Reduction and Robustness via Intermittent Contact

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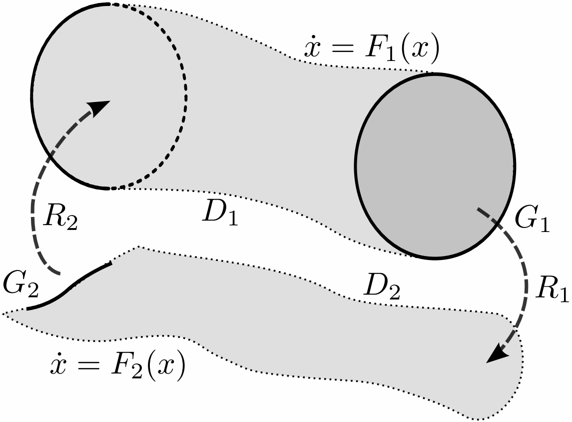
Motivation

Dynamic interaction involves intermittent contact

Nao humanoid robot

video courtesy of Aldeberan Robotics, `http://www.aldebaran-robotics.com/en/`

Contact yields a *hybrid* dynamical system



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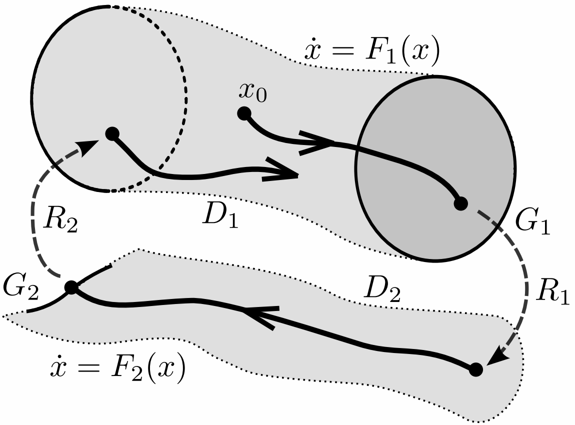

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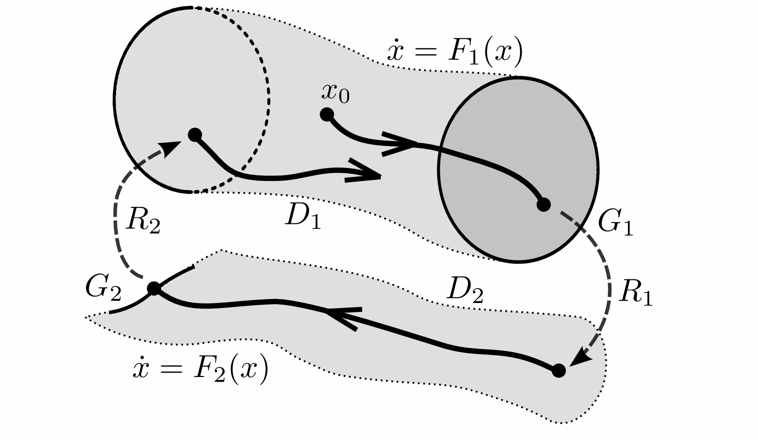
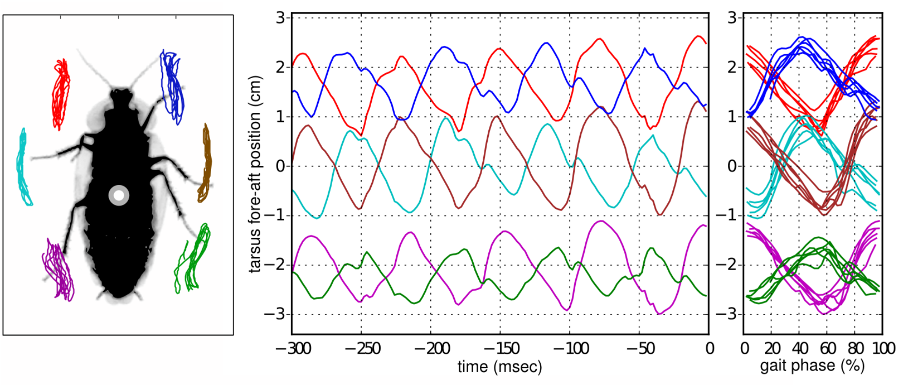


Combinatorial # of discrete modes,
each generally possessing nonlinear dynamics

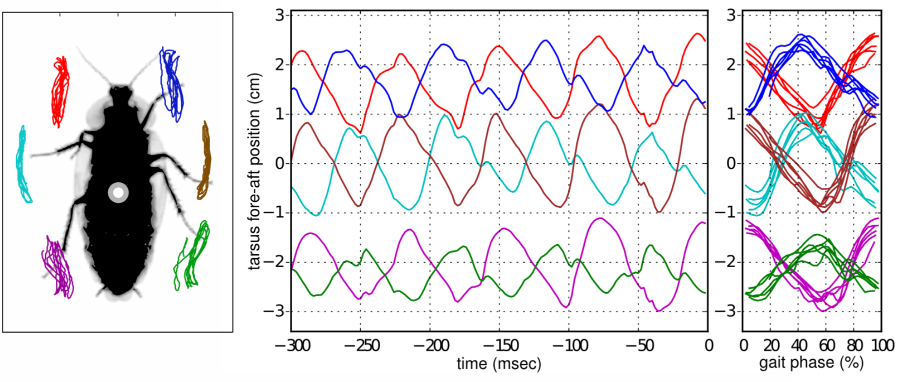
Focus on rhythmic behaviors



Animals utilize rhythmic behaviors for locomotion & manipulation

Grillner, Science 1985

Focus on rhythmic behaviors



Animals utilize rhythmic behaviors for locomotion & manipulation

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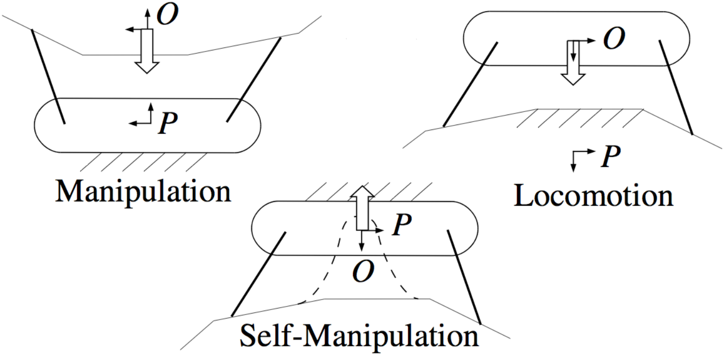
Represented by periodic orbits in hybrid dynamical system

