From Templates to Anchors:

Exact and Approximate Reduction in Models of Legged Locomotion

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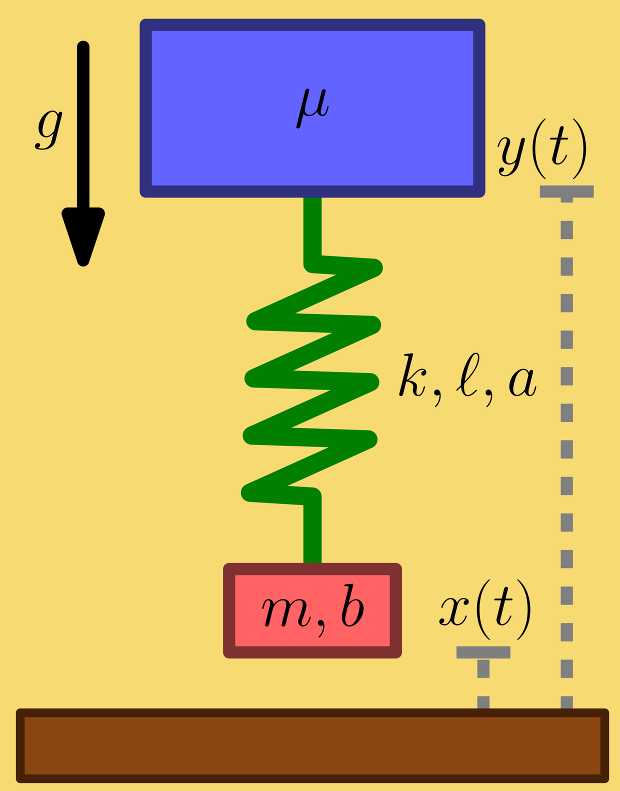
Hybrid Systems Exhibit Model Reduction

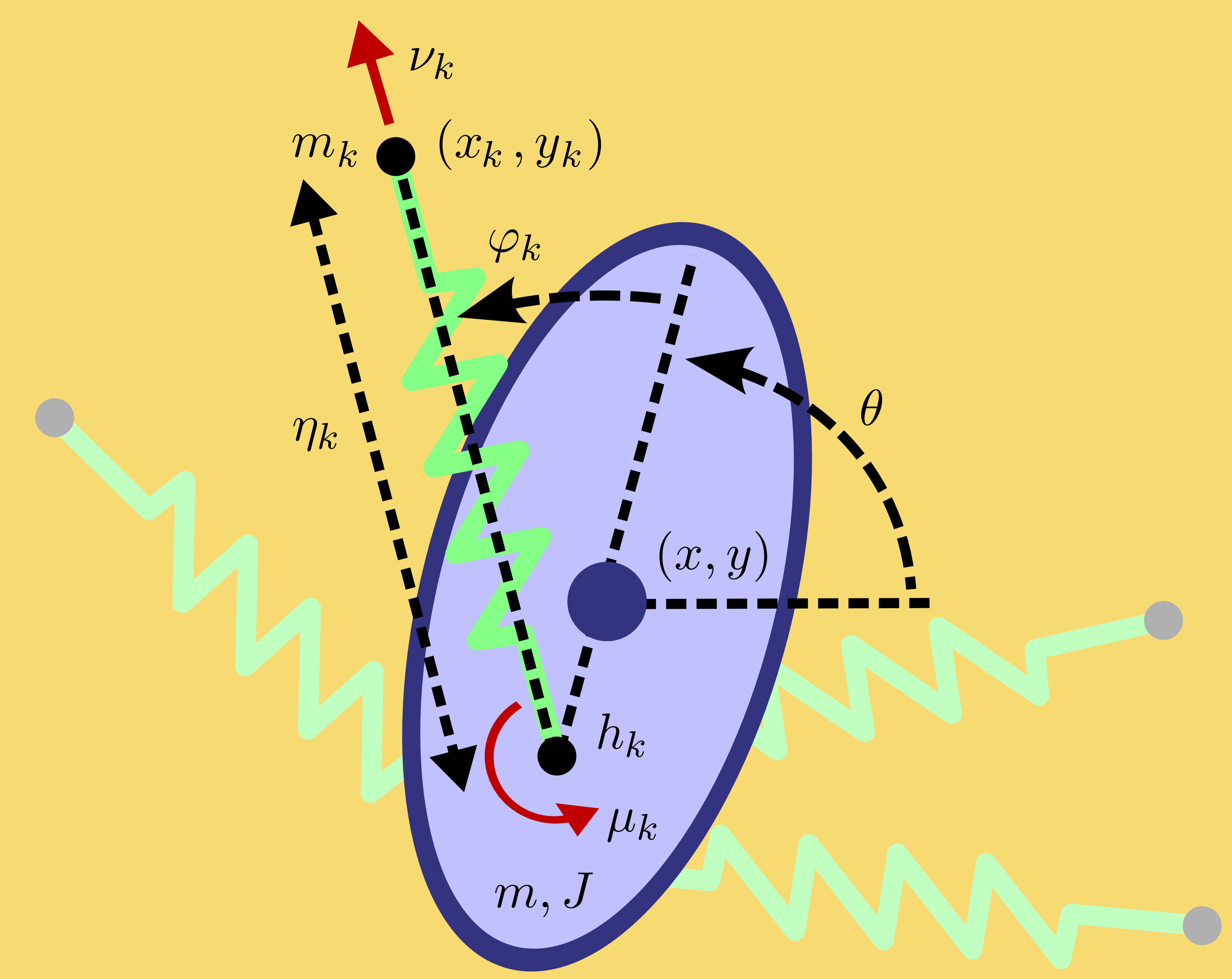
Definition 3. *A* hybrid dynamical system *is specified by a tuple* $H = (D, F, G, R)$ *where:*

