A fractal life

Few people would recognise **Benoit Mandelbrot** in the street, but the intricate pattern of blobs, swirls and spikes that bears his name – the Mandelbrot set – is an icon of science. It has come to symbolise the geometry of fractals, patterns whose shape stays the same whatever scale you view them on. His life has followed a path as jagged as any fractal. Next week he turns 80. He tells **Valerie Jamieson** that he still has plenty of work to do

What is it like seeing the Mandelbrot set emblazoned on T-shirts and posters?
I'm delighted. I always felt that science as the preserve of people from Oxbridge or Ivy League universities – and not for the common mortal – was a very bad idea.

Even though most people view it as a beautiful image and ignore the underlying mathematics?
That's right. Yet there is nothing more to this than a simple iterative formula. It is so simple that most children can program their home computers to produce the Mandelbrot set.

How did you feel when you discovered it?
Its astounding complication was completely out of proportion with what I was expecting. Here is the curious thing: the first night I saw the set, it was just wild. The second night, I became used to it. After a few nights, I became familiar with it. It was as if somehow I had seen it before. Of course I hadn't. No one had seen it. No one had described it. The fact that a certain aspect of its mathematical nature remains mysterious, despite hundreds of brilliant people working on it, is the icing on the cake to me.

What's the mystery?
It relates to a rather subtle mathematical property. In simple terms, there are two ways to define the Mandelbrot set. It is rather like proving that 3+1 and 2+2 give the same result. I have always thought that the two definitions were equivalent. But one is easy to study whereas the other is extremely difficult. So far, the proof has defeated many people. The fact that my conjecture is so simple to state, yet baffles everybody, makes it attractive to mathematicians. The conjecture is the mathematical face of the Mandelbrot set, and the T-shirts are the popular face.

Fractals seem to appear all over nature and in economics. Even the internet is fractal. What does that say about the underlying nature of these phenomena?
Well, it depends on the field. Circles and straight lines also appear everywhere. Does this mean that all those phenomena have something in common? Of course not. The roughly circular trajectory of a planet around the sun is due to gravitational interactions. Berries are round because a sphere has a smaller skin. The beauty of geometry is that it is a language of extraordinary subtlety that serves many purposes.

So fractals don't point to a single rule underlying reality?
There is no single rule that governs the use of geometry. I don't think that one exists.

Your uncle Szolem Mandelbrojt was also a mathematician. How has he influenced your life and work?
In every way imaginable. I was 13 when my uncle became professor at Collège de France in Paris. I learned early on that mathematics is an honourable profession from which you can make a living. My uncle was a pure mathematician on weekdays and a painter and fanatic museum visitor on Sundays. He had a gifted eye. But he felt that beauty and mathematics were completely separate. These two gifts probably existed in the family and I put them together,

Benoit Mandelbrot was born in Warsaw in 1924 to a Jewish family. In 1936 he moved to Paris, where his mathematician uncle had a strong influence on his education. He spent much of the war hiding in the French countryside. Later he discovered fractal geometry. He has shown how many complex phenomena such as coastlines, galaxy clusters and share prices can be described as fractals. He is Sterling Professor of Mathematical Sciences at Yale University, and an emeritus fellow at IBM's Watson Research Center in Yorktown Heights, New York

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which made a big difference to my work. I could relate mathematical formulae to pictures.

Is that where the Mandelbrot set came from?
The Mandelbrot set is the modern development of a theory developed independently in 1918 by Gaston Julia and Pierre Fatou. Julia wrote an enormous book – several hundred pages long – and was very hostile to his rival Fatou. That killed the subject for 60 years because nobody had a clue how to go beyond them.
My uncle didn't know either, but he said it was the most beautiful problem imaginable and that it was a shame to neglect it. He insisted that it was important to learn Julia's work and he pushed me hard to understand how equations behave when you iterate them rather than solve them.
At first, I couldn't find anything to say. But later, I decided a computer could take over where Julia had stopped 60 years previously.

You have recently started writing your memoirs. What has that been like?
It has been a strange exercise.

How so?
To realise what one remembers and what one doesn't remember. My life seemed to be a series of events and accidents. Yet when I look back I see a pattern. For a long time that pattern was imposed by catastrophes, namely the fall of Poland and the occupation of France during the second world war.