Focus on locomotion



Note that locomotion is self-manipulation

Johnson, Haynes, & Koditschek, IROS 2012

Overview of this talk

Motivation
interaction with environment involves intermittent contact

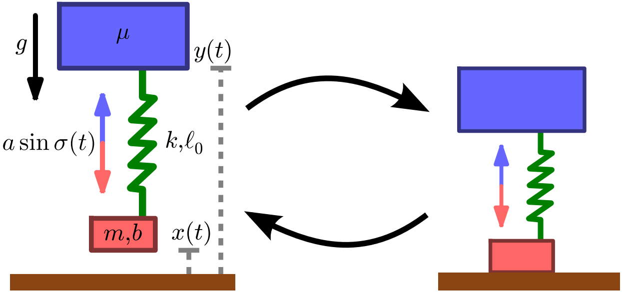
Reduction
low-dimension subsystem appears near hybrid periodic orbit

Robustness
simultaneous hybrid transitions yield robust stability

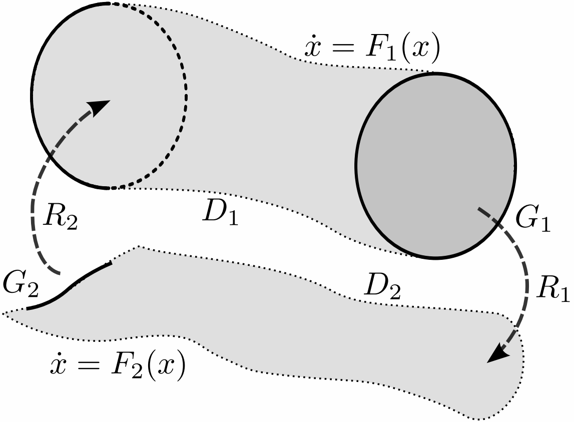
Applications
identification of neuromechanical control architecture in animals
design and optimization of gaits and maneuvers for robots

Reduction

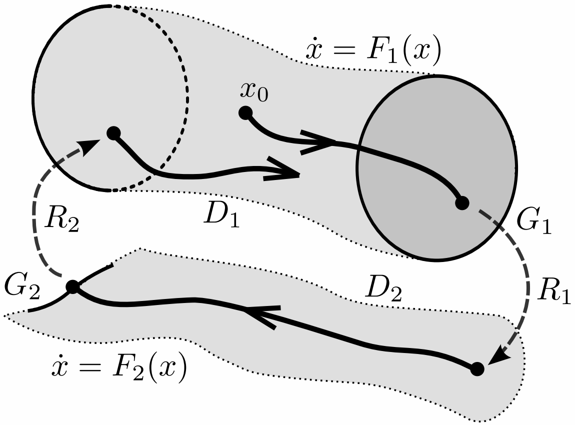
Example (vertical hopper)



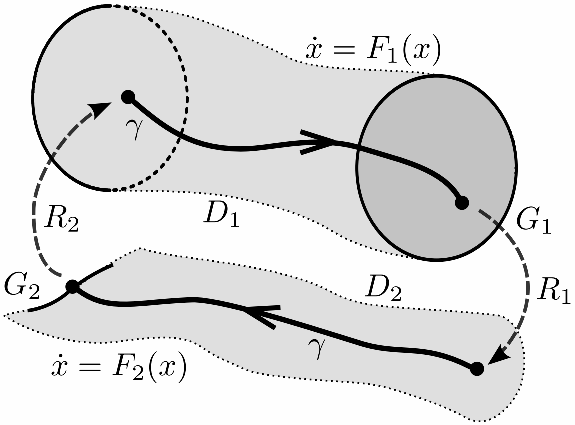
Hybrid dynamical system



Trajectory for a hybrid dynamical system



Periodic orbit γ for a hybrid dynamical system



Assumptions on hybrid periodic orbit γ

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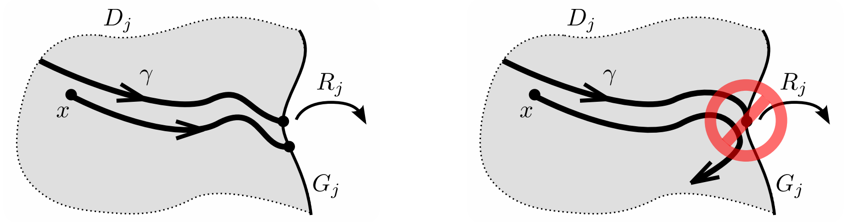

Assumption (transversality)

periodic orbit γ passes transversely through each guard G_j

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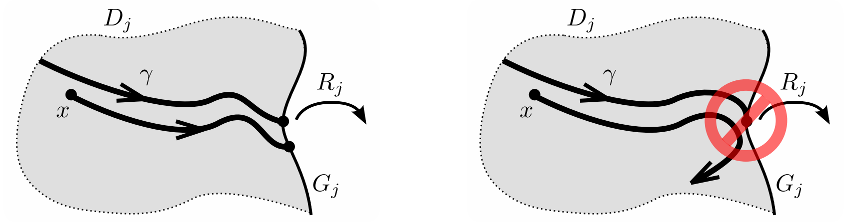


Assumption (transversality)

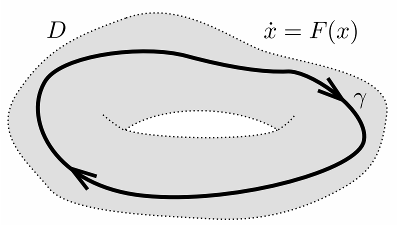
periodic orbit γ *passes transversely through each guard* G_j

Assumption (dwell time)

$\exists \varepsilon > 0$: *periodic orbit* γ *spends at least* ε *time units in each domain* D_j

Poincaré map for periodic orbit γ

smooth dynamical system



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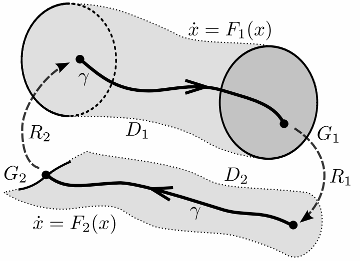