$D = \coprod_{j \in J} D_j$ *is a smooth hybrid manifold;*
$F : D \to TD$ *is a smooth vector field;*
$G \subset \partial D$ *is open;*
$R : G \to D$ *is a smooth map.*



Vertical hopper loses 1 DOF -- we **formalize** & **generalize**

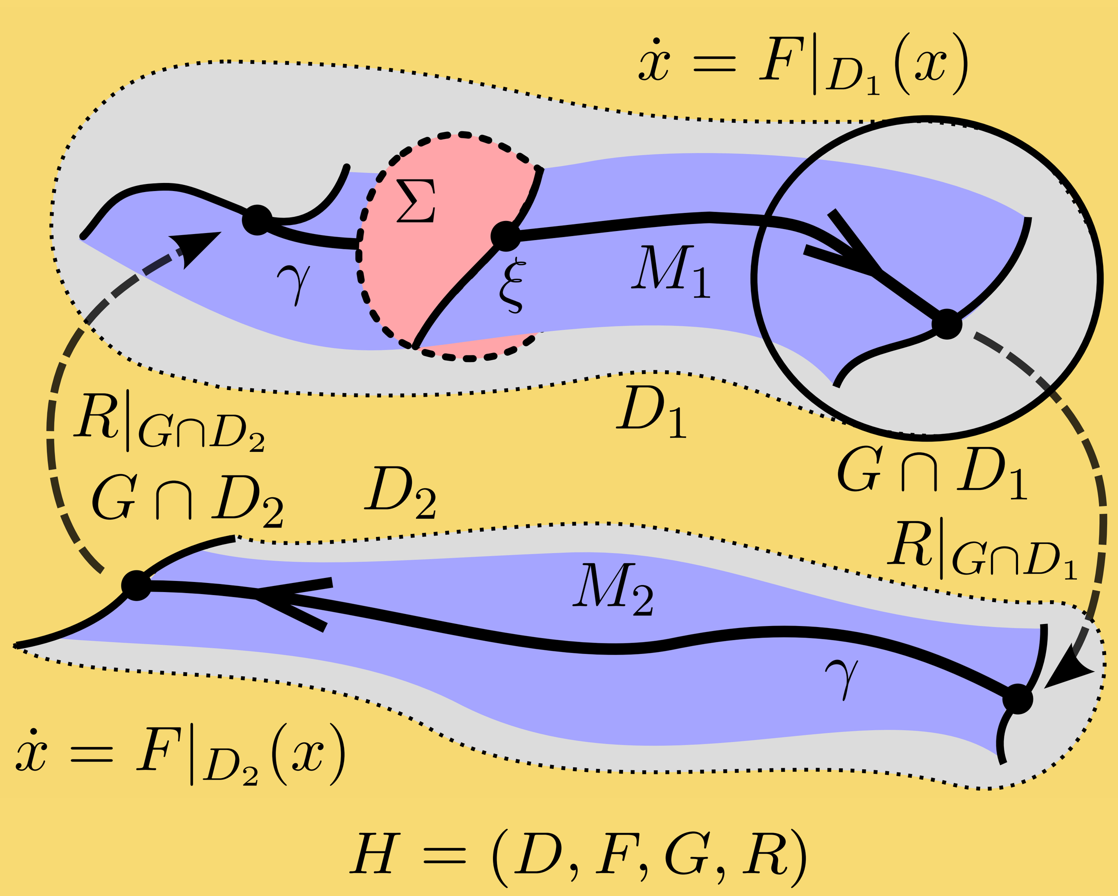
ex: n-leg polyped reduces to 3-DOF LLS

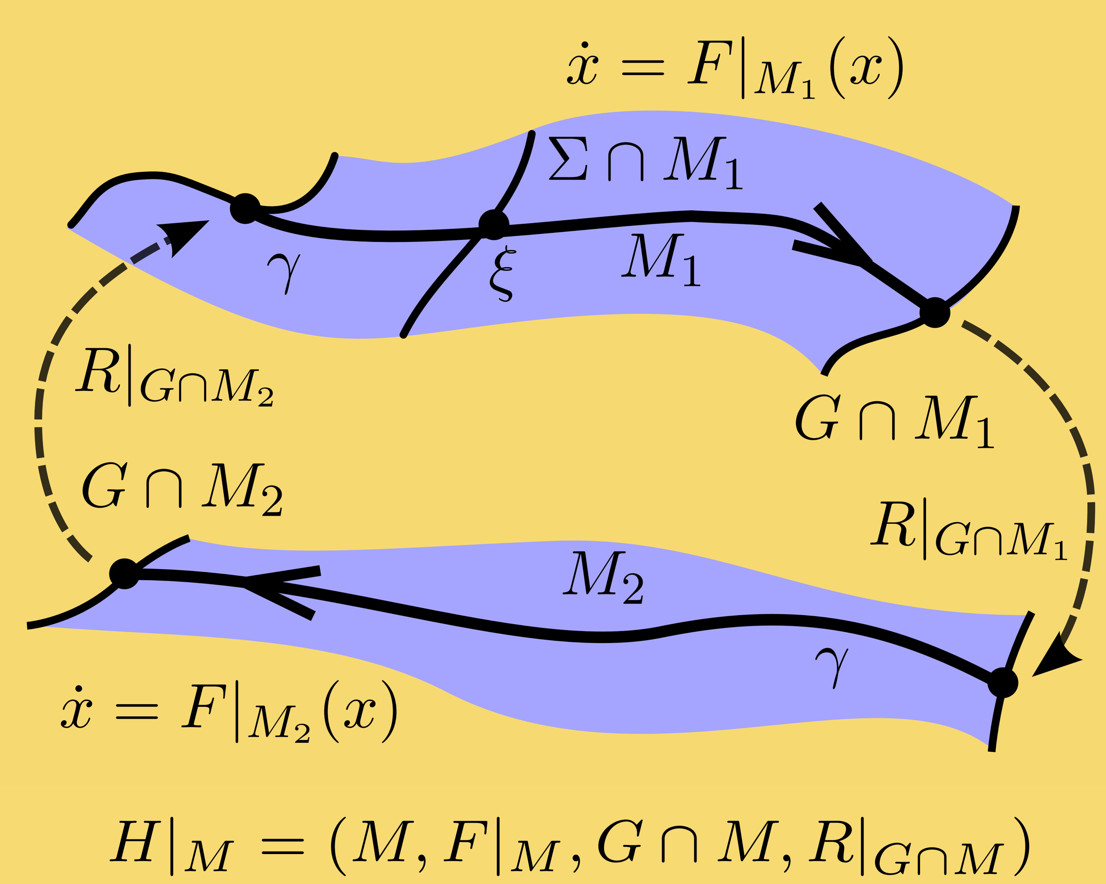


thm: Exact Reduction to a Hybrid Subsystem

Theorem 1. *Let* γ *be a periodic orbit for a hybrid dynamical system* $H = (D, F, G, R)$, $P : U \to \Sigma$ *a Poincaré map for* γ, $n = \min_j \dim D_j - 1$, *and suppose* $\operatorname{rank} DP^n \equiv r \in \mathbb{N}$. *Then there exists an* $(r+1)$*–dimensional hybrid–invariant submanifold* $M \subset D$ *and a hybrid open set* $W \subset D$ *for which* $\gamma \subset M \cap W$ *and trajectories starting in* W *contract to* M *in finite time.*

Corollary 2. $H|_M = (M, F|_M, G \cap M, R|_{G \cap M})$ *is a hybrid dynamical system with periodic orbit* γ.