"The beauty of geometry is that it is a language of extraordinary subtlety"

Those events dictated everything. Different people had different reactions to this kind of youth. A great many people were left with an enormous desire for calm and regularity because life had been rough: they were tired of big events. Somehow I reacted differently. I do not particularly like danger. I didn't like the close encounters with danger. But I found that I could bear them. And they brought other advantages. Being raised under such hair-raising conditions can have a strong effect on someone's personality.

How did the war affect your thinking about maths?
Until mid-1942, my education was only a little disrupted. After that, until early 1944, it was very disrupted and life was very dangerous. The winter of 1944 was awful and at the same time one of the high points of my life. During that time, it became clear that I had a peculiar gift of being able to almost instantaneously transform into geometry everything that I could handle in my head.

Has this been your prime motivation?
What motivates me now are ideas I developed 10, 20 or 30 years ago, and the feeling that these ideas may be lost if I don't push them a little bit further. Does that matter? Most ideas in science that are abandoned are picked up by someone else later, but not all. I have reflected on this issue a great deal. Perhaps I would like to finish my ideas for aesthetic reasons – a feeling of closure.

Are there any neglected parts of maths that you think today's mathematicians should revisit?
There are plenty. I think that mathematics is one of the best fields in that respect. Something that is 150 years old in maths is old but not dead and dried to dust. It is so different from physics, where something that is 100 years old but not in textbooks is, for all practical purposes, dead.

In your latest book you take on the world of finance. What is so attractive about the stock market?
When you've chosen the kind of life I have chosen to live, you must not let opportunities pass by. I recently had the chance to work with Richard Hudson, former managing editor of the European edition of *The Wall Street Journal*, on a book on economics. The most important thing I have done is to combine something esoteric with a practical issue that affects many people. In this spirit, the stock market is one of the most attractive things imaginable. Stock-market data is abundant so I can check everything. Financial markets are very influential and I want to be part of this field now that it is maturing.

In your book, you challenge Alan Greenspan, chairman of the Federal Reserve, and other financiers to set aside $20 million for fundamental research into market dynamics. Why do you feel this is necessary?
There is a problem that is specific to financial markets. In most fields of research, when someone makes an important finding, they publish it. In the case of prices, they set up a firm and sell advice about their discovery. If they can make money from it, they will. So the research into market dynamics is a closed field.

Another reason is that the research carried out so far has proved ineffective because so many fixes and twiddles are added to a theory until it works. That leaves you with a theory that is out of shape. You don't know which is more important – the thing you started with or all the fixes and twiddles. The level of theory in finance has been disappointing.

Do better theories really matter, though?
Financial risks are much underestimated. The effects of wrong business decisions are global. Nobody takes realistic measurements of risk and we should. I think we should take a strongly conservative attitude towards evaluating risks. I have lived all my life skating on thin ice, which does make you conservative. I've met stockbrokers who say that they are perfectly happy that they have judged the financial risks correctly in 95 per cent of their cases. They wonder why they should bother about a few cases that turned out wrong. Well, those are the ones that matter most – such as the Russian market crash of 1998.

I would like scientists, engineers and the whole of society to understand

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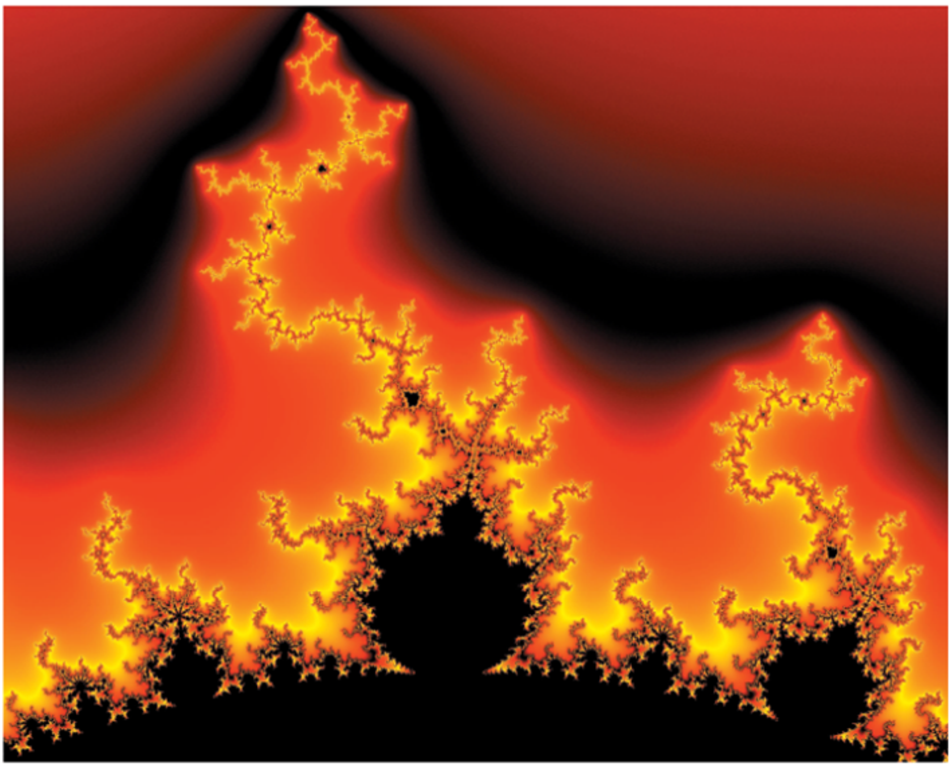