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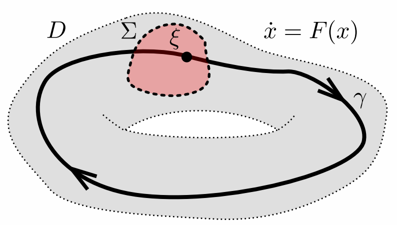

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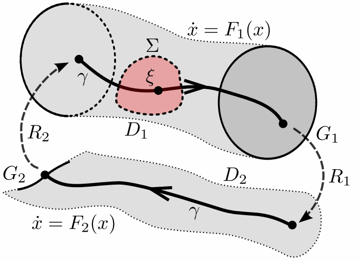

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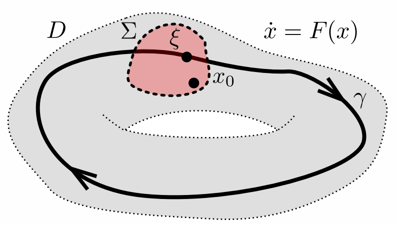

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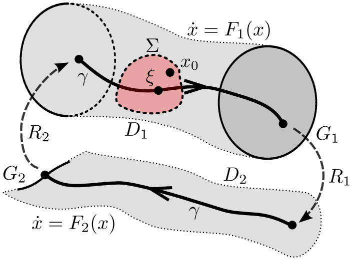

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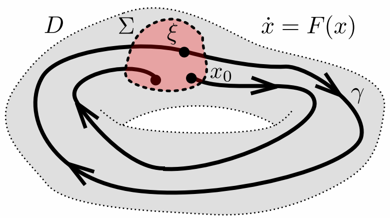

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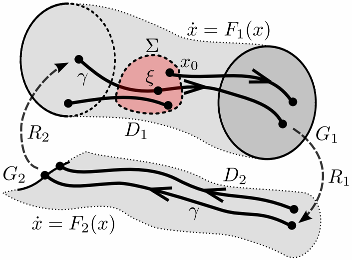

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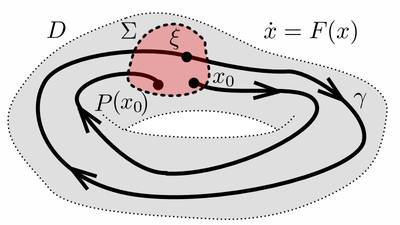

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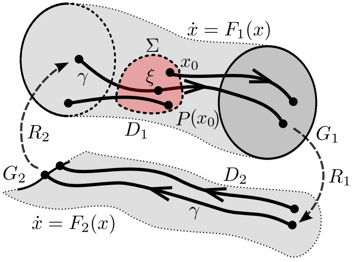

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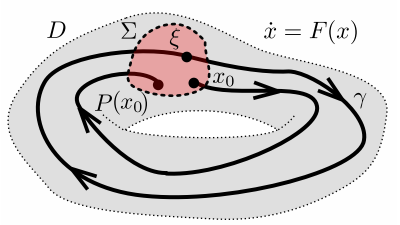

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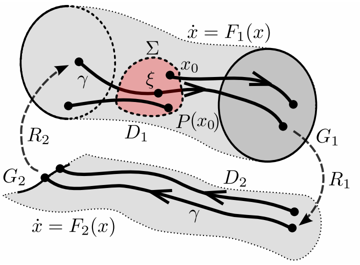
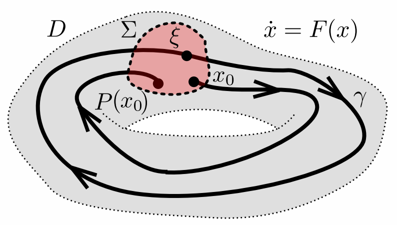


Theorem (Hirsch and Smale 1974, Grizzle *et al.* 2002)

The Poincaré map P is smooth in a neighborhood of ξ.

Rank of Poincaré map P with fixed point $P(\xi) = \xi$

smooth dynamical system



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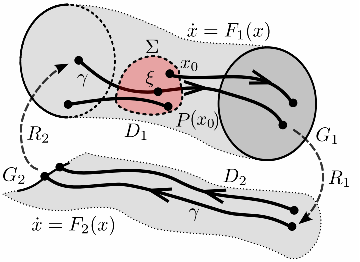

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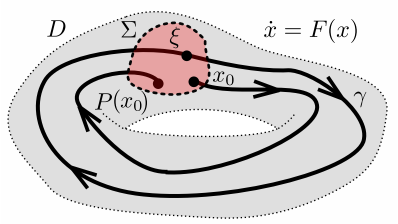

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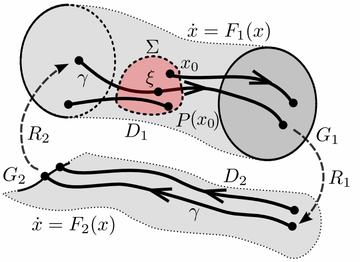

$$\operatorname{rank} DP(\xi) = \dim D - 1$$

Hirsch and Smale 1974

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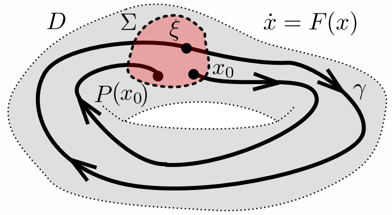

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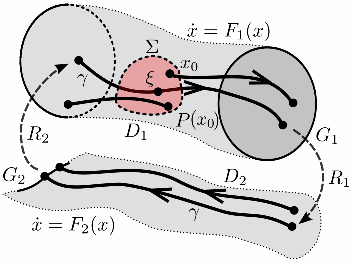
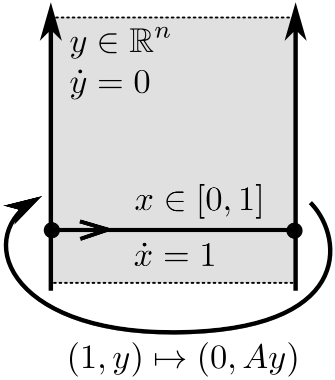


