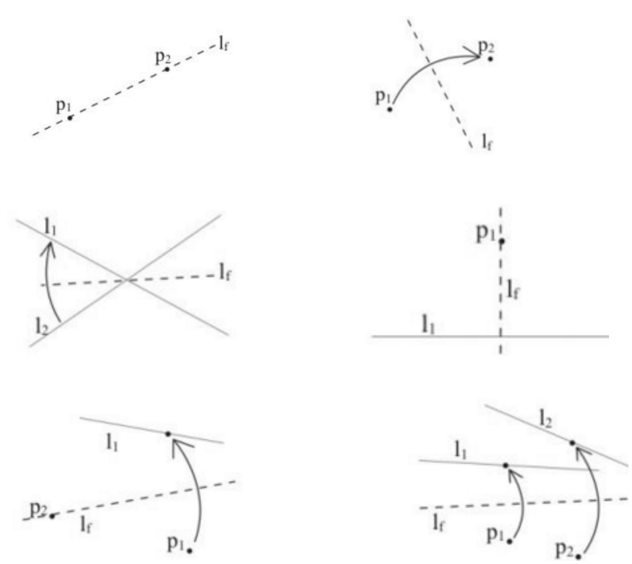
This section is to be taken home, completed, and turned in by 5:00pm Wednesday June 5th. There is no time limit and you do not need to type up your solutions to get full marks although the answers should be well edited and readable.

You may discuss this problem with anyone else from the class and use the class resources posted on Canvas. You may not consult anyone or any resource that is not affiliated with the class such as tutors, websites, or other textbooks.

Consider two points on a piece of paper, P_0 and P_1 with the distance between them defined as 1. We can fold a line that passes through P_0 and P_1 to create an x axis. By folding the line on top of itself and sliding the paper until the new fold passes through P_0 we can create a perpendicular line through P_0 that gives us a y axis. Using only paper folding (which corresponds to the following six axioms) we can identify points on the plan from intersecting folds. We call the set of all possible points on the plan that can be obtained in a finite number of folds, Origami-constructible numbers.

1. Given two points p_1 and p_2 we can fold a line connecting them.
2. Given two points p_1 and p_2 we can fold p_1 onto p_2.
3. Given two lines l_1, and l_2, we can fold line l_1 onto l_2.
4. Given as point p_1 and a line l_1, we can make a fold perpendicular to l_1 passing through the point p_1.
5. Given two points p_1 and p_2 and a line l_1, we can make a fold that places p_1 onto l_1 and passes through the point p_2.
6. (Beloch's Fold) Given two points p_1 and p_2 and two lines l_1 and l_2, we can make a fold that places p_1 onto line l_1 and places p_2 onto line l_2.



1. [2] Verify Origami-constructible numbers form a field. (Note the addition and multiplication defined for constructible numbers will be of use here but we need to verify paper folding is a strong enough tool! Do not worry about verifying distribution.)

2. [1] Origami-constructible numbers are larger than $\mathbb{Q}$. Identify a subfield of Origami-constructible numbers that contains $\mathbb{Q}$.

3. [2] Identify all subfields of $F = \mathbb{Q}(\sqrt{3}, \sqrt{5})$ that are extensions of $\mathbb{Q}$ and arrange these in a lattice.

4. Let $F = \mathbb{Q}(\sqrt{3}, \sqrt{5})$. We write an element of F with the basis $\{1, \sqrt{3}, \sqrt{5}, \sqrt{15}\}$. Define $\tau : F \to F$ by $\tau(a+b\sqrt{3}+c\sqrt{5}+d\sqrt{15}) = a-b\sqrt{3}+c\sqrt{5}-d\sqrt{15}$ for $a, b, c, d \in \mathbb{Q}$. Define $\sigma : F \to F$ by $\sigma(a+b\sqrt{3}+c\sqrt{5}+d\sqrt{15}) = a+b\sqrt{3}-c\sqrt{5}-d\sqrt{15}$ for $a, b, c, d \in \mathbb{Q}$.

   (a) [4] Verify τ is a field isomorphism.

   (b) [3] Verify the set of isomorphisms from F to F forms a group generated by σ and τ. Provide a Cayley table or Cayley Diagram.

   (c) [2] Create a subgroup lattice for the group above.

   (d) [1] Compare the lattice of (c) to the lattice (3).