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**Exercise 5-1** Express the following in the form  $a + bi$  (for real  $a$  and  $b$ ) and also in the form  $Re^{i\theta}$  (for real  $R$  and  $\theta$ ):

- (a)  $\frac{1}{4-3i}$
- (b)  $\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)^4$
- (c)  $i^2, i^3, i^4, i^5, \dots$

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**Exercise 5-2** Find all solutions of

- (a)  $e^z = i$
- (b)  $e^z = -1$

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**Exercise 5-3** Find all solutions of

- (a)  $z^4 = 1$
- (b)  $z^2 = 4i$
- (d)  $z^2 = 1 - i$

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**Exercise 5-4** Verify the following functions  $f(z)$  are analytic for all  $z = x + iy$ . Use the Cauchy-Riemann conditions (Hint: find a way to express these functions as  $f(z) = u(x, y) + iv(x, y)$ ).

- (a)  $f(z) = e^z$
- (b)  $f(z) = \cos(z)$

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**Exercise 5-5** Find all analytic functions  $f = u + iv$  with  $u(x, y) = 2xy$ . Simplify the expression  $f(z)$  as much as possible.

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