is appended for readers who want to go a little further into the early history of our subject than is done in this brief review. It should help establish the approximate sequence of the important steps made in the first two decades. The list does not pretend to be complete, and it contains many items that are not specifically referred to in the main text.

By 1940 this basic work had been about completed, and from there on the work on noise took different directions. First, there was the highly mathematical study of how to extract a weak signal from a background of noise. Then came the transistor and the translation of the vacuum-tube data into the semiconductor language. This opened up new fields of applications, such as low-temperature work, rocketry, and space research. A few recent references may open the door to these fields.

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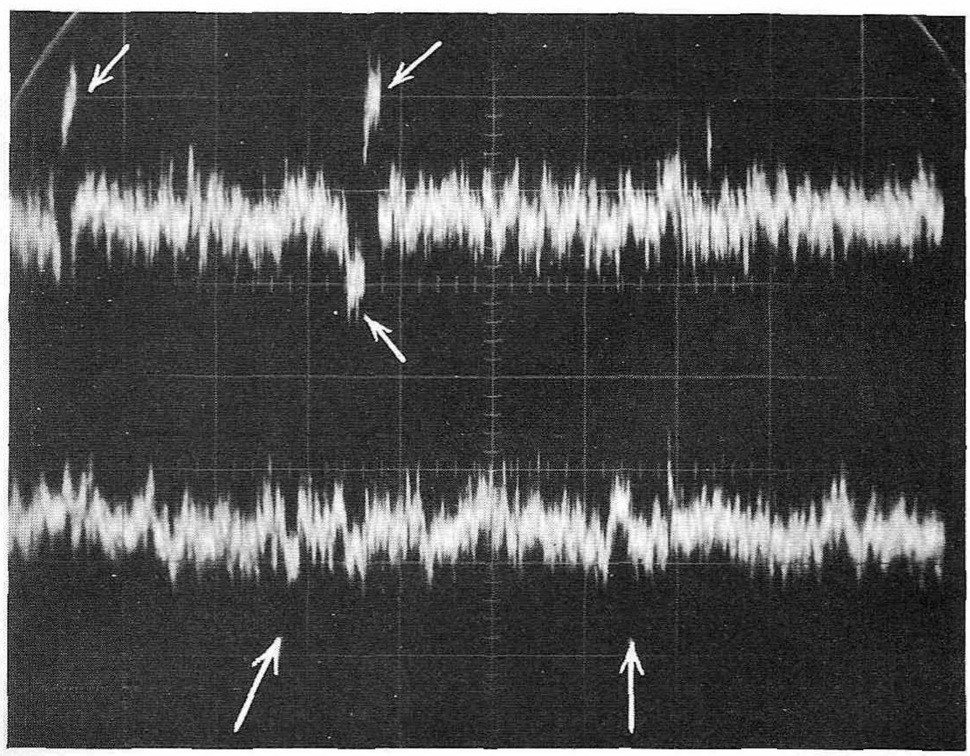
PART I

FUNDAMENTAL CONCEPTS



"Popcorn noise," discussed in Chap. 5, is shown in the traces. The top trace is considered to represent a moderate level of this noise. The bottom trace is a low level. Some devices exhibit popcorn noise with five times the amplitude shown in the top trace. Horizontal sensitivity is 2 ms/cm.

CHAPTER 1

FUNDAMENTAL NOISE MECHANISMS

The problems caused by electrical noise are apparent in the output device of an electrical system, but the sources of noise are unique to the low-signal-level portions of the system. The "snow" that may be observed on a television receiver display is the result of internally generated noise in the first stages of signal amplification.

This chapter defines the fundamental types of noise present in electronic systems and discusses methods of representing these sources for the purpose of noise circuit analysis. In addition, concepts such as noise bandwidth and spectral density are introduced.

1-1 NOISE DEFINITION

Noise, in the broadest sense, can be defined as *any unwanted disturbance that obscures or interferes with a desired signal.* Disturbances often come from sources external to the system being studied and may result from electrostatic or electromagnetic coupling between the circuit and the ac power lines, radio transmitters, or fluorescent lights. Cross-talk between adjacent circuits, hum from dc power supplies, or microphonics caused by the mechanical vibration of components are all examples of unwanted disturbances. With the exception of noise from electrical storms and galactic radiation, most of these types of disturbances are caused by radiation from electrical equipment; they can be eliminated by adequate shielding, filtering, or by changing the layout of circuit components. In extreme cases, changing the physical location of the test system may be warranted.

We use the word "noise" to represent basic random-noise generators or spontaneous fluctuations that result from the physics of the devices and materials that make up the electrical system. Thus the thermal noise apparent in all electrical conductors at temperatures above absolute zero is an example of noise as discussed in this book. This fundamental or true noise cannot be predicted exactly, nor can it be totally eliminated, but it can be manipulated and its effects minimized.

Noise is important. The limit of resolution of a sensor is often determined by noise. The dynamic range of a system is determined by noise. The highest signal level that can be processed is limited by the characteristics of the circuit, but the smallest detectable level is set by noise.

In addition to the familiar effects of noise in communication systems, noise is a problem in digital, control, and computing systems. For example, the presence of spikes of random noise makes it difficult to design a circuit that triggers (switches) at a specific signal amplitude. When noise of varying amplitude is mixed with the signal, noise peaks can cause a level detector to trigger falsely. To reduce the probability of false triggering, noise reduction is necessary.

Suppose that we have a system that is too noisy, but are uncertain whether the noisiness is caused by electrical equipment disturbances or by fundamental noise. We add shielding. A general rule for frequencies above 1000 Hz or impedance levels over 1000 Ω is to use conductive shielding (aluminum or copper). For low frequencies and lower impedances, we can use magnetic shielding (super-malloy, mu-metal) and twisted-wire pairs. We can also put the preamplifier on a separate battery supply. If these efforts help, we can try more shielding. The work may be moved to another location, or measurements can be made during the quieter evening hours. If these techniques do not reduce the disturbance, then look to fundamental noise mechanisms. Fundamental or true noise is the type considered almost exclusively in this book.

1-2 NOISE PROPERTIES

Noise is a totally random signal. It consists of frequency components that are random in both amplitude and phase. Although the long-term rms value can be measured, the exact amplitude at any instant of time cannot be predicted. If the instantaneous amplitude of noise could be predicted, noise would not be a problem.

It is possible to predict the randomness of noise. Much noise has a Gaussian or normal distribution of instantaneous amplitudes with time [1]. The common Gaussian curve is depicted in Fig. 1-1 along with a photograph of the associated electrical noise as obtained from an oscilloscope.

The Gaussian distribution predicts the probability of the measured noise signal having a specific value at a specific point in time. A noise signal with a

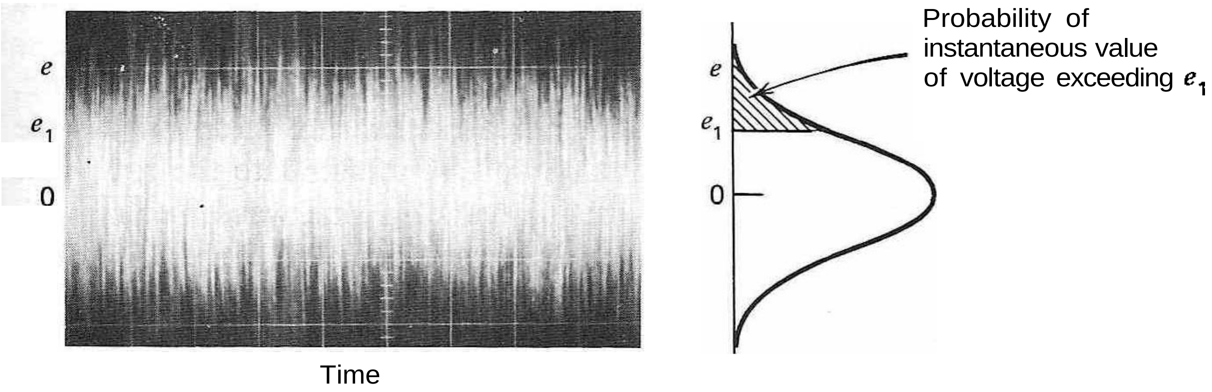


Figure 1-1 Noise waveform and Gaussian distribution of amplitudes.

zero mean Gaussian distribution has the highest probability of having a value of zero at any instant of time. The Gaussian curve is the limiting case produced by overlaying an imaginary coordinate grid structure on the noise waveform. If one could sample a large collection of data points and tally the number of occurrences when the noise voltage level is equal to or greater than a particular level, the Gaussian curve would result. Mathematically, the distribution can be described as

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp - \left[\frac{(x-\mu)^2}{2\sigma^2} \right] \tag{1-1}$$

where μ is the mean or average value and σ is the standard deviation or root mean square (rms) value of the variable x. The function $f(x)$ is referred to as the probability density function, or pdf.

The area under the Gaussian curve represents the probability that a particular event will occur. Since probability can only take on values from 0 to 1, the total area must equal unity. The waveform is centered about a mean or average voltage level μ, corresponding to a probability of .5 that the instantaneous value of the noise waveform is either above or below ***p.*** If we consider a value such as e_1, the probability of exceeding that level at any instant in time is shown by the cross-hatched area in Fig. 1-1. To a good engincering approximation, common electrical noise lies within $\pm 3\sigma$ of the mean μ. In other words, the peak-to-peak value of the noise wave is less than six times the rms value for 99.7% of the time.

The rms definition is based on the equivalent heating effect. True rms voltmeters measure the applied time-dependent voltage $v(t)$ according to

$$V_{\text{rms}} = \sqrt{\frac{1}{T_p} \int_0^T v^2(t)\, dt} \tag{1-2}$$

where T_p is the period of the voltage. Applying Eq. 1-2 to a sine wave of peak value V_m volts gives the familiar result $V_{\mathrm{rms}} = 0.707V_m$.

The most common type of ac volmeter rectifies the wave to be measured, measures the average or dc value, and indicates the rms value on a scale calibrated by multiplying the average value by 1.11 to simulate the rms. This type of meter correctly indicates the rms values of a sine wave, *but noise* is *not sinusoidal,* and the reading of a noise waveform will be 11.5% low. Correction can be made by multiplying the reading by 1.13. Chapter 15 discusses specific noise measurement instrumentation and these correction factors in much greater detail.

1-3 THERMAL NOISE

Three main types of fundamental noise mechanisms are thermal noise, shot noise, and low-frequency $(1/f)$ noise. Thermal noise is the most often encountered and is considered first. The other two types of noise are defined in later sections of this chapter. **A** special case of thermal noise limited by shunt capacitance called kT/C noise is also defined. Additional discussions of the effects of these types of noise in devices and circuits will be found throughout this book.

Thermal noise is caused by the random thermally excited vibration of the charge carriers in a conductor. This carrier motion is similar to the Brownian motion of particles. From studies of Brownian motion, thermal noise was predicted. It was first observed by J. B. Johnson of Bell Telephone Laboratories in 1927, and a theoretical analysis was provided by H. Nyquist in 1928. Because of their work thermal noise is called Johnson noise or Nyquist noise.

