The GCD & LCD

Definition 1. *Let a and b be integers, not both zero. The largest integer d such that $d|a$ and $d|b$ is called the* greatest common divisor *of a and b. The greatest common divisor of a and b is denoted by* $\gcd(a, b)$.

Definition 2. *The* least common multiple *of the positive integers a and b is the smallest positive integer that is divisible by both a and b. The least common multiple of a and b is denoted by* $\text{lcm}(a, b)$.

1. For each a and b given below, find $\gcd(a, b)$ and $\text{lcm}(a, b)$.

   (a) $a = 24$ and $b = 36$

   (b) $a = 17$ and $b = 22$

   (c) $a = 2^3 \cdot 3 \cdot 5$ and $b = 2^2 \cdot 5$.

2. In general, if you have two positive integers n and m with prime decompositions

$$n = p_1^{r_1} \cdot p_2^{r_2} \cdot ...p_n^{r_n} \qquad\qquad m = p_1^{s_1} \cdot p_2^{s_2} \cdot ...p_n^{s_n}$$

   where each p_i is a distinct prime and the exponents are greater than or equal to zero. Write down the prime decomposition for

   (a) the $\gcd(n, m)$

   (b) the $\text{lcm}(n, m)$

Check your answers to questions 1 by consulting §4.3 example 10, 11, & #14.