Adaptive Piecewise–Affine Inverse Modeling of Hybrid Dynamical Systems

Ehsan Elhamifar, **Sam Burden**, and S. Shankar Sastry

Department of Electrical Engineering and Computer Sciences
University of California, Berkeley, CA, USA

IFAC World Congress
August 29, 2014

Complex sensorimotor control systems

Modeling and control of sensorimotor systems

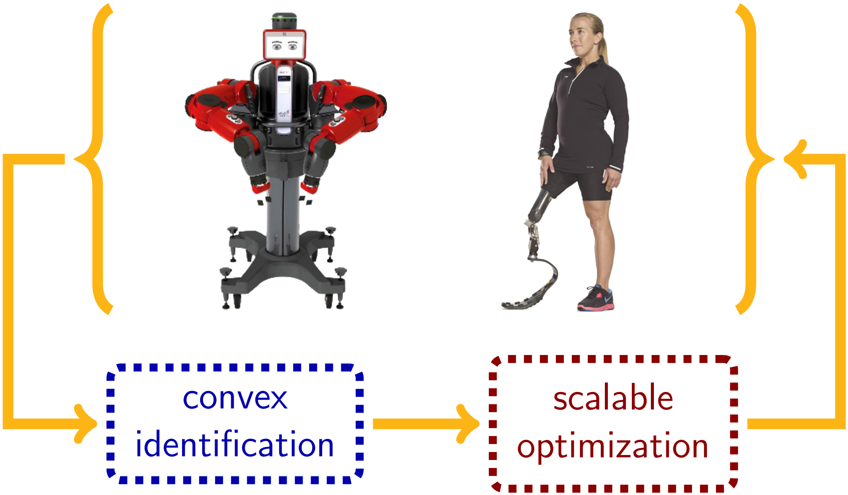
Modeling and control of sensorimotor systems

interaction of aero–, hydro–, and terra–dynamics

intractable to model and control from first principles

Tractable framework for modeling and control

Tractable framework for modeling and control



Overview of today's talk

1. Identification
   convex identification for piecewise–affine systems

2. Control
   scalable optimization for hybrid dynamical systems

Example
   identification and control of jumping dynamics

Future Directions
   ongoing and future work in sensorimotor control theory

Piecewise–affine system identification

$$\min_{\substack{\{z_{it}\}\\ \{\boldsymbol{\beta}_i\}}} \quad \sum_{t=c}^{T} \sum_{i=1}^{s} \ell(\boldsymbol{y}(t), \boldsymbol{u}(t); \boldsymbol{\beta}_i)\, z_{it}$$

$$\text{s. t.} \qquad \sum_{i=1}^{s} z_{it} = 1,\ z_{it} \in \{0, 1\},$$

$i \in \mathbb{N}$: submodel index
$s \in \mathbb{N}$: # of submodels
$t \in \mathbb{N}$: time index
$c \in \mathbb{N}$: start time
$T \in \mathbb{N}$: final time

$\boldsymbol{u}(t) \in \mathbb{R}^p$: input
$\boldsymbol{y}(t) \in \mathbb{R}^q$: output
$\boldsymbol{\beta}_i$: model i parameters
ℓ : loss function
$z_{it} \in \{0, 1\}$: model i active at time t

Piecewise–affine system identification

sum over time samples

sum over submodels $\boldsymbol{\beta}$

$$\min_{\substack{\{z_{it}\}\\ \{\boldsymbol{\beta}_i\}}} \sum_{t=c}^{T} \sum_{i=1}^{s} \underbrace{\ell(\boldsymbol{y}(t), \boldsymbol{u}(t); \boldsymbol{\beta}_i)\, z_{it}}_{\text{cumulative loss}}$$

$$\text{s. t.} \quad \underbrace{\sum_{i=1}^{s} z_{it} = 1,\ z_{it} \in \{0,1\},}_{\text{one submodel per sample}}$$