In every conductor or resistor at a temperature above absolute zero, the electrons are in random motion, and this vibration is dependent on temperature. Since each electron carries a charge of 1.602×10^{-19} ***C,*** there are many little current surges as electrons randomly move about in the material. Although the average current in the conductor resulting from these movements is zero, instantaneously there is a current fluctuation that gives rise to a voltage across the terminals of the conductor.

The *available noise power* in a conductor, N_t, is found to be proportional to the absolute temperature and to the bandwidth of the measuring system. In equation form this is

$$N_t = kT\Delta f \tag{1-3}$$

where k is Boltzmann's constant (1.38×10^{-23} W-s/K), T is the temperature of the conductor in kelvins (K), and Af is the *noise bandwidth* of the measuring system in hertz (Hz).

At room temperature (17°C or 290 K), for a 1.0-Hz bandwidth, evaluation of Eq. 1-3 gives $N_t = 4 \times 10^{-21}$ W. This is -204 dB when referenced to

1 W. In RF communications, 1 mW is often taken as the reference standard and dB, is used to indicate this standard.

$$\text{Noise power in dB,} = 10\log_{10}\left(\frac{4 \times 10^{-21}}{10^{-3}}\right)$$

$$= -174 \text{ dB,} \tag{1-4}$$

This level of -174 dB, is often referred to as the "noise floor" or minimum noise level that is practically achievable in a system operating at room temperature. It is not possible to achieve any lower noise unless the temperature is lowered.

The noise power predicted by Eq. 1-3 is that caused by thermal agitation of the carriers. Other noise mechanisms can exist in a conductor, but they are excluded from consideration here. Thus the thermal noise represents a minimum level of noise in a restrictive element.

In Eq. 1-3 the noise power is proportional to the noise bandwidth. There is equal noise power in each hertz of bandwidth; the power in the band from 1 to 2 Hz is equal to that from 1000 to 1001 Hz. This results in thermal noise being called "white" noise. "White" implies that the noise is made up of many frequency components just as white light is made up of many colors. A Fourier analysis gives a flat plot of noise versus frequency. The comparison to white light is not exact, for white light consists of equal energy per wavelength, not per hertz. Thermal noise ultimately limits the resolution of any measurement system. Even if an amplifier could be built perfectly noise-free, the resistance of the signal source would still contribute noise.

It is considerably easier to measure noise voltage than noise power. Consider the circuit shown in Fig. 1-2. *The available noise power is the power that can be supplied by a resistive source when it is feeding a noiseless resistive load equal to the source resistance.* Therefore, $R_S = R_L$ and $E_o = E_t/2$ represents the true rms noise voltage. The power supplied to R_L is N_t and is given by Eqs. 1-5 and 1-3:

$$N_t = \frac{E_o^2}{R_L} = \frac{E_t^2}{4R_L} = \frac{E_t^2}{4R_S} = kT\,\Delta f \tag{1-5}$$

Solving Eq. 1-5 for E_t, the rms thermal noise voltage E_t of a resistance

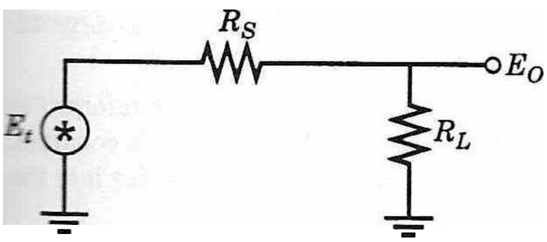


Figure 1-2 Circuit for determination of noise voltage.

R = R_S is*

$$E_t = \sqrt{4kTR\,\Delta f} \tag{1-6}$$

where R is the resistance or real part of the conductor's impedance, T is the temperature in kelvins, (room temperature = 17°C = 290 K), and k is Boltzmann's constant (1.38×10^{-23} W-s/K). Solving for $4kT$,

$$4kT = 1.61 \times 10^{-20} \qquad \text{(at 290 K)} \tag{1-7}$$

Example 1-1 Using Eq. 1-6, the thermal noise of a 1-kΩ resistor produces a noise voltage of 4 nV rms in a noise bandwidth of 1 Hz. This is a good number to memorize for a reference level. Using *4* nV as a standard, we can easily scale up or down by the square root of the resistance and/or the bandwidth.

This discussion might lead one to consider using a large-valued resistor and a wide bandwidth (which can produce several volts) in series with a diode in an attempt to power a load such as a transistor radio. It should be obvious that this will not work, but can you explain the flaw in the reasoning?

Several important observations can be made from Eq. 1-6. Noise voltage is proportional to the square root of the bandwidth, no matter where the frequency band is centered. Reactive components do not generate thermal noise. The resistance used in the equation is not simply the dc resistance of the device or component, but is more exactly defined as the real part of the complex impedance. In the case of inductance, it may include eddy current losses. For a capacitor, it can be caused by dielectric losses. It is obvious that cooling a conductor decreases its thermal noise.

Equation 1-6 is very important in noise analysis. It provides the noise limit that must always be watched. Today, low-noise amplifying devices are so quiet that system performance is often limited by thermal noise. We shall see that the measure of an amplifier's performance, its signal-to-noise ratio (S/N) and its noise figure (NF) are only measures of the noise the amplifier adds to the thermal noise of the source resistance.

The effect of broadband thermal noise must be minimized. Equation 1-6 implies that there are several practical ways to do this. The sensor resistance must be kept as low as possible, and additional series resistance elements must be avoided. Also, it is desirable to keep the system bandwidth as narrow as possible, while maintaining enough bandwidth to pass the desired signal.

*A more complete expression for thermal noise is $E_t^2 = 4kTRp(f)\,df$, where $p(f)$ is referred to as the Planck factor: $p(f) = (hf/kT)(e^{hf/kt} - 1)^{-1}$. $h = 6.62 \times 10^{-34}$ J-s is Planck's constant. The term $p(f)$ is usually ignored since $hf/kT \ll 1$ at room temperature for frequencies into the microwave band. Therefore, $p(f) = 1$ for most purposes [2].

When designing a system, frequency limiting should be incorporated in one of the later stages. For laboratory applications, frequency limiting is usually obtained with spectrum analyzers or tuned filters. It is normally undesirable to do the frequency limiting at the sensor or the input coupling network. This tends to decrease both the signal and the sensor noise but it does not attenuate the amplifier noise that is generated following the coupling network.

Even though we have shown that there is a time-varying current and available power in every conductor, this is not a new power source! Recalling the previous teaser question, you cannot put a diode in series with a noisy resistor and use it to power a transistor radio. If the conductor were connected to a load (another conductor), the noise power of each would merely be transferred to the other. If a resistor at room temperature were connected in parallel with a resistor at absolute zero, 0 K, there would indeed be a power transfer from the higher temperature resistor to the lower. The warmer resistor would try to cool down and the other would try to warm up until they came into thermal equilibrium. At that point there would be no further power transfer.

Thermal noise has been extensively studied. Expressions are available for predicting the number of maxima per second present in thermal noise, and also the number of zero crossings expected per second present in thermal noise, and also the number of zero crossings expected per second in the noise waveform. These quantities are dependent on the width of the passband. Formulas are given in Prob. 1-17.

1-4 NOISE BANDWIDTH

Noise bandwidth is not the same as the commonly used -3-dB bandwidth. There is one definition of bandwidth for signals and another for noise. The bandwidth of an amplifier or a tuned circuit is classically defined as the frequency span between half-power points, the points on the frequency axis where the signal transmission has been reduced by 3 dB from the central or midrange reference value. A -3-dB reduction represents' a loss of 50% in the power level and corresponds to a voltage level equal to 0.707 of the voltage at the center frequency reference.

The noise bandwidth, **Af**, is the frequency span of a rectangularly shaped ***power*** gain curve equal in area to the area of the actual power gain versus frequency curve. Noise bandwidth is the area under the power curve, the integral of power gain versus frequency, divided by the peak amplitude of the curve. This can be stated in equation form as

$$\Delta f = \frac{1}{G_o}\int_0^\infty G(f)\,df \qquad (1\text{-}8)$$

where $G(f)$ is the power gain as a function of frequency and G_o is the peak power gain. Generally, we only know the frequency behavior of the ***voltage*** gain of the system and since power gain is proportional to the network voltage gain squared, the equivalent noise bandwidth can also be written as

$$\Delta f = \frac{1}{A_{vo}^2} \int_0^\infty |A_v(f)|^2 \, df \tag{1-9}$$

where **A,,** is the peak magnitude of the voltage gain and $|A_v(f)|^2$ is the square of the magnitude of the voltage gain over frequency—the square of the magnitude of a Bode plot. Equation 1-9 is a more useful expression for Af.

The plot shown in Fig. ***1-3a*** is typical of a broadband amplifier with maximum gain at dc. The shape of the curve may appear strange because it

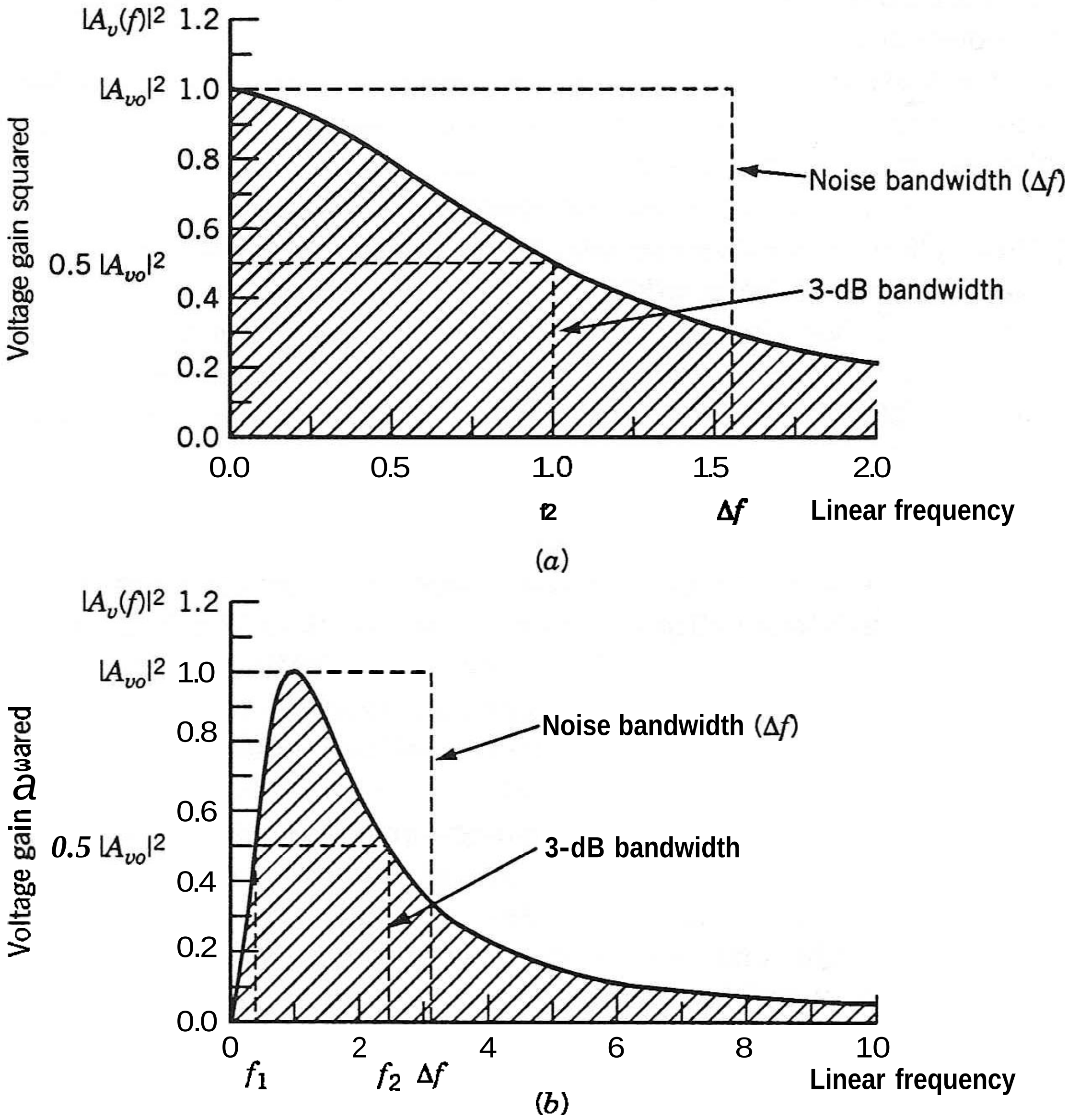


Figure 1-3 Definition of noise bandwidth.

has a linear frequency scale instead of the more common logarithmic scale. If the gain peak does not occur at dc such as with the bandpass amplifier shown in Fig. 1-3*b*, the maximum voltage gain must be found and used to normalize the noise bandwidth calculation. The area of the dashed rectangle is equal to the area of the integration in Eq. 1-9. Thus the noise bandwidth, Af, is not equal to the half-power or -3-dB bandwidth, f_2. The noise bandwidth will always be greater than f_2.