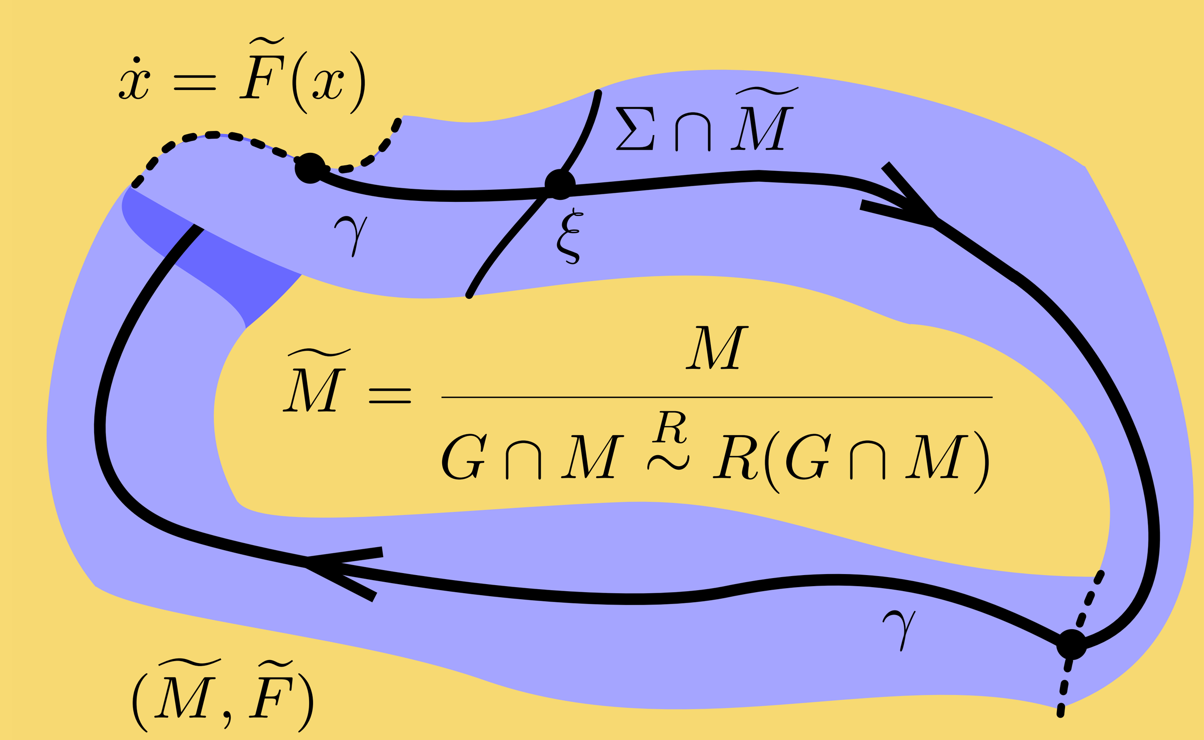


thm: Transitions can be Smoothed

Theorem 3. *Let* $H = (M, F, G, R)$ *be a hybrid dynamical system with* $M = \coprod_{j \in J} M_j$. *Suppose* $\dim M_j = n$ *for all* $j \in J$, $R(G) \subset \partial M$, $\partial M = G \coprod R(G)$, R *is a hybrid diffeomorphism onto its image, and* F *is inward–pointing along* $R(G)$. *Then the topological quotient* $\widetilde{M} = \frac{M}{G \overset{R}{\sim} R(G)}$ *may be endowed with the structure of a smooth manifold:*

1) *the quotient projection* $\pi : M \to \widetilde{M}$ *restricts to a smooth embedding* $\pi|_{M_j} : M_j \to \widetilde{M}$ *for each* $j \in J$;
2) *there is a smooth vector field* $\widetilde{F} \in \mathfrak{T}(\widetilde{M})$ *such that any execution* $x : T \to M$ *of* H *descends to an integral curve of* $\widetilde{F}$ *on* $\widetilde{M}$ *via* $\pi : M \to \widetilde{M}$:

$$\forall t \in T : \frac{\partial}{\partial t} \pi \circ x(t) = \widetilde{F}(\pi \circ x(t)).$$



If periodic orbit is exponentially stable, dynamics **generically** reduce **approximately**

[1] R Full and D Koditschek. Templates and Anchors: Neuromechanical Hypothesis of Legged Locomotion on Land. Journal of Experimental Biology, 1999.
[2] S Burden, S Revzen, S Sastry. Dimension Reduction Near Periodic Orbits of Hybrid Systems. IEEE CDC, 2011.
[3] S Burden, S Revzen, S Sastry. Model Reduction Near Periodic Orbits in Hybrid Dynamical Systems. ArXiv e-print ____.____, 2013.
[4] S Revzen, S Burden, D Koditschek, S Sastry. Pinned Equilibria Provide Robustly Stable Multilegged Locomotion. Dynamic Walking, 2013.