$i \in \mathbb{N}$: submodel index

$s \in \mathbb{N}$: # of submodels

$t \in \mathbb{N}$: time index

$c \in \mathbb{N}$: start time

$T \in \mathbb{N}$: final time

$\boldsymbol{u}(t) \in \mathbb{R}^p$: input

$\boldsymbol{y}(t) \in \mathbb{R}^q$: output

$\boldsymbol{\beta}_i$: model i parameters

ℓ : loss function

$z_{it} \in \{0,1\}$: model i active at time t

Simplification: select from estimated submodels

sum over time samples

sum over *estimated* submodels $\widehat{\beta}$

$$\min_{\{z_{kt}\}} \sum_{t=c}^{T} \sum_{k=1}^{N} \underbrace{\ell(\boldsymbol{y}(t), \boldsymbol{u}(t); \widehat{\boldsymbol{\beta}}_k)\, z_{kt}}_{\text{cumulative loss}}$$

$$\text{s. t.} \quad \underbrace{\sum_{k=1}^{N} z_{kt} = 1,\ z_{kt} \in \{0, 1\},}_{\text{one submodel per sample}} \qquad \sum_{k=1}^{N} \|\boldsymbol{z}_k\|_\infty \leq s$$

$k \in \mathbb{N}$: submodel index

$s \in \mathbb{N}$: # of submodels

$t \in \mathbb{N}$: time index

$c \in \mathbb{N}$: start time

$T \in \mathbb{N}$: final time

$\boldsymbol{u}(t) \in \mathbb{R}^p$: input

$\boldsymbol{y}(t) \in \mathbb{R}^q$: output

$\boldsymbol{\beta}_k$: model k parameters

ℓ : loss function

$z_{kt} \in \{0, 1\}$: model k active at time t

Simplification: select from estimated submodels

sum over time samples

sum over *estimated* submodels $\widehat{\beta}$

$$\min_{\{z_{kt}\}} \quad \sum_{t=c}^{T} \sum_{k=1}^{N} \underbrace{\ell(\boldsymbol{y}(t), \boldsymbol{u}(t); \widehat{\boldsymbol{\beta}}_k)\, z_{kt}}_{\text{cumulative loss}}$$

$$\text{s. t.} \quad \underbrace{\sum_{k=1}^{N} z_{kt} = 1,\ z_{kt} \in \{0,1\},}_{\text{one submodel per sample}} \qquad \underbrace{\sum_{k=1}^{N} \|\boldsymbol{z}_k\|_\infty \leq s}_{\leq s \text{ submodels}}$$

$k \in \mathbb{N}$: submodel index

$s \in \mathbb{N}$: # of submodels

$t \in \mathbb{N}$: time index

$c \in \mathbb{N}$: start time

$T \in \mathbb{N}$: final time

$\boldsymbol{u}(t) \in \mathbb{R}^p$: input

$\boldsymbol{y}(t) \in \mathbb{R}^q$: output

$\boldsymbol{\beta}_k$: model k parameters

ℓ : loss function

$z_{kt} \in \{0,1\}$: model k active at time t

Relaxation: assign continuous score for submodels

sum over time samples

sum over *estimated* submodels $\widehat{\beta}$

$$\min_{\{z_{kt}\}} \quad \sum_{t=c}^{T} \sum_{k=1}^{N} \underbrace{\ell(\boldsymbol{y}(t), \boldsymbol{u}(t); \widehat{\boldsymbol{\beta}}_k)\, z_{kt}}_{\text{cumulative loss}}$$

$$\text{s. t.} \quad \sum_{k=1}^{N} z_{kt} = 1, \; z_{kt} \geq 0, \qquad \underbrace{\sum_{k=1}^{N} \|\boldsymbol{z}_k\|_\infty \leq s}_{\leq s \text{ submodels}}$$

$k \in \mathbb{N}$: submodel index

$s \in \mathbb{N}$: # of submodels

$t \in \mathbb{N}$: time index

$c \in \mathbb{N}$: start time

$T \in \mathbb{N}$: final time

$\boldsymbol{u}(t) \in \mathbb{R}^p$: input

$\boldsymbol{y}(t) \in \mathbb{R}^q$: output

$\boldsymbol{\beta}_k$: model k parameters

ℓ : loss function

$z_{kt} \in \{0, 1\}$: model k active at time t

Relaxation: assign continuous score for submodels

sum over time samples

sum over *estimated* submodels $\widehat{\beta}$

$$\min_{\{z_{kt}\}} \sum_{t=c}^{T} \sum_{k=1}^{N} \underbrace{\ell(\boldsymbol{y}(t), \boldsymbol{u}(t); \widehat{\boldsymbol{\beta}}_k)\, z_{kt}}_{\text{cumulative loss}}$$

$$\text{s.t.} \quad \underbrace{\sum_{k=1}^{N} z_{kt} = 1,\ z_{kt} \geq 0,}_{\text{submodel score per sample}} \qquad \underbrace{\sum_{k=1}^{N} \|\boldsymbol{z}_k\|_\infty \leq s}_{\leq s \text{ submodels}}$$