As an example of Af determination, consider a fist-order low-pass filter whose signal transmission varies with frequency according to

$$A_v(f) = \frac{1}{1 + jf/f_2} \tag{1-10}$$

where f_2 is the conventional -3-dB cutoff frequency and the low-frequency and midband voltage gain has been normalized to unity. The magnitude of the voltage gain is

$$|A_v(f)| = \frac{1}{\sqrt{1 + (f/f_2)^2}} \tag{1-11}$$

Then from Eq. 1-9 the noise bandwidth is

$$\Delta f = \int_0^\infty \frac{df}{1 + (f/f_2)^2} \tag{1-12}$$

Now change variables so that

$$f = f_2 \tan\theta \qquad \text{and} \qquad df = f_2 \sec^2\theta\, dB$$

The new limits of integration become 0 to $\pi/2$ such that

$$\Delta f = \int_0^{\pi/2} \frac{f_2 \sec^2\theta\, d\theta}{1 + \tan^2\theta}$$

$$\Delta f = f_2 \int_0^{\pi/2} d\theta = \frac{\pi f_2}{2} = 1.571 f_2 \tag{1-13}$$

Note that the noise bandwidth is 57% larger than the conventional -3-dB bandwidth for the first-order low-pass filter.

As a second example consider two identical, first-order, low-pass filters cascaded with appropriate buffering to prevent loading as shown in Fig. 1-4.

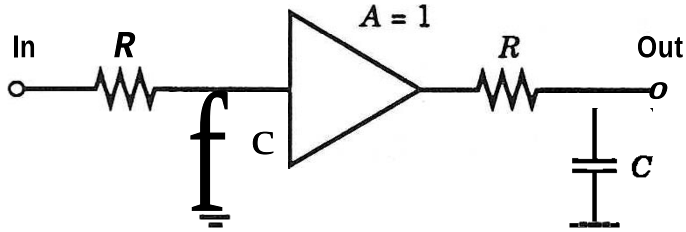


Figure 1-4 Cascaded low-pass filter.

The voltage transfer function for this circuit is

$$A_v(f) = \left| \frac{1}{1 + jf/f_2} \right|^2 \tag{1-14}$$

where f_2 is the -3-dB high-frequency corner of each RC time constant. The noise bandwidth is now

$$\Delta f = \int_0^\infty \left| \frac{1}{1 + (f/f_2)^2} \right|^2 df \tag{1-15}$$

Again making the same change of variable and change of limit substitution,

$$\Delta f = \int_0^{\pi/2} \frac{f_2\, d\theta}{1 + \tan^2 \theta} = \frac{\pi f_2}{4} = 0.785 f_2 \tag{1-16}$$

However, it must be remembered here that f_2 is the conventional -3-dB cutoff frequency of each stage and not the system's -3-dB cutoff frequency which we will denote as f_a. The amplifier system cutoff frequency can be found from

$$\frac{1}{\sqrt{2}} = \frac{1}{1 + (f_a/f_2)^2} \tag{1-17}$$

Solving gives

$$f_a = 0.6436 f_2 \tag{1-18}$$

Therefore, the noise bandwidth of the amplifier system is

$$\Delta f = \frac{\pi f_2}{4} = \frac{\pi f_a}{4 \times 0.6436} = 1.222 f_a \tag{1-19}$$

Note that the noise bandwidth is 22% larger than the conventional -3-dB bandwidth for the system. **As** the high-frequency roll-off becomes sharper by

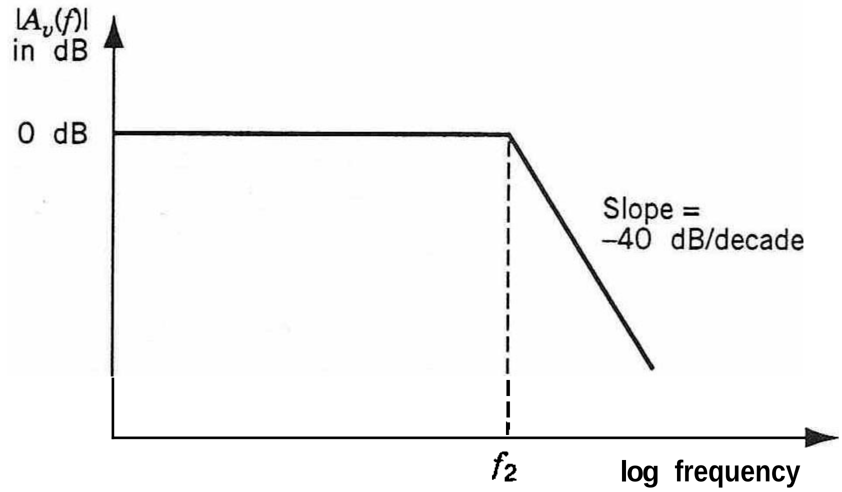


Figure 1-5 Frequency response of the cascaded low-pass filter.

using a large number of cascaded stages, Af approaches the -3-dB bandwidth. For example, Prob. 1-8 shows that a cascade of three identical low-pass stages produces a **Af** 1.155 times the system's -3-dB bandwidth.

Often an exact evaluation of the integration necessary to find the noise bandwidth in a convenient closed form is not possible and an approximation technique becomes necessary. To illustrate one such approximation, consider again the cascaded low-pass filter example of Fig. 1-4. The magnitude of its frequency response is as shown in Fig. 1-5.

The equation for the asymptotic frequency response of the magnitude of $A_v(f)$ in decibels is

$$|A_v(f)|_{\mathrm{dB}} = \begin{cases} 0 \text{ dB} & \text{for} \quad 0 \le f \le f_2 \\ -40\log_{10}(f/f_2) & \text{for} \quad f_2 \le f \le \infty \end{cases} \tag{1-20}$$

The approximate noise bandwidth for the asymptotic response is

$$\Delta f = \int_0^{f_2} df + \int_{f_2}^{\infty} \left(\frac{f_2}{f}\right)^4 df = 1.333 f_2 \tag{1-21}$$

It was found previously in Eq. 1-19 that **Af** $= 1.222 f_a = 0.785 f_2$ for a double time constant circuit, where f_2 is the time constant of each pole. In this example, the approximate analysis technique introduced an error into the Af evaluation of $1.333/0.785 = 1.70$ or 70% error, which dramatically shows that this approximation method cannot be used!

Often the frequency response equations become very complicated and it is very difficult to determine the noise bandwidth using direct mathematical integration for large or complex electronic systems. In these cases, other approximation techniques may be successfully employed using numerical integration routines available in MathCAD, DERIVE, or similar computer programs. Alternatively, the noise bandwidth can be found using circuit simulators like SPICE and the analysis techniques explained in Chap. 4. Finally, a graphical approach may also be a viable method. Here $|A_v(f)|^2$ is plotted on linear graph paper and the number of squares underneath the curve counted. The noise bandwidth is found by dividing this total by $|A_{vo}|^2$.

Care must be exercised in measurements pertaining to noise. We must not allow the measuring equipment to change the bandwidth of the system. Also, we must bear in mind that to arrive at useful results, the network response must continue to fall off as we reach higher and higher frequencies.

The term *spectral density* is used to describe the noise content in a 1 Hz unit of bandwidth. It can be related to narrowband noise as will be presented later. Spectral density has units of volts^2 per hertz and is symbolized as $S(f)$ to show that in general it varies with frequency. For a thermal noise source the spectral density $S(f)$ is

$$S(f) = \frac{E_t^2}{\Delta f} = 4kTR \qquad \text{V}^2/\text{Hz} \tag{1-22}$$

It is characteristic of white-noise sources that the plot of $S(f)$ versus frequency is a simple horizontal line.

When measuring noise we work with the rms value of a noise quantity. Thus we obtain the spectral density by dividing the mean square value of a noise voltage by the noise bandwidth. If we take the square root of this mathematical operation, it can be interpreted as simply the rms noise voltage in 1 Hz of bandwidth. Note that the square root of the spectral density (symbolized as $E/\sqrt{\Delta f}$) is a quantity that can be measured; the units are volts per hertz$^{1/2}$. Often this density function is symbolized by $E/\sqrt{\sim}$, or in the case of a current, $I/\sqrt{\sim}$, using the cycle symbol $(\sim)$ to indicate frequency. This is not correct since frequency is in cycles per second not cycles, so volts per hertz$^{1/2}$ should be used. Since a bandwidth of 1 Hz is almost always used, the units for these functions are referred to as "volts per root hertz" and "amps per root hertz."

