

L24: Nov. 19, 2014

ME 564, Fall 2014

Overview of Topics

- ① Directional derivative
- ② Continuity Eqⁿ and $\nabla \cdot \vec{v} = 0$
- ③ Examples of vector fields \vec{v}
and what we can do with them.

The gradient can be used to take a directional derivative of a function f in the direction \vec{v} :

$$D_{\vec{v}} f = \frac{1}{\|\vec{v}\|} \vec{v} \cdot \nabla f$$

$$\frac{d}{dt} \rho + \nabla \cdot (\rho \vec{v})$$

$$\frac{\partial}{\partial x} \rho v_1 + \frac{\partial}{\partial y} \rho v_2 + \frac{\partial}{\partial z} \rho v_3$$

$$= \rho_x v_1 + \rho v_{1x} + \rho_y v_2 + \rho v_{2y} + \rho_z v_3 + \rho v_{3z}$$

$$= \nabla \rho \cdot \vec{v} + \rho \nabla \cdot \vec{v}$$

If ρ constant everywhere (I.E., like water...)

$$\frac{d}{dt} \rho = 0 \text{ and } \nabla \rho = \vec{0}.$$

$$\frac{d}{dt} \rho + \nabla \cdot (\rho \vec{v}) = 0 \implies \rho \nabla \cdot \vec{v} = 0$$

$$\implies \boxed{\nabla \cdot \vec{v} = 0}$$

incompressible!!!

Examples of Vector Fields \vec{V}

① \vec{V} is often the solution to a partial differential equation (PDE)

A. The PDE is essentially a constraint equation that enforces basic physical laws (such as conservation of mass, momentum, or energy).

B. Deriving a PDE from conservation laws typically involves using vector integral identities (Gauss, Green, Stokes) to relate path, surface, and volume integrals.

C. Solving the PDE may be difficult ... ME565

② Given a vector field \vec{V} , we may analyze what happens to particles \underline{x} in this field:

$$\dot{\underline{x}} = \vec{V}(\underline{x}, t) \quad (\text{note } \vec{V} \text{ varies in space and time})$$

induced ODE

$$\underline{x}(t) = \underline{x}_0 + \int_0^t \vec{V}(\underline{x}(\tau), \tau) d\tau$$

