

Exercise 5-1: Use the Laplace transform to solve the following ODEs:

(a) $\ddot{x} + 3\dot{x} + 2x = u(t)$

(b) $\ddot{x} + 2\dot{x} + 2x = u(t)$

Solve each of these for the following initial conditions and forcing functions (hint: use the Laplace transforms from above so simplify the expression in the frequency domain. I would not recommend using convolution, if you can avoid it):

(i) Step response: A step input (i.e., $u(t)$ is a Heaviside function) with zero initial conditions

(ii) Impulse response: An impulsive input (i.e., $u(t)$ is a Delta function) with zero initial conditions

[OPTIONAL] For each case, plot your solution and also plot the solution using the `step` and `impulse` commands in Matlab or Python.

(iii) Initial condition response: $u(t) = 0$ with initial conditions $x(0) = 1$ and $\dot{x}(0) = 0$.

(iv) Initial condition response: $u(t) = 0$ with initial conditions $x(0) = 0$ and $\dot{x}(0) = 1$.

Summary: Solve both equations (a) and (b) using the forcing and initial conditions from (i)-(iv).

Exercise 5-2: [OPTIONAL] Compute the following Laplace transforms by hand:

(a) Delta function: $f(t) = \delta(t)$ (remember to integrate from 0^- to ∞)

(b) Heaviside function: $f(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$

(c) Time-shifted delta function: $f(t) = \delta(t - a)$.