Spectral density is a narrowband noise and generally varies with frequency. In order to obtain the *total* wideband noise, the spectral density function must be integrated over the frequency band of interest. Consider the system shown in Fig. 1-6 where a resistor is used as a noise source which is amplified by an ideal bandpass amplifier. The total output noise measured by a true rms voltmeter is given by

$$E_{no}^2 = \int_0^\infty 4kTR|A_v(f)|^2\,df = 4kTR\int_0^\infty |A_v(f)|^2\,df$$

$$E_{no}^2 = 4kTRA_{vo}^2\left[\frac{1}{A_{vo}^2}\int_0^\infty |A_v(f)|^2\,df\right]$$

$$E_{no}^2 = 4kTRA_{vo}^2\,\Delta f \tag{1-23}$$

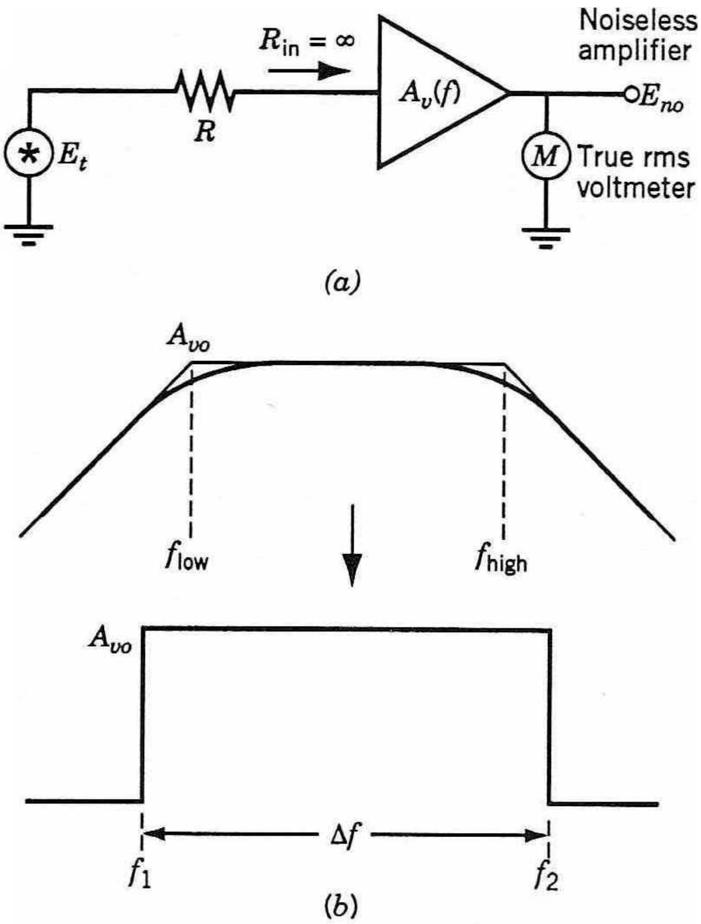


Figure 1-6 Circuit for showing narrowband and wideband noises.

Here E_{no}^2 represents the wideband mean square output noise voltage. The narrowband output noise voltage spectrum or spectral density is given by

$$E_{no}^2/\Delta f = S(f) = 4kTRA_{vo}^2 \tag{1-24}$$

1-5 THERMAL NOISE EQUIVALENT CIRCUITS

In order to perform a noise analysis of an electronic system, every element that generates thermal noise is represented by an equivalent circuit composed of a noise voltage generator in series with a noiseless resistance. Suppose, then, that we have a noisy resistance R connected between terminals a and b. For analysis, we substitute the equivalent shown in Fig. 1-7*a*, a noiseless resistance of the same ohmic value, and a series noise generator with rms value E_t equal to $(4kTR$ **Af** $)^{1/2}$. This generator is supplying the circuit with multifrequency noise; it is specified by the rms value of its total output. The * symbols representing the voltage and current generators are used for noise sources exclusively.

According to Norton's theorem, the series arrangement shown in Fig. 1-7a can be replaced by an equivalent constant-current generator in parallel with a resistance. The noise current generator **I,** will have a rms value equal to

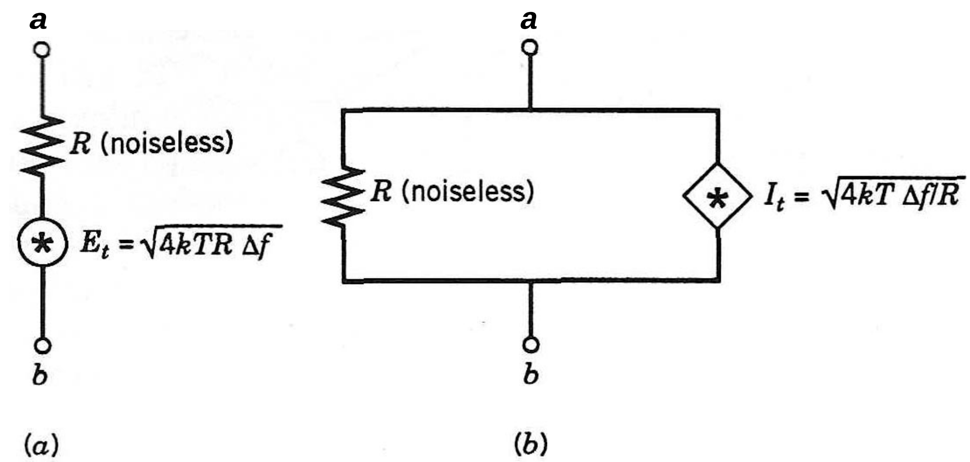


Figure 1-7 Equivalent circuits for thermal noise: (a) Thevenin equivalent circuit and (*b*) Norton equivalent circuit.

E_t/R, or in this instance

$$I_t = \sqrt{\frac{4kT\,\Delta f}{R}} = \sqrt{4kTG\,\Delta f} \tag{1-25}$$

where $G = 1/R$ is the conductance in siemens.

If a voltmeter with infinite input impedance and zero self-noise were connected between *a* and b, the thermal noise voltage could be measured. However, because most voltmeters also contribute noise, a direct reading will usually be too large.

The system of symbols that we employ in noise analysis uses the letters E and **I** to represent noise quantities. The letter V is reserved for signal voltage. Because noise generators do not have an instantaneous phase characteristic as is attributed to sine waves in the phasor method of representation, no specific polarity indication is included in the noise source symbols in Fig. 1-7. Polarity of noise sources (correlation) is discussed in Chap. 2.

1-6 ADDITION OF NOISE VOLTAGES

When two sinusoidal signal voltage sources of equal amplitude and the *same frequency and phase* are connected in series, the resultant voltage has twice the common amplitude, and combined they can deliver four times the power of one source. If, on the other hand, they differ in phase by 180°, the net voltage and power from the pair is zero. For other phase conditions they may be combined using the familiar rules of phasor algebra.

If two sinusoidal signal voltage sources of different non-harmonic frequencies with rms amplitudes V_1 and V_2 are connected in series, the resultant voltage has an rms amplitude equal to $(V_1^2 + V_2^2)^{1/2}$. The mean square value

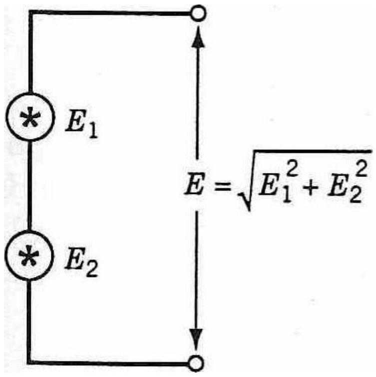


Figure 1-8 Addition of uncorrelated noise voltages.

of the resultant wave V_r^2 is the sum of the mean square values of the components ($V_r^2 = V_1^2 + V_2^2$).

Equivalent noise generators represent a very large number of component frequencies with a random distribution of amplitudes and phases. When independent noise generators are series connected, the separate sources neither help nor hinder one another. The output power is the sum of the separate output powers, and, consequently, it is valid to combine such sources so that the resultant mean square voltage is the sum of the mean square voltages of the individual generators. This statement can be extended to noise current sources in parallel.

The generators E_1 and E_2 shown in Fig. 1-8 represent uncorrelated noise sources. We form the sum of these voltages by adding their mean square values. Thus the mean square of the sum, E^2, is given by

$$E^2 = E_1^2 + E_2^2 \tag{1-26}$$

Taking the square root of the quantity such as E^2 represents the rms. It is not valid to linearly sum the rms voltages of series noise sources, they must be rms summed.

To a good engineering approximation, one can often neglect the smaller of the two noise signals when their rms values are in a 10:1 ratio. In this case, the smaller signal adds less than 1% to the overall voltage. A 3:1 ratio has only a 10% effect on the total. If two resistors are connected in parallel, the total thermal noise voltage is that of the equivalent resistance. Similarly, with two resistors in series, the total noise voltage is determined by the arithmetic sum of the resistances.

As an example, consider two simple circuits composed of resistive elements as shown in Fig. 1-9a and b. In both circuits we want to determine the output noise voltage, E_{no}, due to the thermal noise of the source resistor or resistors. For simplicity, we neglect the thermal noise of the load resistor. If we apply conventional linear circuit techniques, the 4-nV/Hz$^{1/2}$ thermal noise produced by R_S in Fig. 1-9a will be attenuated by a factor of 2 and produce an output noise voltage of

$$E_{no} = R_L E_t/(R_S + R_L) = 0.5(4\ \text{nV/Hz}^{1/2}) = 2\ \text{nV/Hz}^{1/2} \tag{1-27}$$

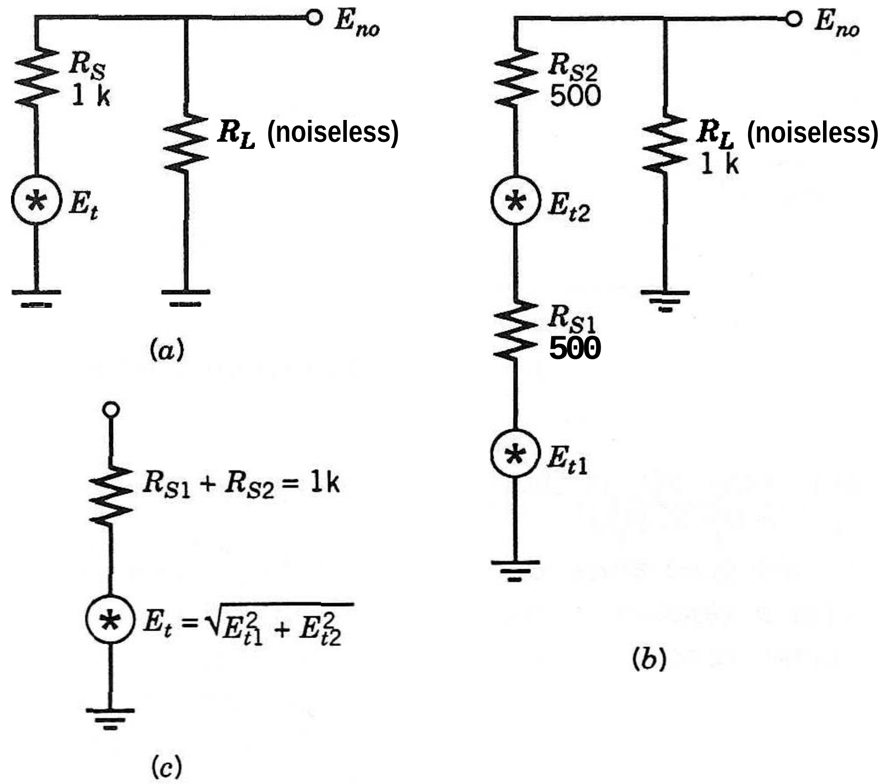


Figure 1-9 Circuits with noise voltages: (a) simple circuit, (*b*) equivalent circuit, and (*c*) correct resultant circuit.