$\operatorname{rank} DP(\xi) \leq \min_j \dim D_j - 1$

Wendel and Ames 2010

Example (rank-deficient Poincaré map)



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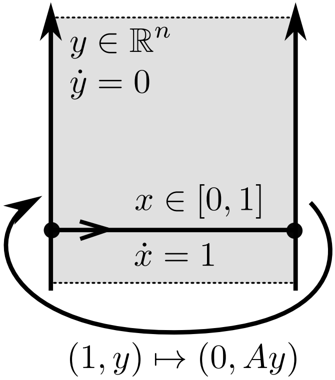
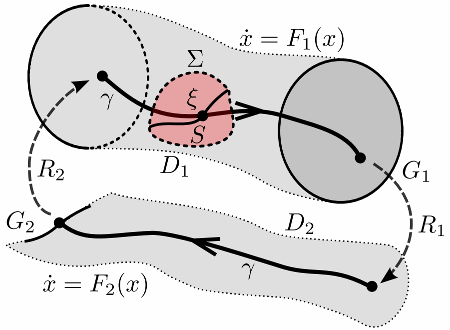


If $A \in \mathbb{R}^{n\times n}$ is nilpotent (i.e. $A^n = 0_{n\times n}$), then $\operatorname{rank} DP^n = 0$.

Exact model reduction near hybrid periodic orbit γ



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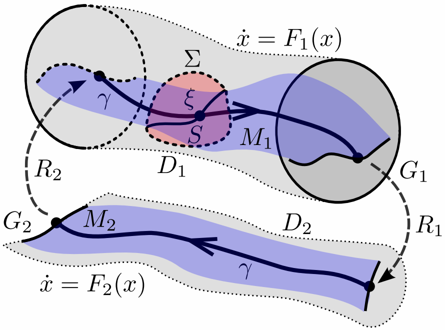

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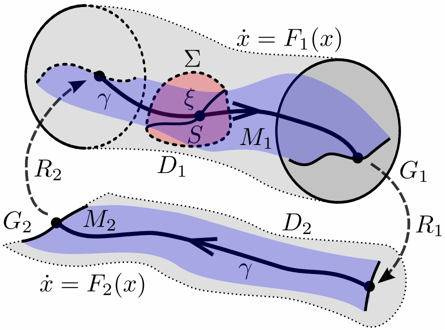
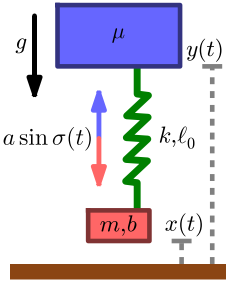


Theorem (Burden, Revzen, Sastry CDC 2011)

Let $n = \min_j \dim D_j$. If $\operatorname{rank} DP^n = r$ near ξ, then trajectories starting near γ contract to a collection of hybrid-invariant $(r+1)-$dimensional submanifolds $M_j \subset D_j$ in finite time.

Example (exact model reduction in vertical hopper)



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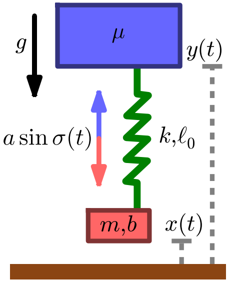

Numerically linearizing Poincaré map P on ground,
we find $DP(\xi)$ has eigenvalues $\simeq -0.25 \pm 0.70j$,
therefore DP^2 is constant rank near ξ.

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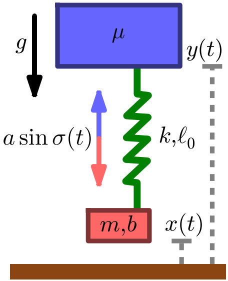

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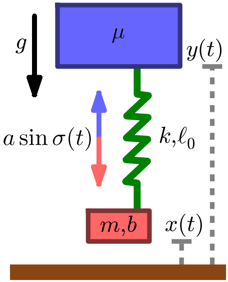

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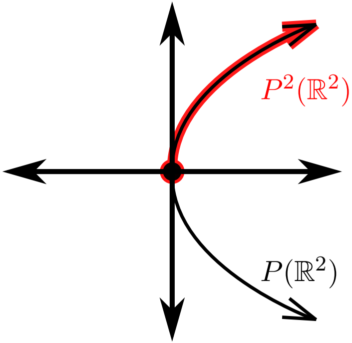
Theorem $\implies$ dynamics collapse to 1-DOF hopper

Interpretation: unilateral (Lagrangian) constraint appears after one "hop"

Example ($\mathrm{rank}\, DP^n$ generically non-constant)

$P(x, y) = (x^2, x)$



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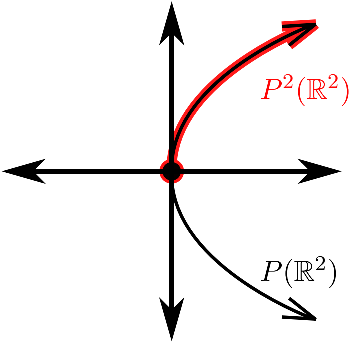

$$DP(x,y) = \begin{pmatrix} 2x & 0 \\ 1 & 0 \end{pmatrix}$$

$$\implies \operatorname{rank} DP = 1$$

$$DP^2(x,y) = \begin{pmatrix} 4x^3 & 0 \\ 2x & 0 \end{pmatrix}$$

$$\implies \operatorname{rank} DP^2(x,y) = \begin{cases} 0, & x = y = 0 \\ 1, & else \end{cases}$$

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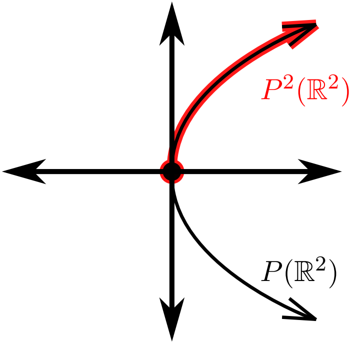