$k \in \mathbb{N}$: submodel index

$s \in \mathbb{N}$: # of submodels

$t \in \mathbb{N}$: time index

$c \in \mathbb{N}$: start time

$T \in \mathbb{N}$: final time

$\boldsymbol{u}(t) \in \mathbb{R}^p$: input

$\boldsymbol{y}(t) \in \mathbb{R}^q$: output

$\boldsymbol{\beta}_k$: model k parameters

ℓ : loss function

$z_{kt} \in \{0, 1\}$: model k active at time t

Algorithm: dissimilarity–based sparse subset selection

$$\min_{\boldsymbol{Z}} \quad \lambda \|\boldsymbol{Z}\|_{1,q} \quad + \quad \operatorname{tr}(\boldsymbol{D}^\top \boldsymbol{Z})$$

$$\text{s. t.} \quad \boldsymbol{Z} \geq 0,\ \mathbf{1}^\top \boldsymbol{Z} = \mathbf{1}^\top$$

$\boldsymbol{D}$: dissimilarity
$\boldsymbol{Z}$: submodel assignments
λ : regularization parameter
$\mathbf{1}^\top = (1, \ldots, 1)$

Elhamifar, Sapiro, Sastry, *under review in* TPAMI (arXiv:1407.6810)
http://purl.org/sburden/IFAC2014

Algorithm: dissimilarity–based sparse subset selection

$$\min_{\boldsymbol{Z}} \quad \underbrace{\lambda \|\boldsymbol{Z}\|_{1,q}}_{\text{relaxation of } \|\boldsymbol{Z}\|_{0,q}} + \underbrace{\mathrm{tr}(\boldsymbol{D}^\top \boldsymbol{Z})}_{\text{cumulative loss}}$$

$$\text{s.t.} \quad \underbrace{\boldsymbol{Z} \geq 0,\ \mathbf{1}^\top \boldsymbol{Z} = \mathbf{1}^\top}_{\text{submodel scores}}$$

$\boldsymbol{D}$: dissimilarity

$\boldsymbol{Z}$: submodel assignments

λ : regularization parameter

$\mathbf{1}^\top = (1, \dots, 1)$

Elhamifar, Sapiro, Sastry, *under review in* TPAMI (arXiv:1407.6810)

`http://purl.org/sburden/IFAC2014`

Identification contribution

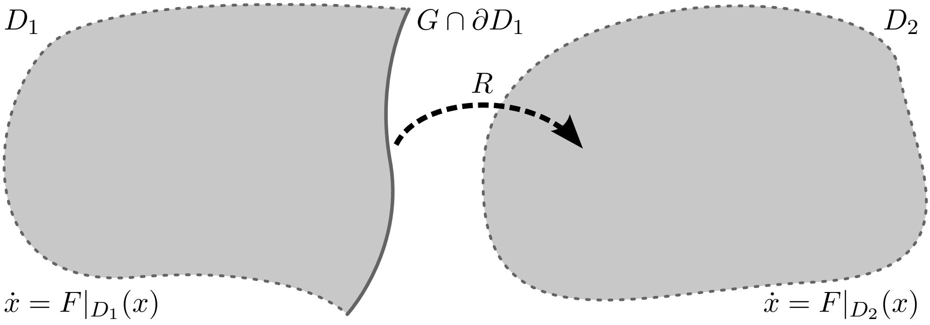
1. Identification
   convex identification for piecewise–affine systems

2. Control
   scalable optimization for hybrid dynamical systems

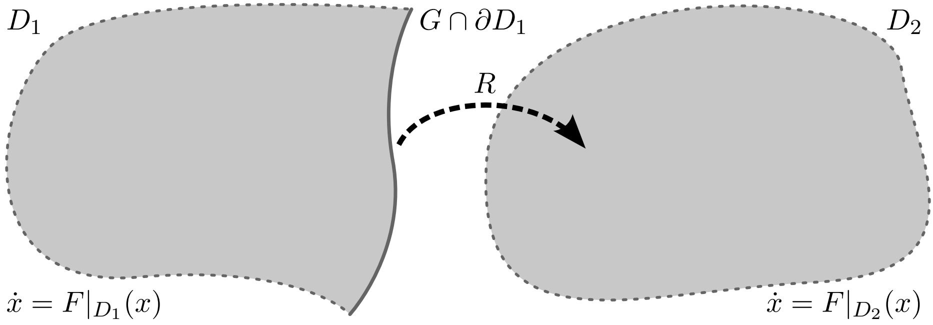
Example
   identification and control of jumping dynamics