If we apply the same linear analysis approach to the equivalent circuit of Fig. 1-9b which has two source resistors in series totaling 1 kΩ, we get an entirely different result

$$E_{no} = R_L E_{t1}/(R_{S1} + R_{S2} + R_L) + R_L E_{t2}/(R_{S1} + R_{S2} + R_L)$$

$$= 0.5(2.82\ \text{nV/Hz}^{1/2}) + 0.5(2.82\ \text{nV/Hz}^{1/2}) = 2.82\ \text{nV/Hz}^{1/2} \tag{1-28}$$

The difficulty with the second approach is that noise voltages do not combine in a linear manner and hence the principle of superposition does not apply here. The correct analysis approach for the circuit of Fig. 1-9b is

$$E_{no}^2 = R_L^2 E_{t1}^2/(R_{S1} + R_{S2} + R_L)^2 + R_L^2 E_{t2}^2/(R_{S1} + R_{S2} + R_L)^2$$

$$= (0.5)^2(2.82\ \text{nV/Hz}^{1/2})^2 + (0.5)^2(2.82\ \text{nV/Hz}^{1/2})^2$$

$$= (0.5)(2.82\ \text{nV/Hz}^{1/2})^2 = 4 \times 10^{-18}\ \text{V}^2/\text{Hz}$$

$$E_{no} = 2\ \text{nV/Hz}^{1/2} \tag{1-29}$$

This example illustrates a common problem when combining noise sources. To avoid this problem, first combine any series or parallel elements into a single equivalent element. Then calculate the noise contribution due to the equivalent element. For example, the two resistors and their noise sources in Fig. 1-9b combine to the correct equivalent network as shown in Fig. 1-9c. Just remember, if you try adding noise sources such as

$$(E_{t1} + E_{t2})^2 = E_{t1}^2 + E_{t1}^2 + 2E_{t1}E_{t2} \tag{1-30}$$

you will get an extra cross-product term which defines the correlation coefficient which is presented in the next section.

1-7 CORRELATION

When noise voltages are produced independently and there is no relationship between the instantaneous values of the voltages, they are uncorrelated. Uncorrelated voltages are treated according to the discussion of the preceding section.

Two waveforms that are of identical shape are said to be 100% correlated even if their amplitudes differ. An example of correlated signals would be two sine waves of the same frequency and phase. The instantaneous or rms values of fully correlated waveforms can be added arithmetically.

A problem arises when we have noise voltages that are partially correlated. This can happen when each contains some noise that arises from a common phenomenon, as well as some independently generated noise. In order to sum partially correlated waves, the general expression is

$$E^2 = E_1^2 + E_2^2 + 2CE_1E_2 \tag{1-31}$$

The term C is called the correlation coefficient and can have any value between -1 and $+1$, including 0. When $C = 0$, the voltages are uncorrelated, and the equation is the same as given in Fig. 1-8. When $C = 1$, the signals are totally correlated. Then rms values E_1 and E_2 can be added linearly. A -1 value for C implies subtraction of correlated signals, for the waveforms are then 180" out of phase.

Very often one can assume the correlation to be zero with little error. The maximum error will occur when the two voltages are equal and fully correlated. Summing gives 2 times their separate rms values, whereas the uncorrelated summing is 1.4 times their separate rms values. Thus the maximum error caused by the assumption of statistical independence is 30%. If the signals are partially correlated or one is much larger than the other, the error is smaller. When one signal is 10 times the other, the error is 8.6% maximum which is pretty good accuracy for noise measurements.

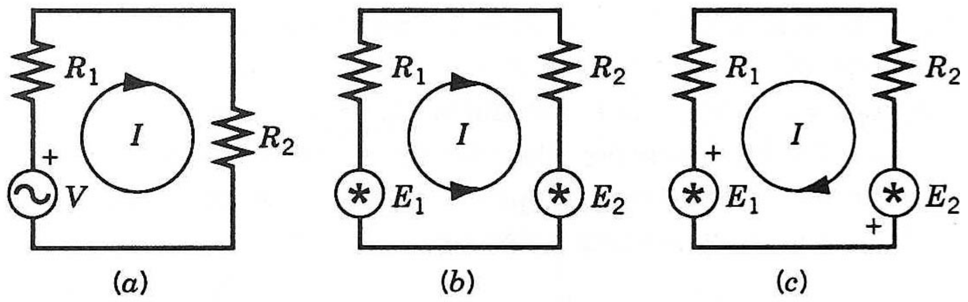


Figure 1-10 Circuits for analysis examples.

1-8 NOISE CIRCUIT ANALYSIS

An introduction to the circuit analysis of noisy networks was given in the preceding two sections. Here we expand on these discussions in order to clarify the theory and extend it to further applications.

Refer to Fig. 1-10*a*. A sinusoidal voltage source is feeding two noiseless resistances. Kirchhoff's voltage law allows us to write

$$V = IR_1 + IR_2 \tag{1-32}$$

Now suppose that we wanted to equate the mean square values of the three terms in Eq. *1-32.* Let us square each term.

$$V^2 = (IR_1)^2 + (IR_2)^2 \tag{1-33}$$

This operation is not valid! Why? Because there is *100%* correlation between IR_1 and IR_2, for they contain the same current I. Therefore, a correlation term must be present, and the correct expression is

$$V^2 = (IR_1)^2 + (IR_2)^2 + 2CIR_1IR_2 \tag{1-34}$$

Since $\boldsymbol{C}$ must equal unity here because only one current exists, the equation becomes

$$V^2 = I^2(R_1 + R_2)^2 \tag{1-35}$$

The rule for series circuit analysis is simply that *when resistances or impedances are series-connected, they should be summed first, and then, when dealing with mean square quantities, the sum should be squared.*

If V had been a noise source E, the same rule applies, for there is only one current in the circuit.

Now consider the circuit shown in Fig. 1-10*b*. Two uncorrelated noise voltage sources (or sinusoidal sources of different frequencies) are in series with two noiseless resistances. The current in this circuit must be expressed in mean square terms:

$$I^2 = \frac{E_1^2 + E_2^2}{(R_1 + R_2)^2} \tag{1-36}$$

No correlation term is present.

A convenient method for noise circuit analysis, when more than one source is present, employs superposition. The superposition principle states: *In a linear network the response for two or more sources acting simultaneously* is *the sum of the responses for each source acting alone with the other voltage sources short-circuited and the other current sources open-circuited.* Let us use superposition on the circuit of Fig. 1-10*b*. The loop currents caused by E_1 and E_2, each acting independently, are

$$I_1 = \frac{E_1}{R_1 + R_2} \qquad I_2 = \frac{E_2}{R_1 + R_2} \tag{1-37}$$

And, for *uncorrelated* quantities,

$$I^2 = I_1^2 + I_2^2 \tag{1-38}$$

Therefore,

$$I^2 = \frac{E_1^2}{(R_1 + R_2)^2} + \frac{E_2^2}{(R_1 + R_2)^2} = \frac{E_1^2 + E_2^2}{(R_1 + R_2)^2} \tag{1-39}$$

This agrees with Eq. 1-36.

In Fig. 1-10*c* sources E_1 and E_2 are correlated. Polarity symbols have been added to show that the generators are aiding. Then

$$I^2 = \frac{E_1^2 + E_2^2 + 2CE_1E_2}{(R_1 + R_2)^2} \tag{1-40}$$

where $0 < C \leq +1$ for aiding generators. If the polarity symbol on either generator were at its opposite terminal, C would take on values between 0 and -1. Note that when full correlation exists, it is valid to equate rms quantities ($E = IR_1 + IR_2$).

Suppose that we have a circuit such as shown in Fig. *1-11* in which there are several uncorrelated noise currents. We wish to determine the total current I_1 through R_1. For this example superposition is used; the contribution of E_1 to **I,** is termed I_{11}, and the contribution of E_2 to **I,** is **I,,.** It

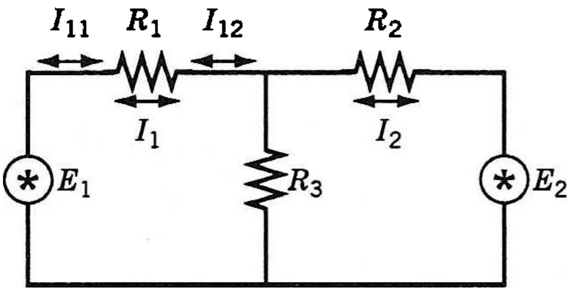


Figure 1-11 Two-loop circuit.

follows that

$$E_1^2 = I_{11}^2\left[R_1 + \frac{R_2R_3}{(R_2 + R_3)}\right]^2 \quad \text{and} \quad E_2^2 = I_2^2\left[R_2 + \frac{R_1R_3}{(R_1 + R_3)}\right]^2 \tag{1-41}$$

Next observe that

$$I_{12} = I_2R_3/(R_1 + R_3) \tag{1-42}$$

Therefore, we can write

$$I_{11}^2 = \frac{E_1^2(R_2 + R_3)^2}{(R_1R_2 + R_1R_3 + R_2R_3)^2}$$

and

$$I_{12}^2 = \frac{E_2^2R_3^2}{(R_1R_2 + R_1R_3 + R_2R_3)^2} \tag{1-43}$$

Hence

$$I_1^2 = \frac{E_1^2(R_2 + R_3)^2 + E_2^2R_3^2}{(R_1R_2 + R_1R_3 + R_2R_3)^2} \tag{1-44}$$

When finding the total current resulting from several uncorrelated noise sources, the contributions from each source must be added in such a way so that the magnitude of the total current is increased by each contribution. Therefore, neither the E_1 nor the E_2 terms in Eq. 1-44 could accept negative signs. An argument based on the heating effect of the currents, or one based on combining currents of different frequencies, can be used to justify this statement.

When performing a noise analysis of multisource networks, it is convenient to ascribe polarity symbols to uncorrelatcd sources in order that the proper addition (and no subtraction) of effects takes place.

1-9 EXCESS NOISE

We previously discussed the fundamental thermal noise in a resistor. Now it is time to point out that there can also be an additional excess noise source in a resistor or semiconductor, but *only when* a direct current is flowing [3]. Excess noise is so named because it is present in addition to the fundamental thermal noise of the resistance. Excess noise usually occurs whenever current flows in a discontinuous medium such as an imperfect semiconductor lattice. For example, in the base region of a BJT there are discontinuities or impurities that act as traps to the current flow and cause fluctuations in the base current.

As described in Chap. 12, many resistors also exhibit excess noise when a dc current is flowing. This noise contribution is greatest in composition carbon resistors and is usually not important in metal film resistors. A carbon resistor is made up of carbon granules squeezed together, and current tends to flow unevenly through the resistor. There are something like microarcs between the carbon granules. Excess noise in a resistor can be measured in terms of a noise index expressed in decibels. *The noise index is the number of microvolts of noise in the resistor per volt of dc drop across the resistor in each decade of frequency.* Thus, even though the noise is caused by current flow, it can be expressed in terms of the direct voltage drop rather than resistance or current. The noise index of some brands of resistors may be as high as 10 dB which corresponds to 3 μV/dc V/decade. This can be a significant contribution.

This excess noise exhibits a $1/f$ noise power spectrum. A $1/f$ spectrum means that the noise power varies inversely with frequency. Thus the noise voltage increases as the square root of the decreasing frequency. By decreasing the frequency by a factor of 10, the noise voltage increases by a factor of approximately 3.

Since excess noise has a $1/f$ power spectrum, most of the noise appears at low frequencies. This is why excess noise is often called low-frequency noise.