the true meaning of statistics. People have generally been indoctrinated to believe that the world is simpler than it is. I'd like people to understand the difference between what I call mild randomness and wild randomness. Mild randomness is the thing that everyone thinks about where things go up and down a little bit in the financial market. Wild randomness is where one bad event in the stockmarket wipes out a long period of favourable events.

Do you think Greenspan and his colleagues will listen to you?
I don't know. A fair number of people with comparable influence are extremely favourable to my ideas.

Your work has covered many areas. Would you describe yourself as a pure or an applied mathematician?
A mathematical scientist. It's the official name of my chair at Yale and it was chosen with care. It is deliberately ambiguous. In a different era, I would have called myself a natural philosopher. All my life, I have enjoyed the reputation of being someone who disrupted prevailing ideas. Now that I'm in my 80th year, I can play on my age and provoke people even more.

Is that a benefit of being an elder statesman of science?
Elder statesmen of science don't produce new results: they only comment on other people's results. I am still active in research.

What are you working on now?
My work is more varied than at any other point in my life. I am still carrying out research in pure mathematics. And I am working on an idea that I had several years ago on negative dimensions.

What are they?
Negative dimensions are a way of measuring how empty something is. In mathematics, only one set is called empty. It contains nothing whatsoever. But I argued that some sets are emptier than others in a certain useful way. It is an idea that almost everyone greets with great suspicion, thinking I've gone soft in the brain in my old age. Then I explain it and people realise it is obvious. Now I'm developing the idea fully with a colleague. I have high hopes that once we write it down properly and give a few lectures about it at suitable places that negative dimensions will become standard in mathematics.

Are there any physical manifestations of negative dimensions?
Oh yes. I described them in a paper I wrote in the late 1960s on how to measure turbulence. For that analysis, I had to consider different forms of turbulence and found that negative dimensions were important under certain conditions. It was only when I joined Yale in 1987 that a colleague who studied turbulence in the laboratory verified the things I'd been saying.

When you were 20, you said that you wanted to be the Johannes Kepler of a new branch of science. What did you mean?
What Kepler did was to make sense of the motion of planets around the sun. He replaced an earlier accumulation of fixes with a beautiful collection of three laws that truly explained the behaviour of planets. Kepler used the mathematics of ellipses, a great achievement of Greek mathematics, for something practical. My childish ambition was to find a field that nobody had studied, then study it using sophisticated mathematical tools which I would create and manipulate if necessary.

And have you succeeded in that ambition?
Yes. Before my first paper on cotton prices in 1963, the model in circulation was pretty bad. I proposed a different model. People came proposing cycles, epicycles and so on which would mimic my model to a point. But they were much more complicated and less complex.

Ludwig Boltzmann's famous formula for entropy is carved on his tombstone. Do you think a Mandelbrot set would be a fitting epitaph?
The Mandelbrot set covers a small space yet carries a large number of different implications. Is it a fitting epitaph? Absolutely. ●

Benoit Mandelbrot and Richard Hudson's **The (Mis)Behaviour of Markets: A fractal view of risk, ruin and reward** *is published this year in the UK by Profile Business and in the US by Basic Books. Mandelbrot's website is at www.math.yale.edu/mandelbrot/*