: Oct. 13, 2014 ME 564, Fall 2014

Overview of Topics:

① Examples of $\dot{\underline{x}} = \underline{\underline{A}}\,\underline{x}$ (w/ eigenvalues & eigenvectors)

- unstable node : $\lambda_1 = 2$, $\lambda_2 = 4$
- Saddle point : $\lambda_{1,2} = \pm 1$

L09

- center : $\lambda_{1,2} = \pm i$
- stable spiral

② Phase portraits

L09

③ Linearization of nonlinear systems

- $\ddot{x} = -\dfrac{\partial V}{\partial x}$ where $V(x)$ is potential

Example

$$A = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}, \quad T = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad D = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$$

to compute T^{-1}:

$$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 1 & -1 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & -2 & -1 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cc|cc} 1 & 0 & .5 & .5 \\ 0 & -2 & -1 & 1 \end{array}\right]$$

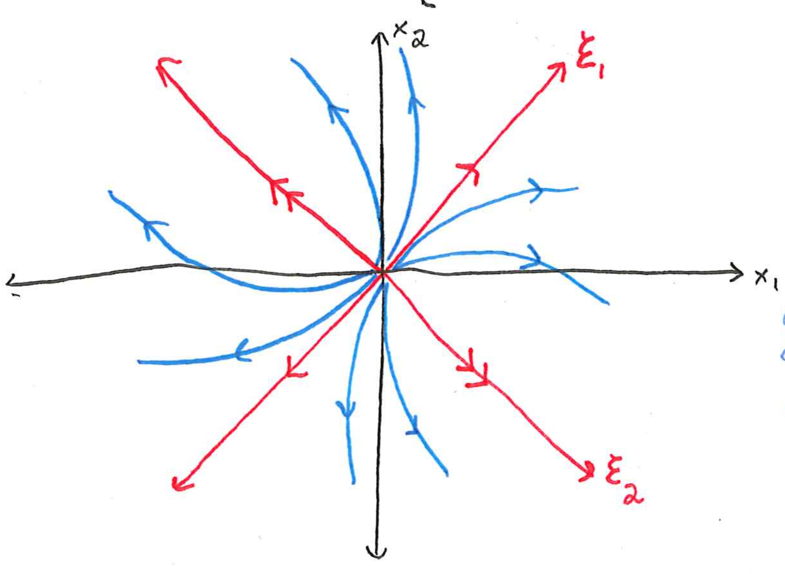
$$\left[\begin{array}{cc|cc} 1 & 0 & .5 & .5 \\ 0 & 1 & .5 & -.5 \end{array}\right] \Longrightarrow T^{-1} = \begin{bmatrix} .5 & .5 \\ .5 & -.5 \end{bmatrix}$$

In Matlab, check $T^{-1}AT = D$

$$\underline{x}(t) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} e^{2t} & 0 \\ 0 & e^{4t} \end{bmatrix} \begin{bmatrix} .5 & .5 \\ .5 & -.5 \end{bmatrix} \underline{X}(0)$$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} .5e^{2t} & .5e^{2t} \\ .5e^{4t} & -.5e^{4t} \end{bmatrix} \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} e^{2t}+e^{4t} & e^{2t}-e^{4t} \\ e^{2t}-e^{4t} & e^{2t}+e^{4t} \end{bmatrix} \begin{bmatrix} X_1(0) \\ X_2(0) \end{bmatrix}$$

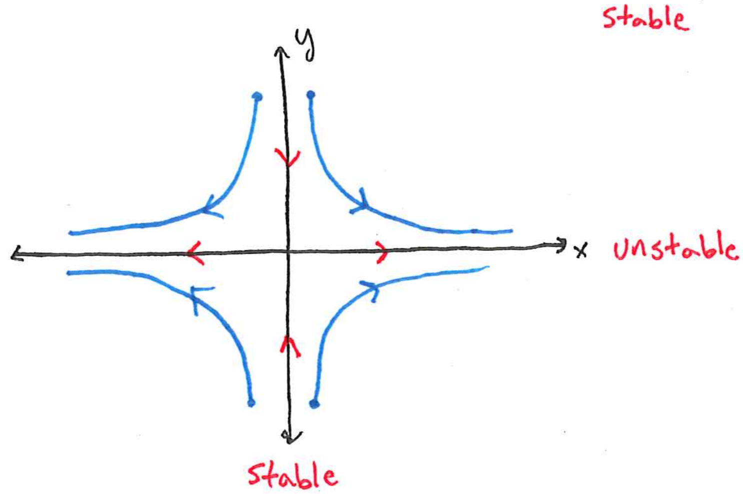


Try "pplane" in Matlab

Unstable source (or node)

Example: $\frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} \overbrace{e^{t}}^{\text{unstable}} & 0 \\ 0 & \underbrace{e^{-t}}_{\text{stable}} \end{bmatrix}\begin{bmatrix} x(0) \\ y(0) \end{bmatrix}$

Already diagonal!



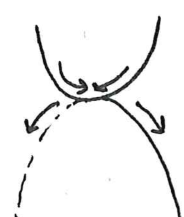
Saddle point

Examples of Saddle Points

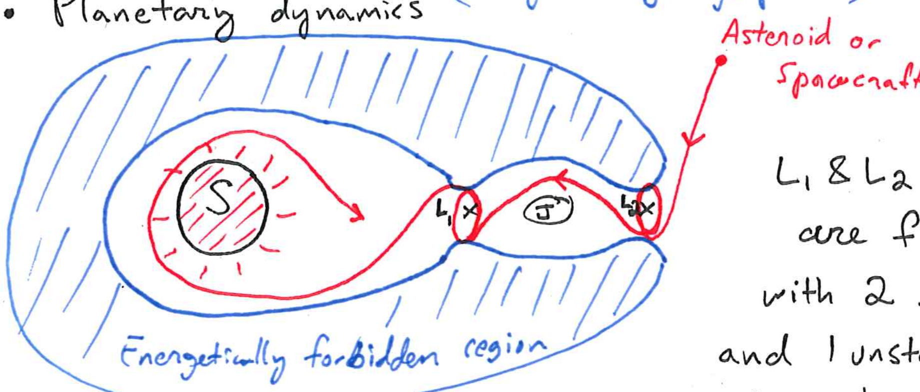
- Person walking  -arms = stable direction
  -body = unstable (inverted pendulum)

  -efficient locomotion

- Drop a bead on a saddle:



- Planetary dynamics (Google: Lagrange points)



L_1 & L_2 are fixed points with 2 stable and 1 unstable eigenvalue.

James Webb space telescope will orbit L_2 (efficient maneuvering).