1-10 LOW-FREQUENCY NOISE

Low-frequency or $1/f$ noise has several unique properties. If it were not such a problem it would be very interesting. The spectral density of this noise increases without limit as frequency decreases. Firle and Winston [4] have measured $1/f$ noise as low as 6×10^{-5} Hz. This frequency is but a few cycles per day. When first observed in vacuum tubes, this noise was called "flicker effect," probably because of the flickering observed in the plate current. Many different names are used some of them uncomplimentary. In the literature, names like excess noise, pink noise, semiconductor noise, low-frequency noise, current noise, and contact noise will be seen. These all refer to the same thing. The term "red noise" is applied to a noise power spectrum that varies as l/f^2.

The noise ***power*** typically follows a $1/f^{\alpha}$ characteristic with α usually unity, but has been observed to take on values from *0.8* to *1.3* in various devices. The major cause of l/f noise in semiconductor devices is traceable to properties of the surface of the material. The generation and recombination of carriers in surface energy states and the density of surface states are important factors. Improved surface treatment in manufacturing has decreased l/f noise, but even the interface between silicon surfaces and grown oxide passivation are centers of noise generation.

As pointed out by Halford [5] and Keshner [6], l/f noise is quite common. Not only is it observed in vacuum tubes, transistors, diodes, and resistors, but it is also present in thermistors, carbon microphones, thin films, and light sources. The fluctuations of a membrane potential in a biological system have been reported to have flicker noise. No electronic amplifier has been found to be free of flicker noise at the lowest frequencies. Halford points out that a = 1 is the most common value, but there are other mechanisms with different values of a. For example, fluctuations of the frequency of rotation of the earth have an a of 2 and the power spectral density of galactic radiation noise has a = 2.7.

Since l/f noise power is inversely proportional to frequency, it is possible to determine the noise content in a frequency band by integration over the range of frequencies in which our interest lies. The result is

$$N_f = K_1 \int_{f_l}^{f_h} \frac{df}{f} = K_1 \ln \frac{f_h}{f_l} \tag{1-45}$$

where N_f is the noise power in watts, K_1 is a dimensional constant also in watts, and f_h and f_l are the upper and lower frequency limits of the band being considered. Now consider the noise power present in any decade of frequency such that $f_h = 10f_l$. Equation *1-45* then simplifies to

$$N_f = 2.3K_1 \tag{1-46}$$

This shows that l/f noise results in equal noise power in each decade of frequency. In other words, the noise power in the band from *10* to *100 Hi* is equal to that of the band from *0.01* to *0.1 Hi*. Since the noise in each of these intervals is uncorrelated, the mean square values must be added. Total noise power increases as the square root of the number of frequency decades.

Since noise power is proportional to the mean square value of the corresponding noise voltage, then the spectral density of the noise voltage for l/f noise is

$$S_f(f) = E_f^2/f \qquad \text{in V}^2/\text{Hz} \tag{1-47}$$

Suppose we know that there is 1 μV of $1/f$ noise in a decade of frequency. This would cause us to write

$$(1\ \mu\text{V})^2 = \int_{f_l}^{10f_l} S_f(f)\,df = \int_{f_l}^{10f_l} \frac{E_f^2\,df}{f} = 2.3E_f^2 \qquad (1\text{-}48)$$

Consequently, for this example the spectral density of the $1/f$ noise reduces to

$$S_f(f) = \frac{(1\ \mu\text{V})^2}{2.3f} \qquad (1\text{-}49)$$

Because $1/f$ noise power continues to increase as the frequency is decreased, we might ask the question, "Why is the noise not infinite at dc?" Although the noise voltage in a 1-Hz band may theoretically be infinite at dc or zero frequency, there are practical considerations that keep the total noise manageable for most applications. The noise power per decade of bandwidth is constant, but a decade such as that from 0.1 to 1 Hz is narrower than the decade from 1 to 10 Hz. But, when considering the $1/f$ noise in a dc amplifier, there is a lower limit to the frequency response set by the length of time the amplifier has been turned on. This low-frequency cutoff attenuates frequency components with periods longer than the "on" time of the equipment.

Example 1-2 A numerical example may be of assistance. Consider a dc amplifier with upper cutoff frequency of 1000 Hz. It has been on for 1 day. Since 1 cycle/day corresponds to about 10^{-5} Hz, its bandwidth can be stated as 8 decades. If it is on for 100 days, we add 2 more decades or $\sqrt{10/8} = \sqrt{1.25} = 1.18$ times its 1-day noise. The noise per hertz approaches infinity, but the total noise does not.

A fact to remember concerning a $1/f$ noise-limited dc amplifier is that measurement accuracy cannot be improved by increasing the length of the measuring time. In contrast, when measuring white noise, the accuracy increases as the square root of the measuring time.

1-11 SHOT NOISE

In transistors, diodes, and vacuum tubes, there is a noise current mechanism called shot noise. Current flowing in these devices is not smooth and continuous, but rather it is the sum of pulses of current caused by the flow of carriers, each carrying one electronic charge. Consider the case of a simple forward-biased silicon diode with electrons and holes crossing the potential barrier. Each electron and hole carries a charge q, and when they arrive at the anode and cathode, respectively, an impulse of current results. This pulsing flow is a granule effect, and the variations are referred to as shot noise. The rms value of the shot noise current is given by

$$\boxed{I_{sh} = \sqrt{2qI_{DC}\,\Delta f}} \tag{1-50}$$

where q is the electronic charge (1.602 X 10^{-19} Coulombs), $I_{„}$ is the direct current in amperes, and Af is the noise bandwidth in hertz. We note that the shot noise current is proportional to the square root of the noise bandwidth. This means that it is white noise containing constant noise power per hertz of bandwidth.

One example of shot noise is a heavy rain on a tin roof. The drops arrive with about equal energy, the inches per hour rate corresponds to the current $I_{„}$ and the area of the roof relates to the noise bandwidth Af.

Shot noise is associated with current flow across a potential barrier. Such a barrier exists in every pn junction in semiconductor devices and in the charge-free space in a vacuum tube. No barrier is present in a simple conductor; therefore, no shot noise is present. The most important barrier is the emitter–base junction in a bipolar transistor and the gate–source junction in a junction field effect transistor (JFET). The V–I behavior of the base–emitter junction is described by the familiar diode equation

$$I_E = I_S\left(e^{qV_{BE}/kT} - 1\right) \tag{1-51}$$

where I_E is the emitter current in amperes, I_S is the reverse saturation current in amperes, and V_{BE} is the voltage between the base and emitter. Suppose that we consider separately the two currents that make up I_E in Eq. 1-51:

$$I_E = I_1 + I_2 \tag{1-52}$$

where $I, = -I_S$ and $I_2 = I_S \exp(V_{BE}/kT)$.

Current I_1 is caused by thermally generated minority carriers, and current I_2 represents the diffusion of majority carriers across the junction. Each of these currents has full shot noise, and even though the direct currents they represent flow oppositely, their mean square noise values are added.

Under reverse biasing, $I_2 \cong 0$ and the shot noise current of $I,$ dominates. On the other hand, when the diode is strongly forward-biased, the shot noise current of I_2 dominates. For zero bias, there is no external direct current, and $I,$ and I_2 are equal and opposite. The mean square value of shot noise is twice the reverse-bias noise current:

$$I_{sh}^2 = 4qI_S\,\Delta f \tag{1-53}$$

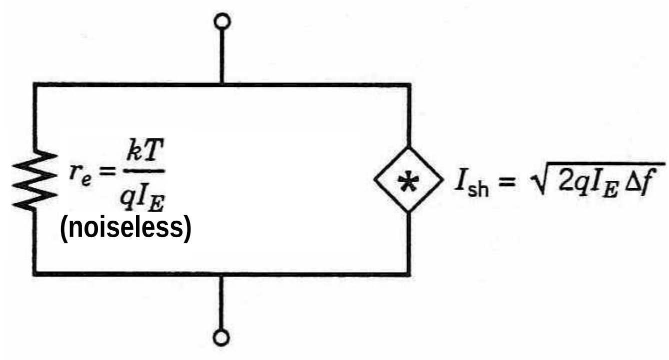


Figure 1-12 Shot noise equivalent circuit for forward-biased *pn* junction.

The equivalent circuit representation for a shot noise source is a current generator as previously noted in **Eq.** 1-50. For the case of the forward-biased pn junction, a noiseless resistance parallels this current generator. By differentiating Eq. 1-51 with respect to V_{BE}, we obtain a conductance. The reciprocal of that conductance is referred to as the Shockley emitter resistance r_e and is given by

$$r_e = kT/qI_E \tag{1-54}$$

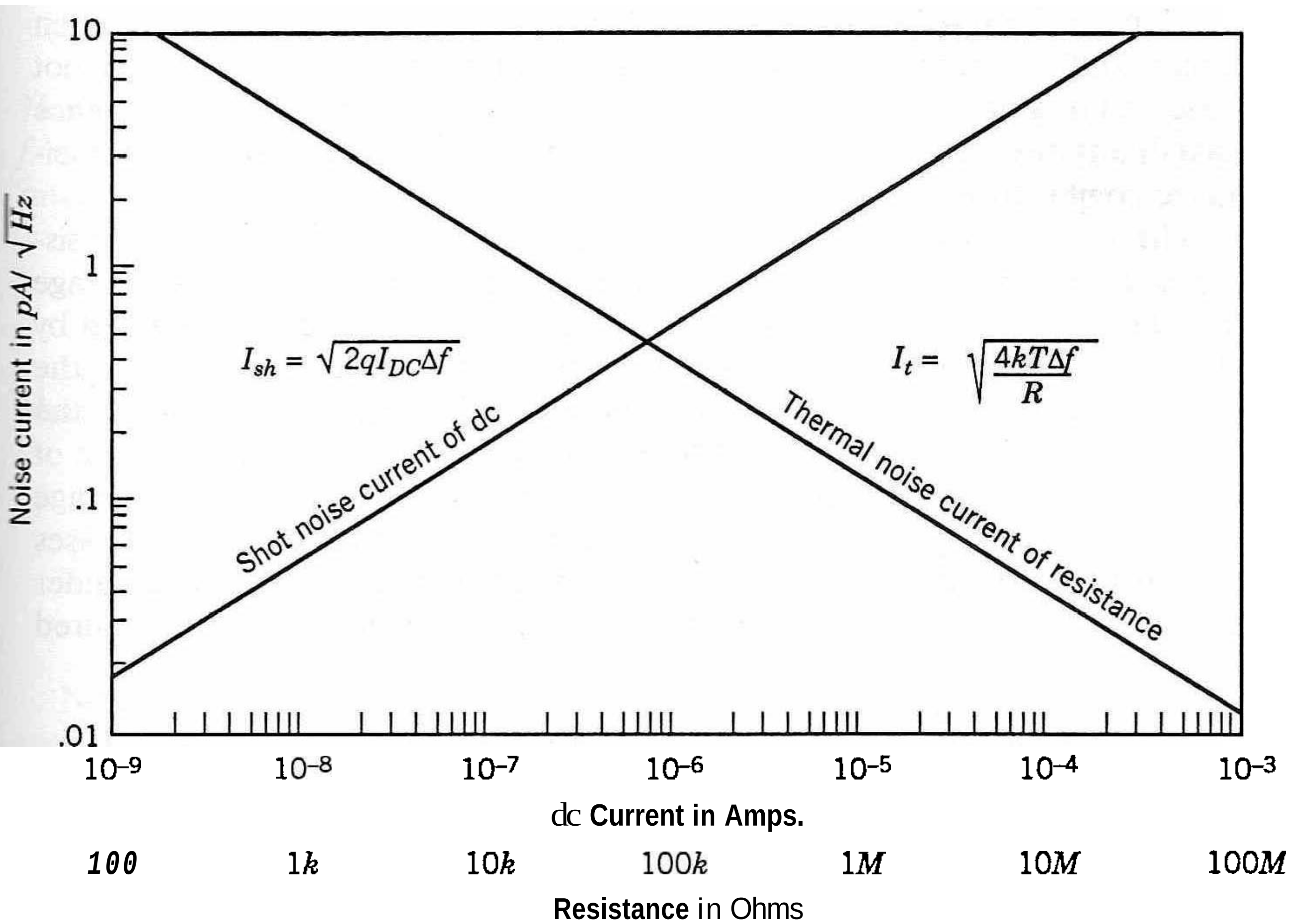


Figure 1-13 Plot of noise current of shot noise and thermal noise.