Future Directions
   ongoing and future work in sensorimotor control theory

Hybrid dynamical system



Hybrid dynamical system



Definition (hybrid dynamical system $H = (D, F, G, R)$)

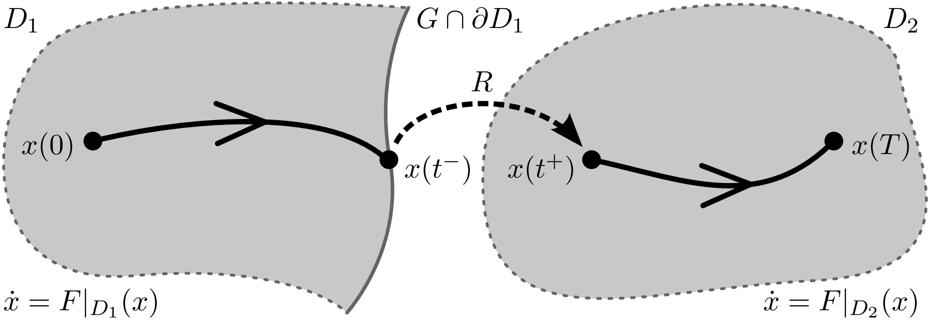
$D = \coprod_{i \in S} D_i$ is a smooth hybrid manifold;

$F : D \to TD$ is a smooth hybrid vector field;

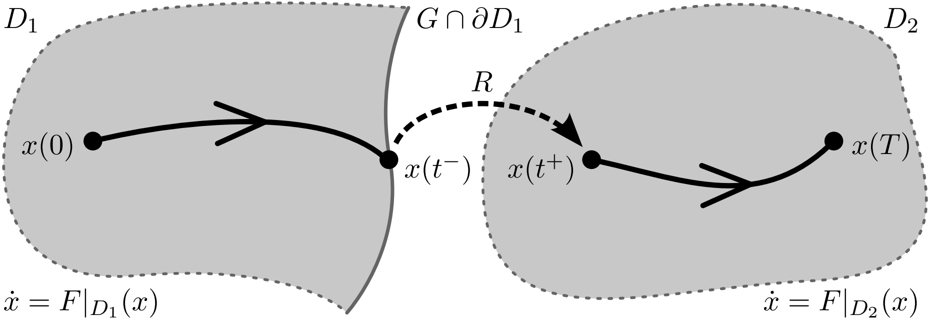
$G \subset \partial D$ is an open subset of ∂D;

$R : G \to D$ is a smooth hybrid map.

State execution



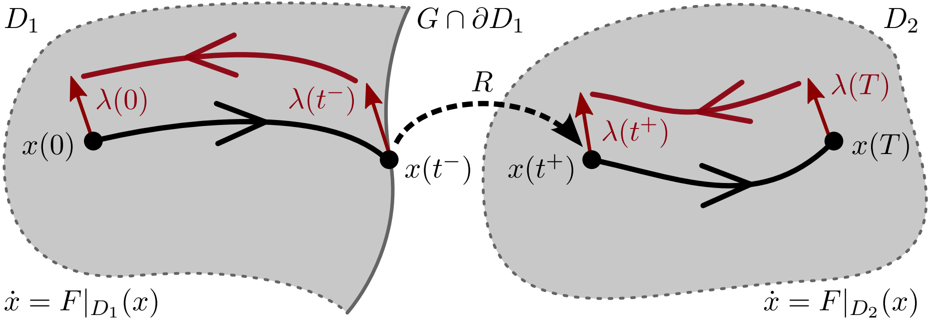
State execution



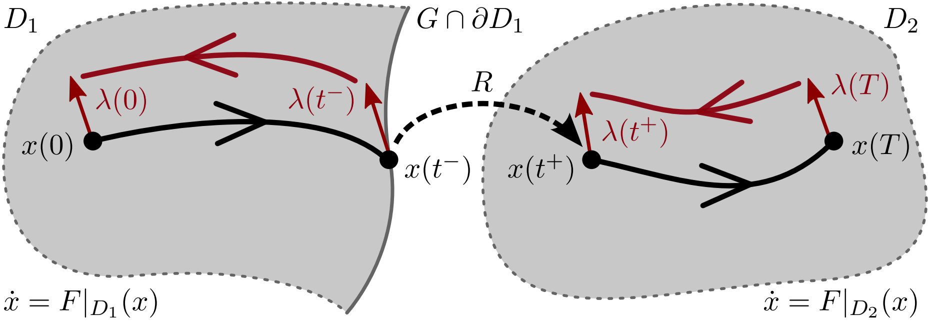
Definition (state execution $x : \tau \to D$ over $\tau = \coprod_{i=1}^{N} \tau_i$)

- $\forall t \in \tau$, in coordinates: $\frac{d}{dt}x(t) = F(x(t))$;
- at transition time t: $x(t^-) \in G,\ R(x(t^-)) = x(t^+)$.

