ME565 - Winter 2021 **Due: 5:00 pm Friday January 29^{th}, 2021**
Homework 2

Exercise 2-1 Solve the one-dimensional Laplace equation for the equilibrium temperature distribution in a finite rod for the following boundary conditions:

1. $u(0) = A$ and $u_x(L) = 0$,
2. $u_x(0) = a$ and $u(L) = B$,
3. $u(0) + u_x(0) = 0$ and $u(L) = B$.

Please also draw a quick sketch for each boundary condition. If you need an initial condition for any reason, you may assume that $u(x, 0) = f(x)$.

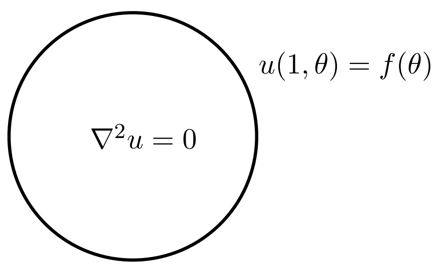
Exercise 2-2 Solve the modified Laplace equation with a source term (Poisson's equation!):

$$u_{xx} = -Q(x), \tag{1}$$

for $Q = x$. Use fixed temperature boundary conditions so that $u(0) = 0$ and $u(L) = 0$.

What can you say about the heat flux at $x = 0$ and $x = L$?

Exercise 2–3: Solve the 2D Laplace's equation in polar coordinates for the equilibrium temperature $u(r, \theta)$ in a circular disk of radius $r = 1$ with the following boundary conditions: $u(1, \theta) = f(\theta)$ (prescribed temperature). Hint: use separation of variables!



Laplace's equation in polar coordinates is given by:

$$\nabla^2 u = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0.$$