At room temperature $r_e = 0.025/I_E$. The element r_e is not a thermally noisy component, for it is the dynamic effect of the junction, and not a bulk or material characteristic.

An equivalent circuit representing shot noise at a forward-biased pn junction is shown in Fig. 1-12. Mathematically, the mean square value of shot noise is equal to the thermal noise for an unbiased junction, and equal to one-half of the resistive thermal noise voltage at a forward-biased junction. The noise voltage E_{sh} of a forward-biased junction is the product of the shot noise current I_{sh} and the diode resistance r_e:

$$E_{sh} = \frac{0.025}{I_E}\sqrt{2qI_E\,\Delta f} = 1.42 \times 10^{-11}\sqrt{\frac{\Delta f}{I_E}} \tag{1-55}$$

which shows that the shot noise voltage E_{sh} will decrease by the square root of the diode current. Further discussion is available in the literature [7, 8].

The shot noise current I_{sh} of a diode and the thermal noise current I_t of a resistor are compared in the plot of Fig. 1-13.

1-12 CAPACITANCE SHUNTING OF THERMAL NOISE: kT/C NOISE

The thermal noise expression $E_t = (4kTR\,\Delta f)^{1/2}$ predicts that an open circuit (infinite resistance) generates an infinite noise voltage. This is not observed in a practical situation since there is always some shunt capacitance that limits the voltage. Consider the actual noisy resistance–shunt capacitance combination as shown in Fig. 1-14.

The thermal noise voltage E_t increases as the square root of the resistance. Low-frequency noise from E_t directly affects the output noise voltage E_{no}. Higher-frequency components from E_t are more effectively shunted by the capacitor C. Increasing R increases the noise voltage, but decreases the cutoff frequency and consequently the noise bandwidth. A plot of the resulting noise voltage versus frequency is shown in Fig. 1-15 for one value of capacitance and resistance values of R, $4R$, and 9R. The noise voltage increases as the square root of the resistance and the bandwidth decreases proportional to the resistance but the integrated mean square noise under each curve is equal. The total output noise voltage which would be measured

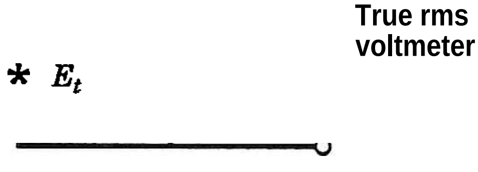


Figure 1-14 Thermal noise of a resistor shunted by a capacitance.

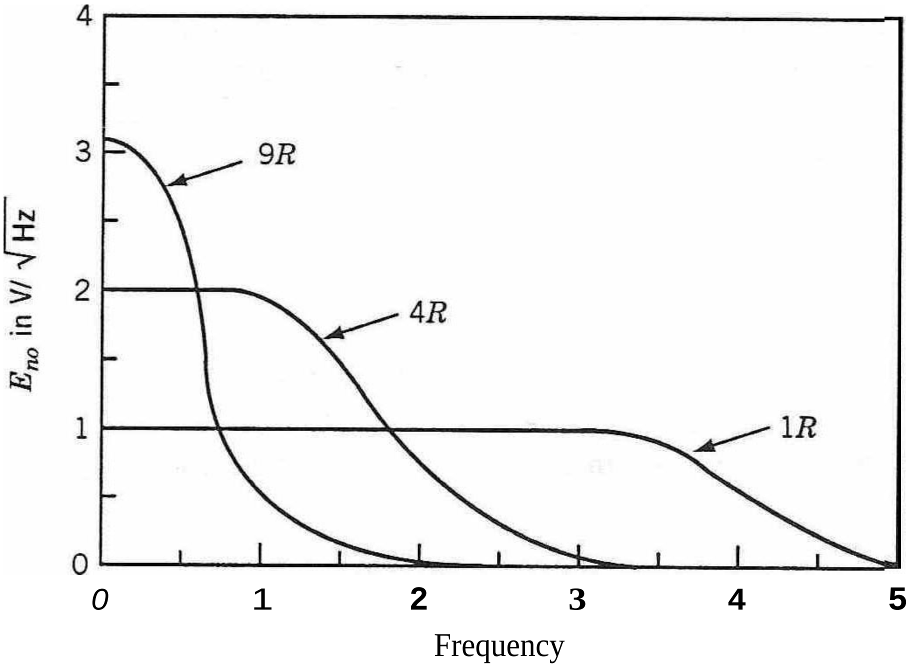


Figure 1-15 Noise spectral density for a resistance shunted by a capacitance.

by a true rms voltmeter having an infinite bandwidth is found from

$$E_{no}^2 = \int_0^\infty E_t^2 \left| \frac{1/j\omega C}{R + 1/j\omega C} \right|^2 df = \int_0^\infty \frac{E_t^2 \, df}{1 + (\omega RC)^2} \tag{1-56}$$

Now, making a change of variable so we can perform the integration, we let $f = f_2 \tan\theta$, $f_2 = 1/2\pi RC$, $df = f_2 \sec^2\theta \, d\theta$, and change the upper limit of integration to $\pi/2$. The output noise voltage now becomes

$$E_{no}^2 = \int_0^{\pi/2} \frac{E_t^2 f_2 \sec^2\theta \, d\theta}{1 + \tan^2\theta} = \int_0^{\pi/2} E_t^2 f_2 \, d\theta \tag{1-57}$$

Next, substituting for the thermal noise voltage E,, we have

$$E_{no}^2 = \int_0^{\pi/2} 4kTRf_2 \, d\theta = 2\pi kTRf_2$$

$$E_{no}^2 = kT/C \qquad \text{in mean squared volts} \tag{1-58}$$

Note that the output rms noise voltage is independent of the source resistance and only depends on the temperature and capacitance. The majority of the energy is contained in the very low frequency region because the shunt capacitance attenuates high frequencies. This noise limit is often referred to simply as kT/C noise. It becomes important in applications where sample and hold circuits are utilized such as with analog-to-digital converters and switched capacitor circuits.

Example 1-3 As a numerical example if $C = 1$ pF of stray shunting capacitance, $E_{no}^2 = 4 \times 10^{-9}$ V^2, or $E_{no} \approx 64$ μV. This is a significant noise voltage level. To minimize noise, reduce the system bandwidth to that which is absolutely necessary for properly processing the desired signals.

SUMMARY

a. Noise is any unwanted disturbance that obscures or interferes with a signal.

b. Thermal noise is present in every electrical conductor, with rms value:

$$E_t = \sqrt{4kTR\,\Delta f}$$

When evaluated this yields 4 nV for 1000 Ω and $Af = 1$ Hz.

c. The noise bandwidth Af is the area under the $|A_v(f)|^2$ curve divided by A_{vo}^2, the reference or maximum value of gain squared.

d. For circuit analysis a noisy resistance can be replaced by a noise voltage generator in series with a noiseless resistance, or a noise current generator in parallel with a noiseless resistance.

e. Noise quantities can be added according to

$$E^2 = E_1^2 + E_2^2 + 2CE_1E_2$$

where C is the correlation coefficient, $-1 \;\mathrm{I}\; C \;\mathrm{I}\; +1$. Usually $C = 0$.

f. Excess noise is generated in most components when direct current is present.

g. l/f noise is especially troublesome at low audio frequencies.

h. Shot noise is present when direct current flows across a potential barrier:

$$I_{sh} = \sqrt{2qI_{DC}\,\Delta f}$$

i. The total thermal noise energy in a resistance is finite and is limited by the effective capacitance across its terminals and the absolute temperature. In the limiting case $E_{no}^2 = kT/C$.

PROBLEMS

1-1. Determine the rms thermal noise voltage of resistances of 1 kΩ, 50 kΩ, and 1 MΩ for each of the following noise bandwidths: 50 kHz, 1 MHz, and 20 MHz. Consider $T = 290$ K.

1-2. Calculate the rms thermal noise voltage of a 100-mH inductance in a 1-Hz noise bandwidth. Consider that the inductor has an impedance of 8 kΩ and that the frequency band is centered around 10 kHz.

1-3. Determine the noise bandwidth of a circuit with $|A_v|^2$ frequency response described as follows:

$0 \le f \le 1$ kHz	$\|A_v\|^2 = f$
1 kHz $\le f \le 20$ kHz	$\|A_v\|^2 = 1000$
20 kHz $\le f \le 1000$ kHz	$\|A_v\|^2 = 1000 - 0.0125(f - 20{,}000)$
100 kHz $< f$	$\|A_v\|^2 = 0$

1-4. The frequency response of the magnitude of the voltage gain for a certain amplifier is

$$A_v(f) = \begin{cases} 3\sin(2\pi f/400) & \text{for } 0 \le f \le 200 \text{ Hz} \\ 0 & \text{elsewhere} \end{cases}$$

Determine the noise bandwidth for this amplifier.

1-5. Calculate E_t for a noisy resistance of 500 **kΩ.** Transform this into the noise current generator form and determine **I,.** Let T = 290 K and Af = 10^5 HZ.

1-6. The resistor in the previous problem is connected in parallel with another noisy resistor of 250 **kΩ.** Determine the mean square and rms values of the noise voltage present at the terminals of the pair.

1-7. Find the resistance of a *pn* junction that exhibits 200 **nV** rms of shot noise. Assume Af = **1** MHz and **I,** = 10 **mA.** Compare your answer with the value of r_e predicted by Eq. 1-54. What conclusions can you reach?

1-8. Find the noise bandwidth for a cascade of three identical low-pass **filter** stages which are buffered by ideal amplifiers. Each single stage has a –3-dB cutoff frequency of f_2.

1-9. Determine the noise bandwidth Af for the filter whose frequency characteristics are shown in Fig. **P1-9.**

(a) First find Af for the asymptotic response.

(b) Then find Af for the exact response. Compare the two values of Af.