Costate execution



Costate execution

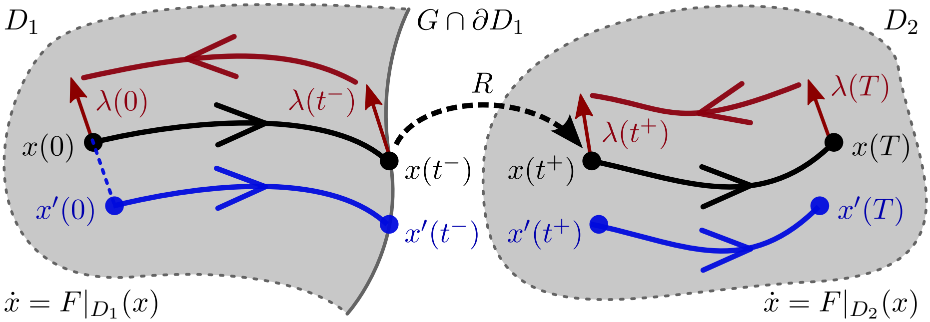


Definition (costate execution $\lambda : \tau \to T^*D$ over $\tau = \coprod_{i=1}^{N} \tau_i$)

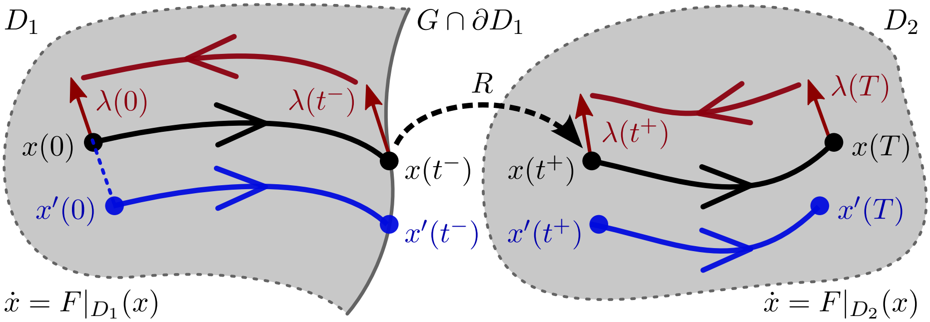
- $\forall t \in \tau$, in coordinates: $\frac{d}{dt}\lambda(t) = \lambda(t) D_x F(x(t))$;
- at transition time t, in coordinates where F is orthogonal to G,

$$\lambda(t^-) = \lambda(t^+) D_x R(x(t^-)) + F(x(t^-)) \frac{\lambda(t^+) F(x(t^+))}{\|F(x(t^-))\|^2}.$$

Optimization using first–order variation of cost



Optimization using first–order variation of cost



Scalable (first–order) optimization algorithm

Solve $\arg\min_{x(0)} J(x(T))$ by iterating $x'(0) = x(0) - \alpha\lambda(0)$.

Control contribution

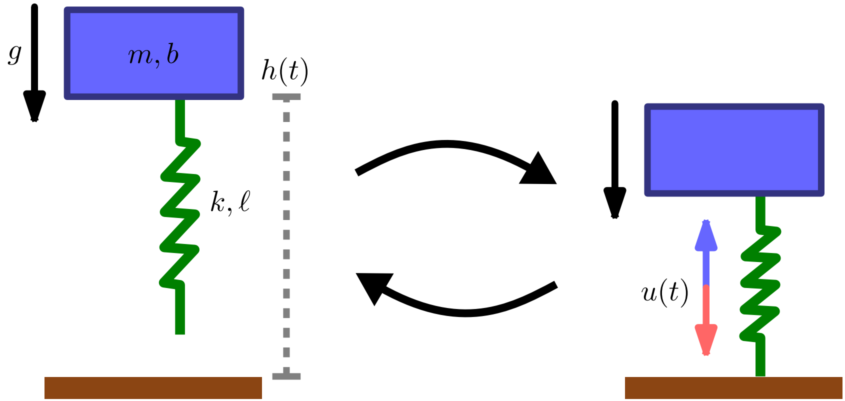
1. Identification
   convex identification for piecewise–affine systems

