Exercise 2-1 Solve the one-dimensional Laplace equation for the equilibrium temperature distribution in a finite rod for the following boundary conditions:

- 1. $u_x(0) = 0$ and u(L) = B,
- 2. u(0) = B and $u_x(L) = -a$,
- 3. $u(0) + u_x(0) = 0$ and u(L) = B.

Please also draw a quick sketch for each boundary condition. If you need an initial condition for any reason, you may assume that u(x, 0) = f(x).

Exercise 2-2 Solve the modified Laplace equation with a source term (Poisson's equation!):

$$u_{xx} = -Q(x),\tag{1}$$

for $Q = x^2$. Use fixed temperature boundary conditions so that u(0) = 0 and u(L) = 0.

What can you say about the heat flux at x = 0 and x = L?

Plot the temperature distribution for L = 10.

Exercise 2-3 Please solve for the equilibrium temperature distribution of the following heat equation, $u_t = u_{xx} + 2$ (i.e. with Q = 2), with the following initial and boundary conditions: u(x,0) = f(x), $u_x(0,t) = a$, and $u_x(L,t) = b$. Does this distribution exist for all values b? If not, then what are the values of b that have solutions?

Exercise 2-4 Derive the 3D Laplacian operator in cylindrical coordinates (R, θ, z) , starting from the Laplacian in Cartesian coordinates (x, y, z).