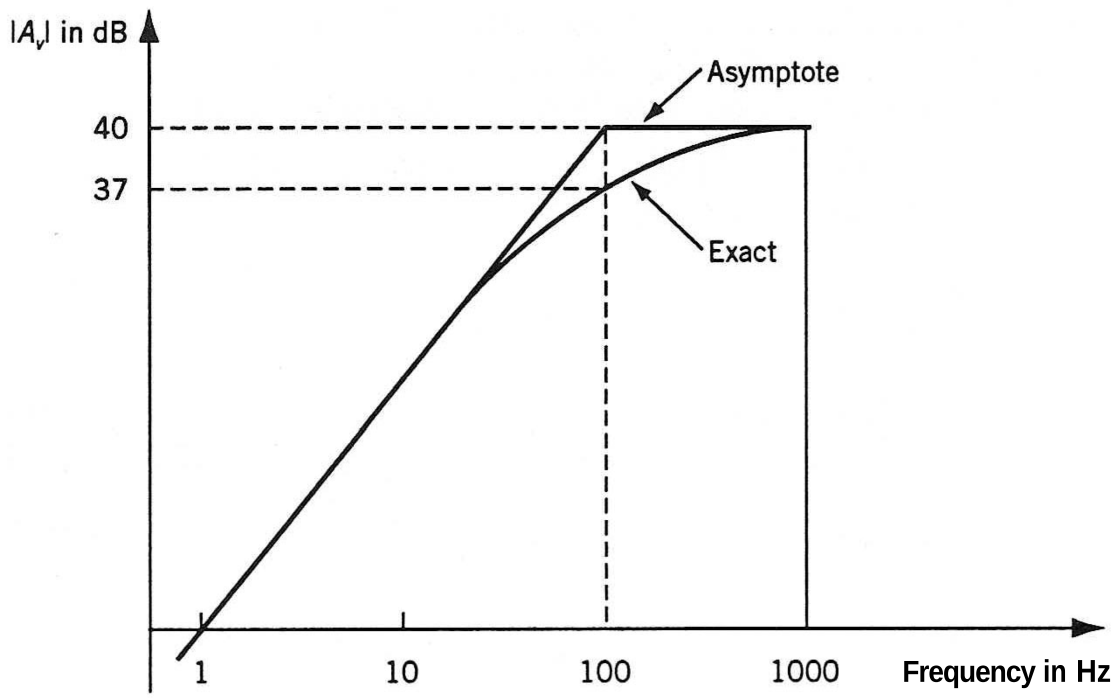


Figure P1-9

1-10. **A** certain noise source is known to have $1/f$ spectral density. The noise voltage in one decade of bandwidth is measured to be 1 μV rms. How many decades would be involved to produce a total noise voltage of 3 μV?

1-11. Determine the noise bandwidth Δf for the circuit shown in Fig. P1-11. The 20 kΩ represents the input resistance of the amplifier which has a voltage gain of 100.

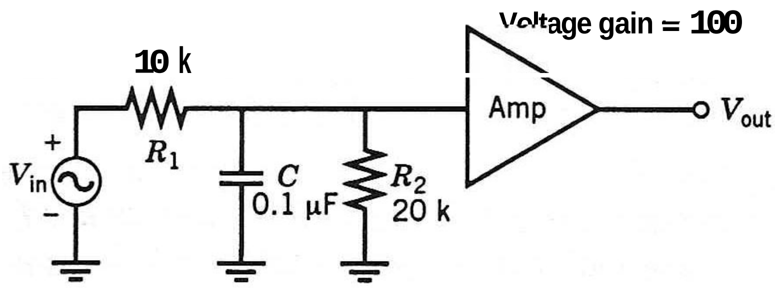


Figure P1-11

1-12. Determine the noise bandwidth Δf for the circuit shown in Fig. P1-12.

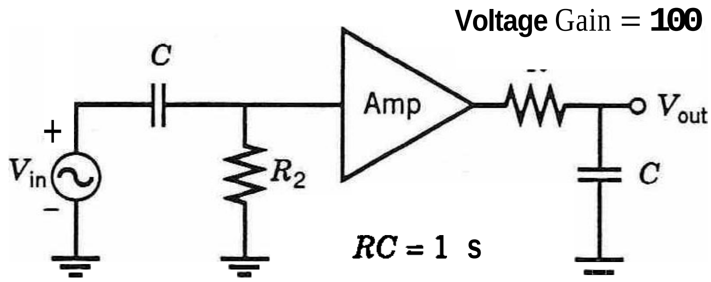


Figure P1-12

1-13. The transfer function for a second-order bandpass active filter is given by

$$T(s) = \frac{GBs}{s^2 + Bs + \omega_o^2}$$

where G is the gain at the radian center frequency, ω_o, and B is the −3-dB radian bandwidth. Prove that the noise bandwidth is given by $\Delta f = B/4 = (\text{BW})\pi/2$, where BW is the −3-dB bandwidth in hertz.

1-14. The transfer function of an active bandpass amplifier is known to be

$$T(s) = \frac{-10sR_2C_1}{(1 + sR_1C_1)(1 + sR_2C_2)}$$

where $R_1 = 15.9$ kΩ, $R_2 = 31.8$ kΩ, $C_1 = 0.1$ μF, and $C_2 = 0.01$ μF. Find the equivalent noise bandwidth, Δf, in units of hertz for this amplifier.

1-15. It is desired to replace the diode in Fig. P1-15*a* with the resistor R_x as in Fig. P1-15*b*.

(a) Determine the value of R_x which will produce the same amount of noise voltage as the diode produces. Assume $\Delta f = 1$ Hz.

(b) Now suppose V_{in} is a 1-mV peak amplitude sinusoidal signal at a frequency of 1000 Hz and it is applied to both circuits. Determine the output signal voltages, V_{o1} and V_{o2}, for both circuits which a true rms voltmeter would measure.

(c) Which circuit gives the better signal-to-noise ratio at the output? Explain your answer. Consider only the effects of substituting the resistor R_x for the diode.

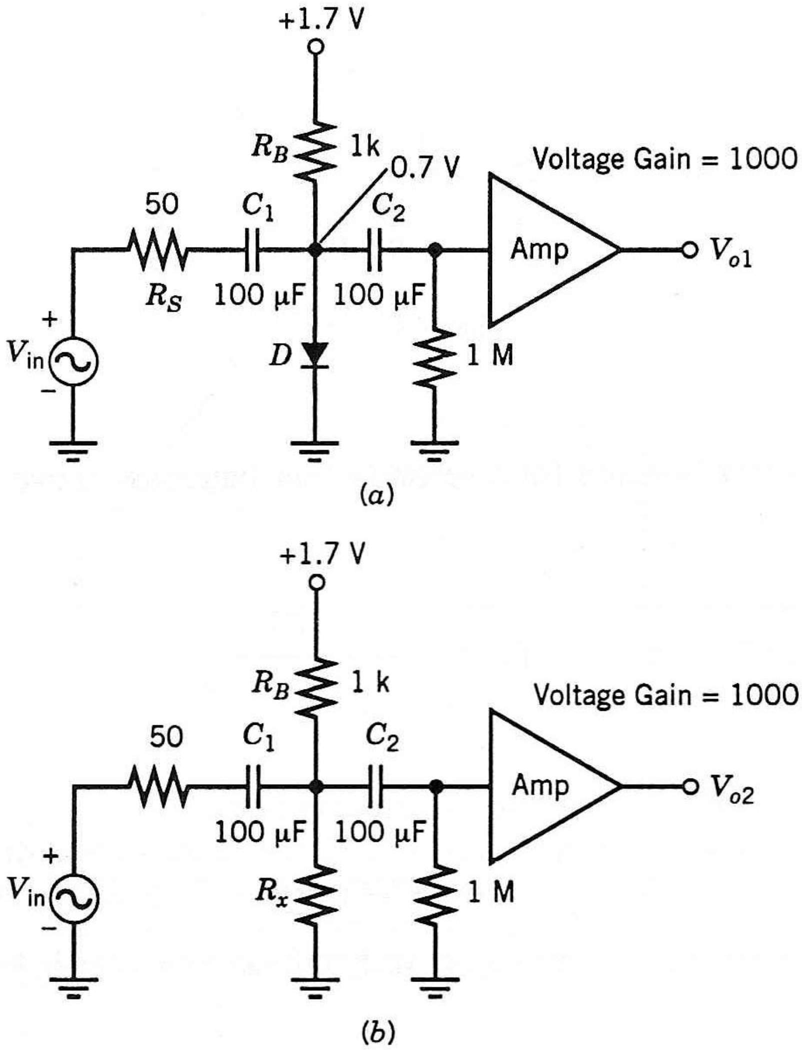


Figure P1-15

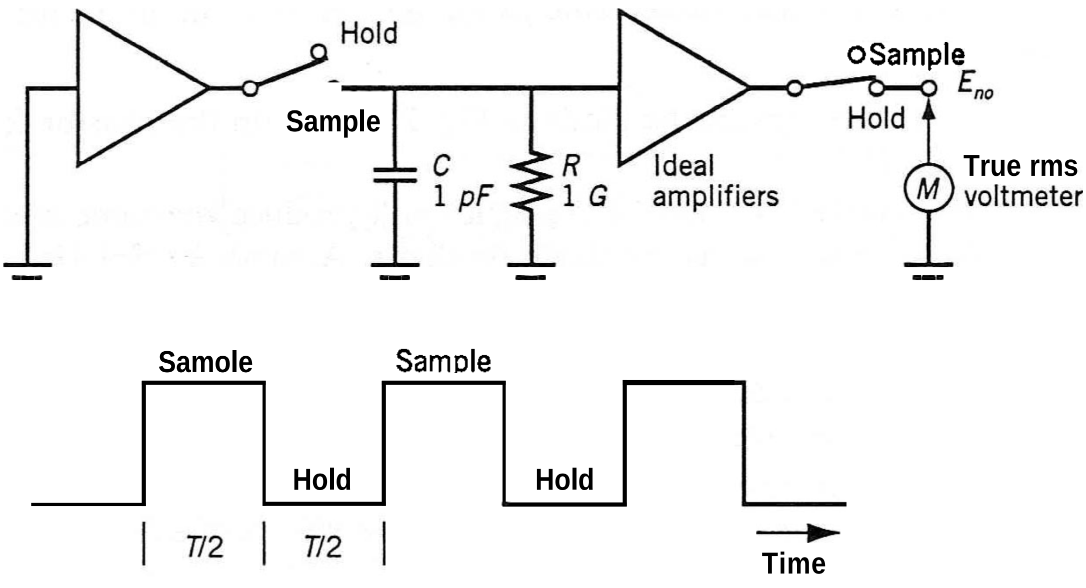


Figure P1-16

1-16. The sample-hold circuit shown in Fig. P1-16 is clocked by a square wave. Determine the output noise voltage, E_{no}, which would be recorded by a true rms voltmeter having infinite bandwidth if (a) T = 1 μs, (b) T = 1 ms, and (c) T = 1 s. For all practical purposes, the 1-GΩ resistor can be considered to be an open circuit.

1-17. a The statistically expected number of maxima per second in white noise with upper and lower frequency limits f_2 and f_1 is [9]:

$$\left[\frac{3(f_2^5 - f_1^5)}{5(f_2^3 - f_1^3)}\right]^{1/2}$$

Find the maxima for $f_1 = 0$ and $f_2 = 10^6$ MHz.

(b) The expected total number of zero crossings per second is given by

$$\left[\frac{4(f_2^2 + f_1 f_2 + f_1^2)}{3}\right]^{1/2}$$

If $f_1 = 0$ and $f_2 = 10$ MHz, evaluate the number of zero crossings. Show that for narrowband noise the assumption that $f_1 = f_2$ yields

$$(f_1 + f_2)$$

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