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# NOTE ON CENSUS-TAKING IN MONTE-CARLO CALCULATIONS

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ABSTRACT

An analysis is given of the proposal, made by investigators at the Argonne Laboratory, that in some Monte Carlo problems census taking should be made on the basis of distance travelled rather then time elapsed. The analysis given is restricted to critical systems, and it is shown how to interpret the resulting neutron distribution in this case.



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We consider a "Monte Carlo" type treatment of a neutron chain reaction, according to which the detailed history of a typical reaction in the given system is produced, whereby the position $\vec{x}$, velocity $\vec{v}$ and genealogy of every neutron is known at every time t. Let $n(\vec{x}, \vec{v}, t)$ be the density in phase-space $\vec{x}, \vec{v}$ of these neutrons at time t. For a stationary population n is independent of t except for statistical fluctuations. In practice n is found by determining the position and velocity of every neutron present at some time t*, called a census time, and observing the distribution of the position and velocity values thus found. This procedure is called taking a census.

This report discusses alternative methods of census taking. They yield distributions simply related to $n(\vec{x}, \vec{v}, t)$ in the case of a stationary population.

While the procedure described above is suited to discussion of fast multiplying systems in which the neutrons essentially involved do not differ greatly in velocity, it is impractical for systems where neutrons of vastly different velocities are of importance. This is the case, for example, in a reactor near critical condition when both fast and thermal neutrons contribute appreciably to the reactivity. A ratio of about 10,000 in the speeds of the two kinds of neutrons would force one in this case to follow up the life history of a fast neutron for an enormous number of generations before any of the slow neutrons has moved a single free path.

In such cases one must change the census procedure. One might, for example, divide the study of a neutron into intervals traversed by the neutron or by its ancestors. This "census distance" procedure is applicable, of course, only to critical systems or to systems that are conventionally made critical by adding a "time absorption." As will be shown, a distribution of neutrons at a given value of the census distance corresponds to the flux distribution instead of the density distribution that one would obtain from the "census time" prescription.

Given that a neutron is present at point $\vec{x}, \vec{v}$ of phase space at time $t > 0$, we can trace a unique trajectory backward to time 0 by following backward the path of this neutron and of each of its direct ancestors in turn until we reach $t = 0$. In particular, the speed $v$ is a unique function of $t$ on this trajectory.

Let $g(v)$ be a given function of $v$ and define

$$\tau = \int_0^t g(v(t'))\, dt' \qquad (1)$$

where $v(t)$ is the speed as function of time on the trajectory defined above. Each neutron has associated with it at each instant a certain value of $\tau$. $\tau$ is called a census parameter. If $g(v) = 1$, it is the elapsed time; if $g(v) = v$, it is the distance traversed.

Now consider all points, in a space $(\vec{x}, \vec{v}, t)$, corresponding to a particular value of $\tau$. These points have various different times $t$ as well as different positions in phase space $(\vec{x}, \vec{v})$, but we focus our attention on their distribution in phase space, ignoring the time differences; and we call $N(\vec{x}, \vec{v}, \tau)$ the density of these points in phase space.

To establish the connection between the distributions $n$ and $N$ for a stationary population, we investigate the change in $N$ caused by a small change in $\tau$, by methods similar to those used in deriving the Boltzmann equation, and in this way obtain an equation for $\frac{\partial N}{\partial \tau}$, which will eventually be equated to zero.

In the neighborhood of a given point of phase space the neutrons move along trajectories $\vec{x} = \vec{v}t + \text{const.}$, $\vec{v} = \text{const.}$, between collisions, and there is a change $\delta N$ in the density, in time $\delta t$, due just to the motions, apart from collisions, amounting to

$$\delta N = - \left[(\vec{v} \cdot \nabla) N\right] \delta t; \qquad (2)$$

but, according to (1), $\delta t = \delta \tau / g(v)$ where $\delta \tau$ is the increment of $\tau$ under consideration. Therefore the motions contribute to $\frac{\partial N}{\partial \tau}$ an amount $-(\vec{v} \cdot \nabla) \frac{N}{g}$.

Other contributions to $\frac{\partial N}{\partial \tau}$ can be similarly obtained. In particular, if $\sigma$ is a cross section for any process, the number of such processes per unit volume of phase space and having values of $\tau$ lying in the interval $\delta \tau$ is

$$v \sigma N \delta t = v \sigma N \frac{\delta \tau}{g(v)} \qquad (3)$$

For a system in which fission, elastic scattering, inelastic scattering and absorption take place, the complete equations is, in customary notation:

$$\frac{\partial N}{\partial \tau} = -(\vec{v} \cdot \nabla)\frac{N}{g} - v\sigma_t \frac{N}{g} + \frac{\nu \Phi(v)}{4\pi} \int_{\substack{\text{all velocity} \\ \text{space}}} v'\sigma_f(v') \frac{N(\vec{x}, \vec{v}', \tau)}{g(v')} d\vec{v}' +$$

$$+ \frac{1}{4\pi} \int_{\substack{\text{all velocity} \\ \text{space}}} v'\sigma_i(v', v) \frac{N(\vec{x}, \vec{v}', \tau)}{g(v')} d\vec{v}' +$$

$$+ \int_{\substack{\text{all directions of} \\ \vec{v}', \text{ with } v' = v}} v\,\sigma_s(v, \alpha)\, \frac{N(\vec{x}, \vec{v}', \tau)}{g(v)}\, d\,\omega(\vec{v}') \qquad (4)$$

where $\alpha$ is the angle between $\vec{v}'$ and $\vec{v}$.

We now suppose that after the calculation has proceeded sufficiently, the distribution represented by $N$ becomes independent of $\tau$, except for statistical fluctuations. We therefore replace the left member of (4) by zero, and if we furthermore replace $\frac{N(\vec{x},\vec{v})}{g(v)}$ by $n(\vec{x},\vec{v})$, equation (4) reduces exactly to the Boltzmann equation for a neutron population $n$ independent of the time $t$, and we are therefore justified in identifying $n$ with $N/g$, because, for systems of the sort considered, there is only one stationary solution of the Boltzmann equation in the absence of sources and sinks. If $g(v)$ is equal to $v$, so that $\tau$ is the census distance, the distribution $N$ represents the distribution of neutron flux, $nv$, in the system.

The application to Monte Carlo calculations is this: one maintains a current record of $\tau$ rather than $t$, for each neutron, and discriminates, at each collision, to see whether $\tau$ has exceeded a preassigned value $\tau^*$, and if so, one calculates where the neutron was when $\tau = \tau^*$ and prints a census card.

As an application we consider the proposal, made sometime ago in the Theoretical Physics Division of the Argonne Laboratory, that for chain reacting systems in which both fast and slow neutrons are important (e.g. both 1 Mev and 1 ev neutrons) one should use a census distance $\tau = \int v\, dt$ rather than a census time. The advantage of this can be seen as follows. The mean free path is of the same order of magnitude for both fast and slow neutrons, so that if one used a census time long enough to allow a

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1 ev neutron to make several collisions, a fast one would make several thousand in each census interval, and the calculation would be prohibitively long. This difficulty is largely avoided by use of a census distance. An even stronger dependence of $g(v)$ on $v$ (e.g. perhaps $g = v^{3/2}$ ) may be indicated in some cases, such as that of a system containing large volumes of graphite or other moderator, in which a neutron may make many collisions (of the order of 100) before emerging as a slow neutron.

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