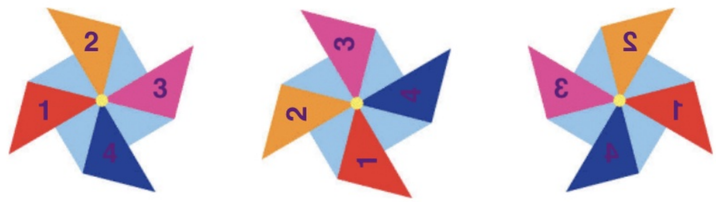


Figure 1: Original state, rotation by 90°, not a symmetry

Pinwheels

Our goal is to capture a certain kind of symmetry of a pinwheel. In particular, we'd like to describe all possible ways that the pinwheel position could be altered though movement and returned to the same "footprint". To be clear, we are not interested in translations, glide reflections, or flips around the vertical line through the center as the pinwheel would have a different "footprint". However, rotating the pinwheel counterclockwise by 90° (denote this by R_{90}) would work as this results in the same "footprint".

1. Find all possible ways that a pinwheel could be repositioned and returned to the same "footprint". Create notation for these actions, similar to what was done for R_{90}.

   Several 'footprints' for the pinwheel are provided below. Note numbers can be used for labels if you do not have colors!

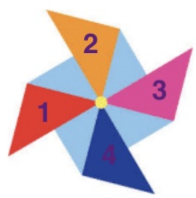
   

2. Can we use (refocus? reorganize? collect together?) your above work to consider actions that result in distinct *configurations* of the pinwheel (instead of *all* re-positionings)? (What mathematical concept from TMath 300 are you using?)

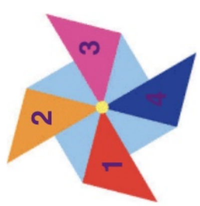
3. Now that we are focused on configurations (instead of all movements), let's consider 'composing' movements, that is, how these actions behave when applied sequentially. We will try to capture this visually in a graph called a Cayley graph. (Named after British mathematician Arthur Cayley 1821-1895.)

   First we place all the configurations of the pinwheel on the page to act as our vertices. Second we add edges between the configurations that correspond to the actions found above. For example, two configurations are provided along with the name of the action that moves one to the other.

   Complete the Cayley graph below.



$\downarrow [R_{90}]$



4. Can you simplify $R_{90} \circ R_{90} \circ R_{90} \circ R_{90} \circ R_{90} \circ R_{90} \circ R_{90} \circ R_{90}$? How?