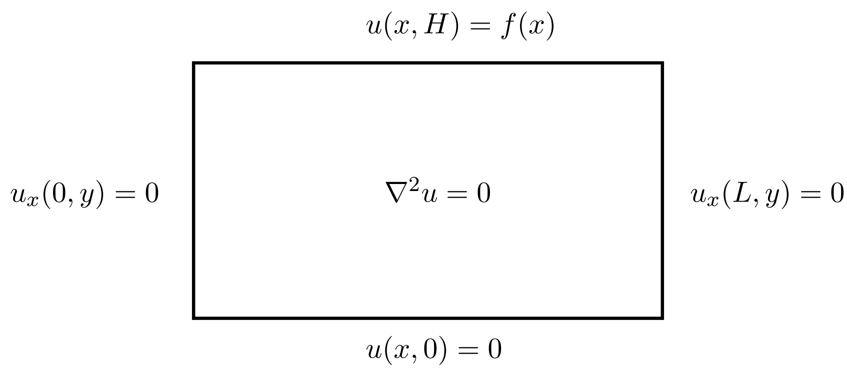
ME565 - Winter 2023 **Due: 5:00 pm Friday February 17^{th}, 2020**
Homework 3

Exercise 3–1: Solve for the equilibrium temperature distribution using the 2D Laplace equation on an $L \times H$ sized rectangular domain with the following boundary conditions:

1. Left: $u_x(0, y) = 0$ (insulating)
2. Bottom $u(x, 0) = 0$ (fixed temperature)
3. Top: $u(x, H) = f(x)$ (zero temperature)
4. Right: $u_x(L, y) = 0$ (insulating)

Solve for a general boundary temperature $f(x)$. Also solve for a particular temperature distribution $f(x)$; you may choose any non-constant distribution you like.



How would this change if the left and right boundaries were fixed at zero temperature? (you don't have to solve this new problem, just explain in words what would change)

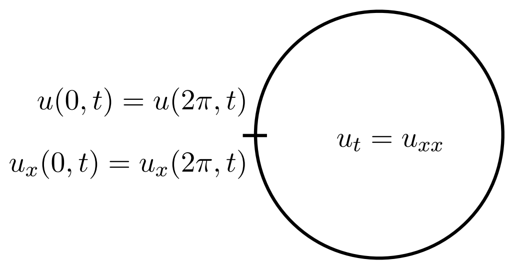
Exercise 3–2: Heat conduction in a thin circular ring is described by the following PDE

$$u_t = u_{xx}, \text{ for } t \geq 0. \tag{1}$$

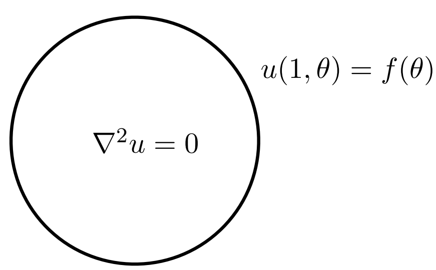
with periodic boundary conditions in space ($x \in [0, 2\pi]$):

$$u(0,t) = u(2\pi,t), u_x(0,t) = u_x(2\pi,t), \text{ etc.} \tag{2}$$

Solve using the separation of variables, i.e. $u(x,t) = X(x)T(t)$. Find a general solution (i.e. for arbitrary initial condition $u(x,0) = f(x)$) and describe the asymptotic behavior of *all* solutions as $t \to \infty$. Provide a physical interpretation of this behavior.



Exercise 3–3: Solve the 2D Laplace's equation in polar coordinates for the equilibrium temperature $u(r,\theta)$ in a circular disk of radius $r = 1$ with the following boundary conditions: $u(1,\theta) = f(\theta)$ (prescribed temperature). Hint: use separation of variables!



Laplace's equation in polar coordinates is given by:

$$\nabla^2 u = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0.$$