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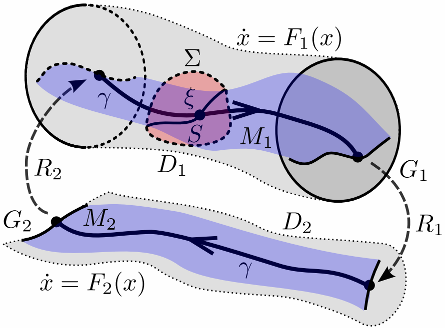
Note that P contracts arbitrarily rapidly since $DP(0,0)$ is nilpotent: for all $\varepsilon > 0$ there exists $\delta > 0$ and $\|\cdot\|_\varepsilon$ such that

$$\|(x,y)\| < \delta \implies \|P(x,y)\| < \varepsilon \|(x,y)\|_\varepsilon$$

Approximate model reduction near hybrid periodic orbit γ



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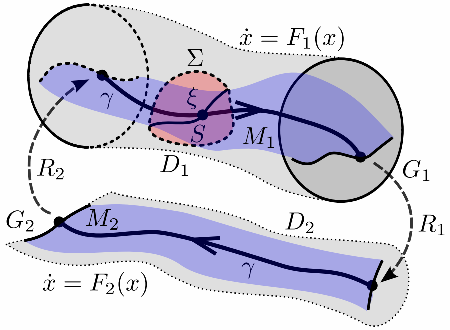
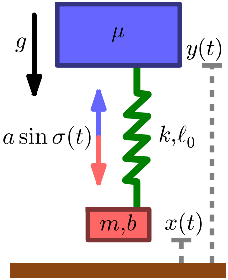


Theorem (Burden, Revzen, Sastry *(in preparation)*)

If $\operatorname{rank} DP^n(\xi) = r$ *and* $\operatorname{spec} DP(\xi) \subset B_1(0) \subset \mathbb{C}$, *then for any* $\varepsilon > 0$ *trajectories starting sufficiently near* γ *contract exponentially fast with rate* ε *to a collection of* $(r+1)-$*dimensional submanifolds* $M_j \subset D_j$.

Example (deadbeat control of vertical hopper)



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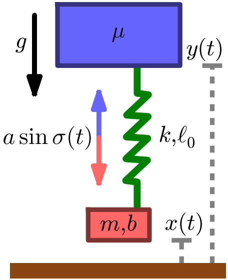

There exists a deadbeat control for vertical hopper,

i.e. smooth actuator feedback law $a(x, y, \dot{x}, \dot{y})$ such that hopper exactly tracks periodic orbit after one "hop"

Carver, Cowen, & Guckenheimer, Chaos 2009

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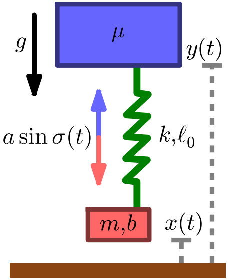

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perturbing parameters k, ℓ_0, m, μ, b yields $\text{rank}\, DP = 2$

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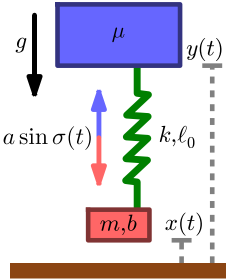

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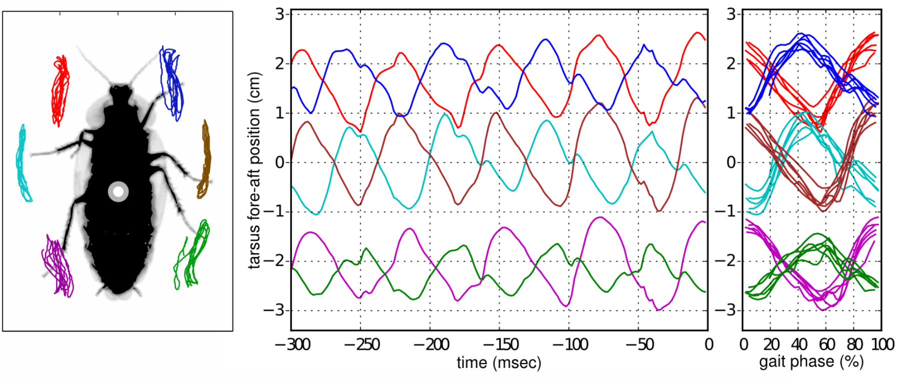
Theorem $\implies$ hopper contracts to orbit at rate bounded by size of parameter perturbation

Robustness

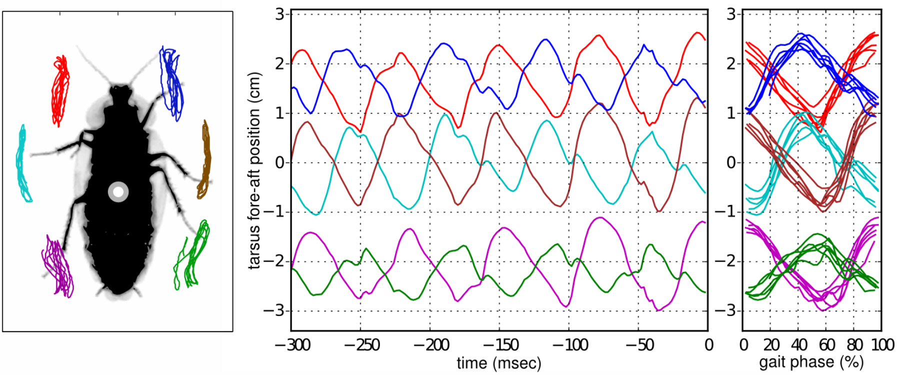
Simultaneous hybrid transitions



Empirically, simultaneous limb touchdown typical for animal gaits

Golubitsky *et al.* Nature 1999

Simultaneous hybrid transitions



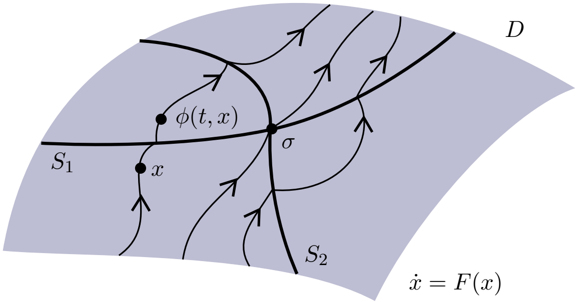
Empirically, simultaneous limb touchdown typical for animal gaits

Golubitsky *et al.* Nature 1999

(Consequently) also typical for polyped robot gaits

Saranli *et al.* IJRR 2001; Kim *et al.* IJRR 2006; Hoover *et al.* IROS 2008

Assumptions on simultaneous hybrid transitions



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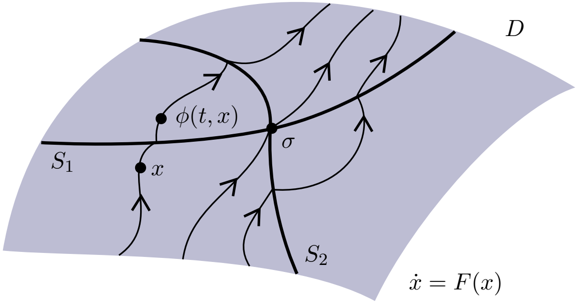

Assumption (transversality)

$n = \dim D$ *transition surfaces* $\{S_j\}_1^n$ *intersect transversely at* $\sigma \in D$.