2. Control
   scalable optimization for hybrid dynamical systems

Example
   identification and control of jumping dynamics

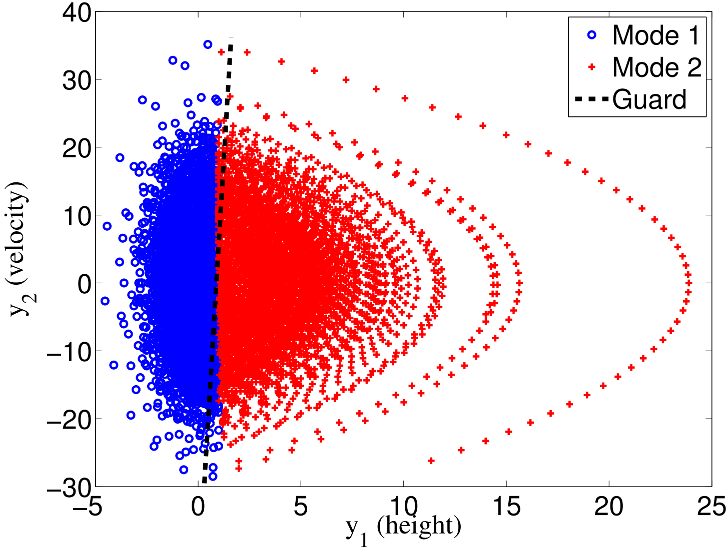
Future Directions
   ongoing and future work in sensorimotor control theory

Piecewise–affine model for jumping robot

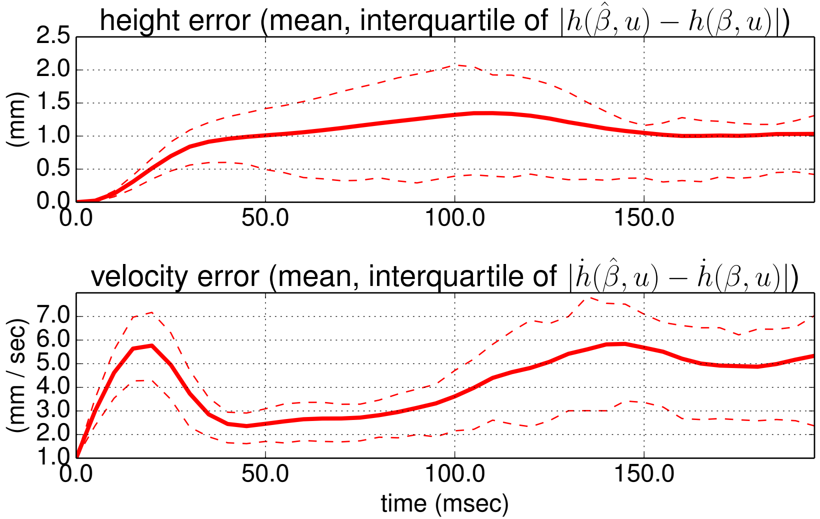


Aguilar, Lesov, Wiesenfeld, Goldman, PRL 2012

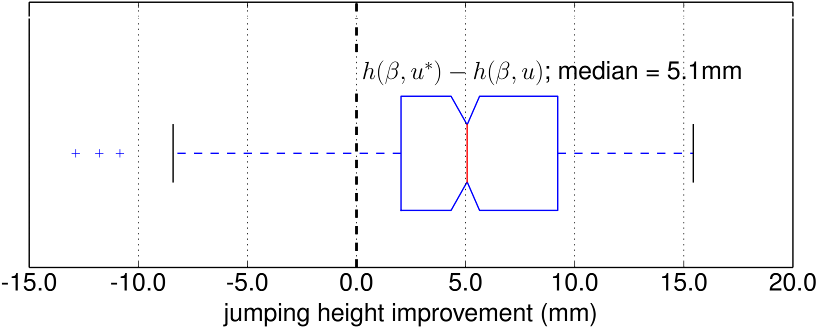
Identification of jumping dynamics



Control of jumping dynamics



Control of jumping dynamics



Ongoing and future work in sensorimotor control theory

identification and optimization of locomotion and manipulation

Discussion & Questions — Thanks for your time!

Our contribution

convex identification and scalable optimization of PWA hybrid systems

Ehsan Elhamifar

Shankar Sastry