Take Home Midterm | # TMath 402 | Autumn 2022

This section is to be taken home, completed, and turned in through Canvas by 8:00pm Tuesday Nov 1st. There is no time limit and you do not need to type up your solutions to get full marks although the answers should be well edited and readable.

You may discuss this problem with anyone else from the class and use the class resources posted on Canvas. You may not consult anyone or any resource that is not affiliated with the class such as tutors, websites, or other textbooks.

1. Consider the group G in $\mathbb{C}^*$ with the generators:
   $\omega = \cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)$ and $\mu = \cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)$.

   (a) [2] Is G cyclic? Provide proof.

   (b) [2] Provide either a Cayley Table or a Cayley Diagram for the group G. You do not need to use the generators given above if you do not want but please carefully identify any elements!

   (c) [2] Find all the non-trivial proper subgroups.

2. Consider Dr. Vanderpool's proof written for the theorem below.

   (a) [2] Identify any logical errors.

   (b) [2] Provide concrete suggestions for how the writing and organization of the proof can be improved.

   Theorem 1 (3.4)**.** *Let $\mathbb{Z}_n$ be the set of all equivalences classes of the integers* $\mod n$*. Let a and b be elements of $\mathbb{Z}_n$. If $a \neq 0$, then* $\gcd(a, n) = 1$ *if and only if there exists a b such that $ab \equiv 1 \mod n$.*

   Proof. Assume $ab \equiv 1 \mod n$. Then $ab - kn = 1$. Recall $\gcd(a, n)$ divides a which means that $\gcd(a, n)$ divides ab, similarly we know $\gcd(a, n)$ divides kn, thus $\gcd(a, n)$ divides $ab - kn$ which equals 1. The only number that divides 1 is 1, so $\gcd(a, n) = 1$. If $\gcd(a, n) = 1$ then we need to show that $ab \equiv 1 \mod n$. Our assumption means there is a k such that $ab = 1 + kn$ which we can reduce $\mod n$ to get $ab \equiv 1 \mod n$. □