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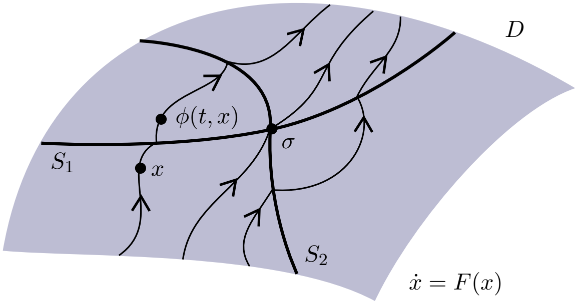

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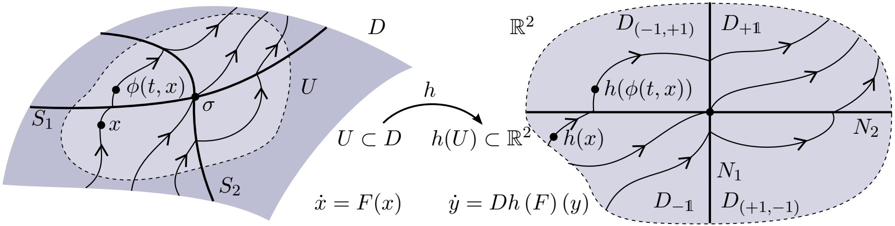
Assumption (piecewise smooth vector field)

All points where F *is discontinuous or nonsmooth are contained in* $\bigcup_j S_j$.

Assumptions on simultaneous hybrid transitions



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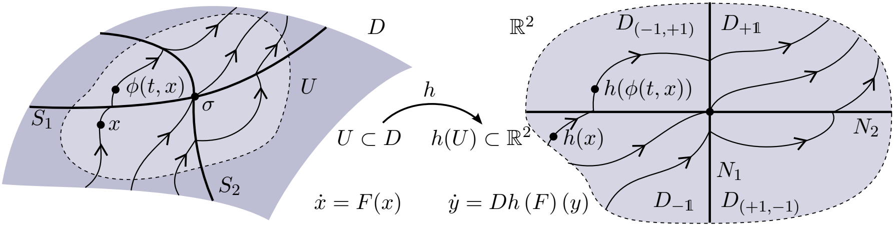
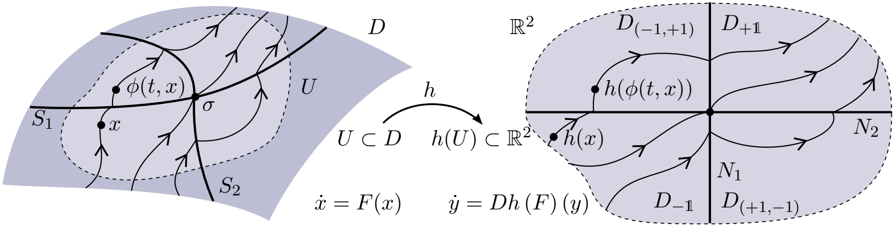


Assumption (no sliding modes)

For $q \in \{-1,+1\}^n$, *let* $F_q = \lim_{\substack{y \to 0 \\ y \in D_q}} Dh(F)(y)$ *and assume* $F_q \in \operatorname{Int} D_{+\mathbb{1}}$.

Assumptions on simultaneous hybrid transitions



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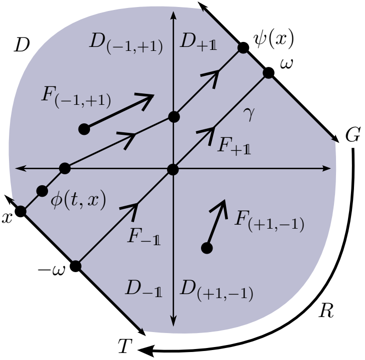
Theorem (Fillipov 1988)

The flow ϕ *is well-defined and continuous in a neighborhood of* σ.

Normal form for simultaneous hybrid transitions



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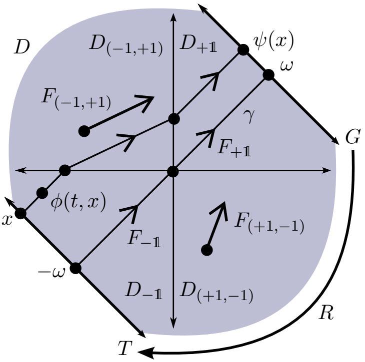
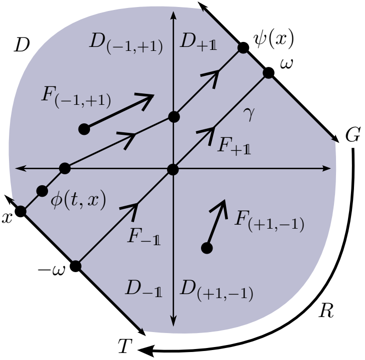


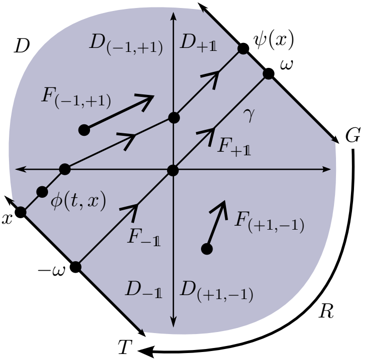
$R : G \to T$ continuous, $\psi : T \to G$ obtained by integrating flow ϕ.

Normal form for simultaneous hybrid transitions



$R : G \to T$ continuous, $\psi : T \to G$ obtained by integrating flow ϕ.
With $\omega := \frac{1}{\sqrt{n}}\mathbb{1}$, let $\Pi := I - \omega\omega^T$ be orthogonal projection onto $\ker \omega^T$.

Sufficient condition for stability



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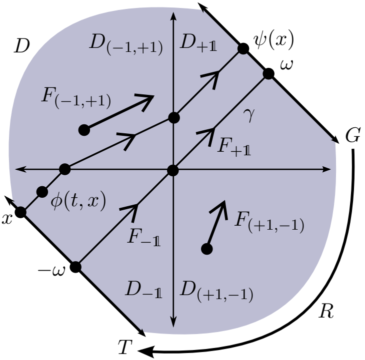
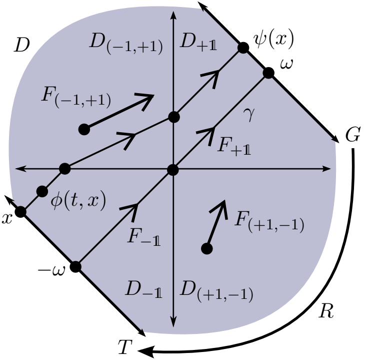


Theorem (Burden, Revzen, Koditschek, Sastry *(in preparation)*)

$\Pi F_q \in \operatorname{Int} D_{-q}$ *for all* $q \neq \pm\mathbb{1} \implies \exists c \in (0,1) : \|\Pi\psi(x)\| < c\,\|\Pi x\|$.

Sufficient condition for stability



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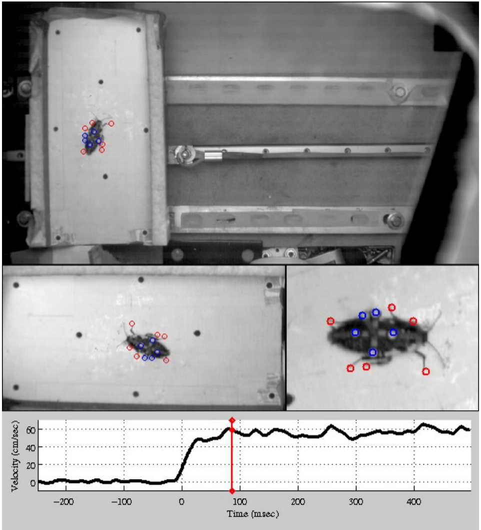
If R *Lipschitz with constant* $1/c$ *and* $R(\omega) = -\omega$ *then* γ *exp. stable.*

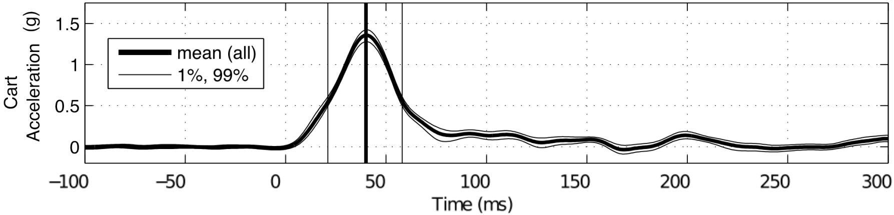
Applications

Identify neuromechanical control architecture in animals



Revzen *et al.* (in review) 2012

Mechanical self-stabilization vs. neural feedback



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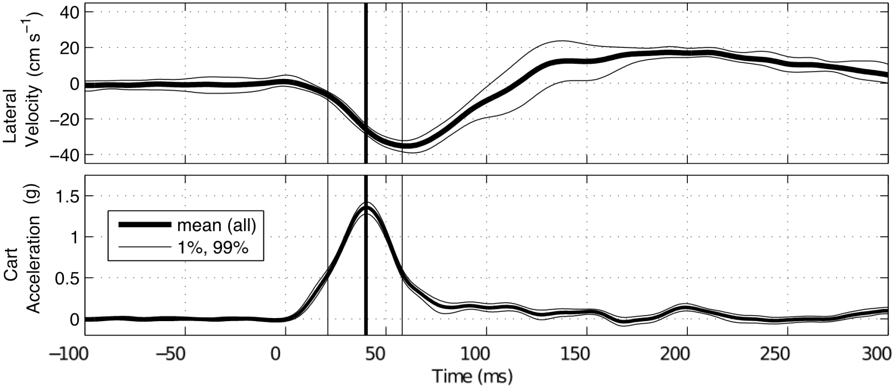

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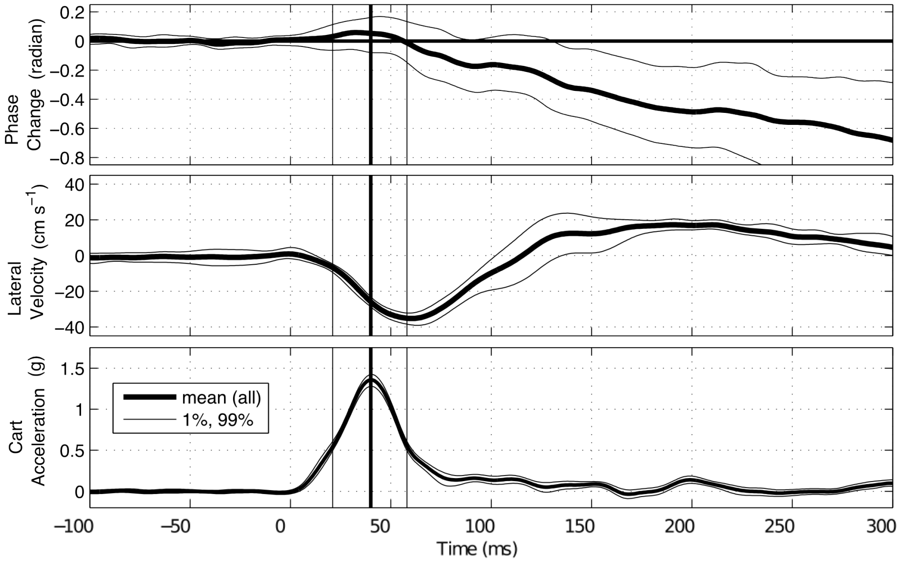
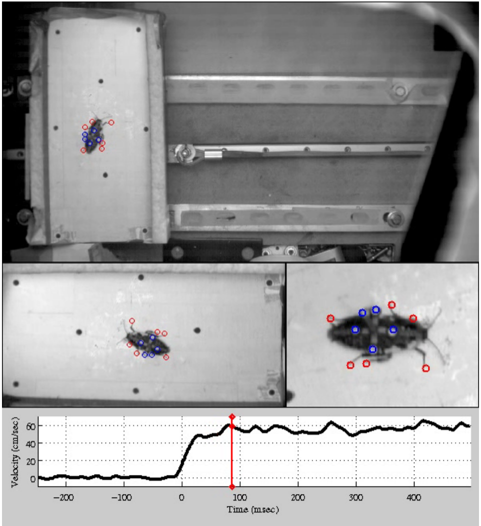
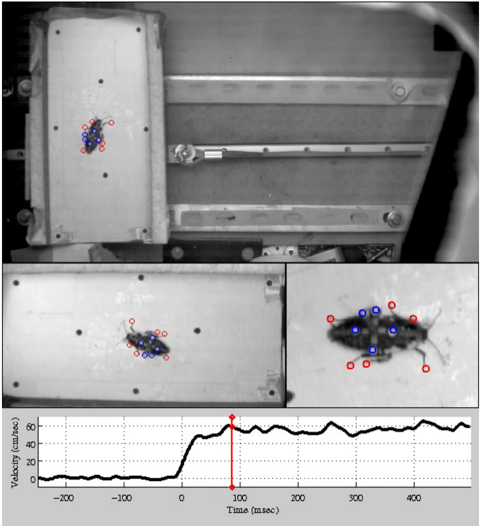


Identify neuromechanical control architecture in animals



Burden *et al.* SysID 2012; Burden *et al.* SICB 2013

Identify neuromechanical control architecture in animals

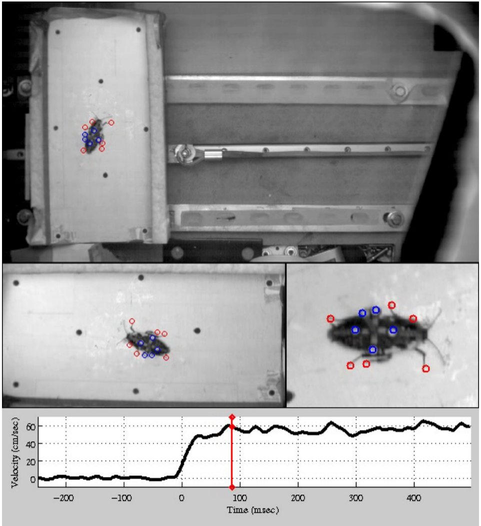


Identification problem

$$\arg\min_{z \in D_j} \varepsilon\left(z, \{\eta_i\}\right)$$

Burden *et al.* SysID 2012; Burden *et al.* SICB 2013

Identify neuromechanical control architecture in animals



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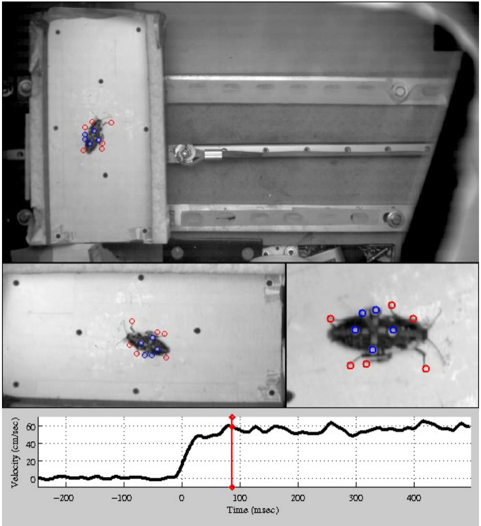
$$\arg\min_{z \in D_j} \varepsilon\left(z, \{\eta_i\}\right)$$

- $\nabla\varepsilon$ undefined on $G_j \subset D_j$

global optimization needed

Burden *et al.* SysID 2012; Burden *et al.* SICB 2013

Identify neuromechanical control architecture in animals



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global optimization needed

Identification on $\bigcup_j M_j$

$$\arg\min_{z \in M_j} \varepsilon\left(z, \{\eta_i\}\right)$$

- $\nabla\varepsilon$ defined on $G_j \cap M_j$

first-order methods apply

Burden *et al.* SysID 2012; Burden *et al.* SICB 2013

Design and optimize gaits and maneuvers for robots

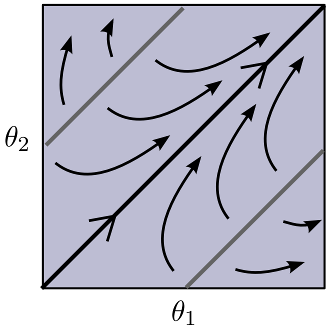
RHex robot

video courtesy of KodLab, http://kodlab.seas.upenn.edu/

Exploit hybrid transitions for robust stability of gaits

smooth leg coordination

$T^2 = S^1 \times S^1$



Burden *et al.* (in preparation)

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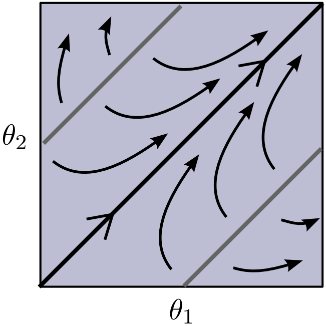

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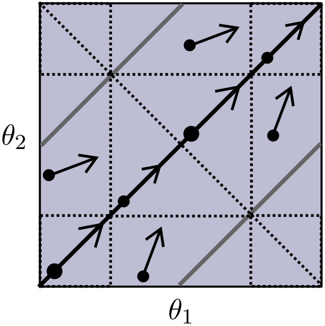

Burden *et al.* (in preparation)

Discussion & Questions — Thanks for your time!

Reduction

Hybrid dynamics generically reduce dimensionality near a periodic orbit.

Robustness

Simultaneous hybrid transitions can lend robust stability to a periodic orbit.

Collaborators

- Prof. Shankar Sastry
- Prof. Dan Koditschek
- Prof. Shai Revzen
- Prof. Robert Full
- Prof. Henrik Ohlsson
- Prof. Aaron Hoover
- Talia Moore
- Justin Starr
- Mike Choi