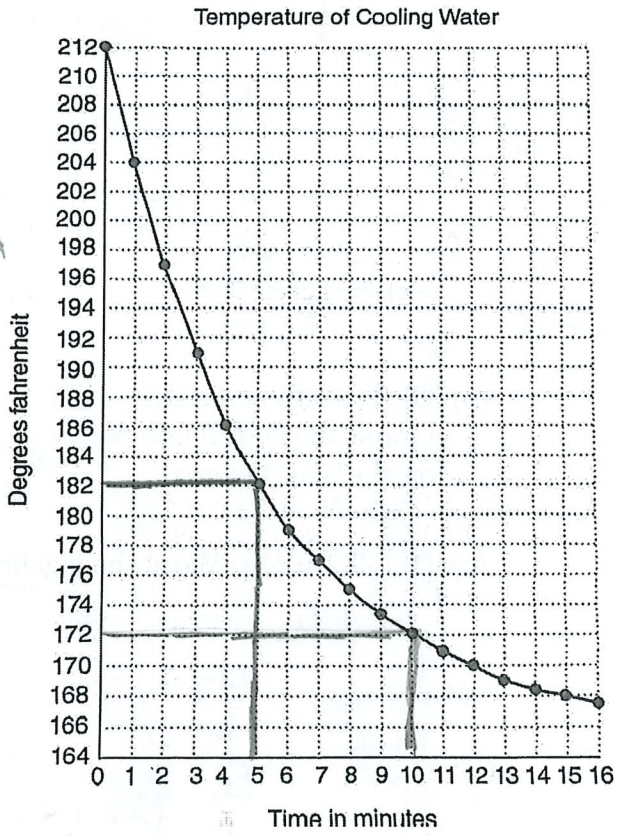
Key

Final **TMath 171** Autumn 2024

As a reminder, you are welcome to use a non-internet accessing calculator (which includes Desmos Test Mode) and one 2-sided 8.5 in by 11 in sheet of notes. Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

1. Fourth graders conducted an experiment to observe the rate at which water cools. They placed a thermometer in a beaker of water and heated the water to boiling. They recorded the water temperature every minute until the temperature dropped to just below 168° F. Then they plotted the results on the grid like the one below.



(a) [2] (Suggested §9.3 #5) Does the rule described above and graphed to the right determine a function or not? Briefly justify your answer.

reason +1
sense/complete +1
+1 Yes. Each minute provides at most one temperature/output OR Vertical line test

(b) [2] (Function2Activity #2) What is the temperature of the water, 10 minutes after boiling?

+1 172°F

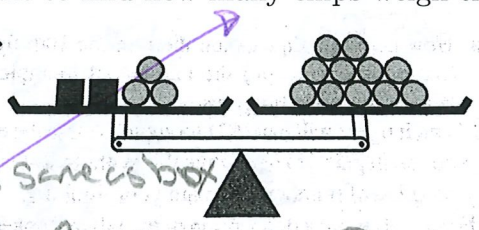
graph reading +1

(c) [2] (Function2Activity #2) Approximately how many minutes does it take for water to cool from 212° F to 182° F?

about 5 min +1

graph reading +1

2. Consider the balanced scales below. We want to find how many chips weigh the same as a box.



(a) [3] (Quiz8 #3) Define variables and use the variables in an equation that corresponds to the scale shown.

+1 let x = # of chips that weigh same as box

notation +1 ~~sketch +1~~ involves +1

$2x+3=11$ +1

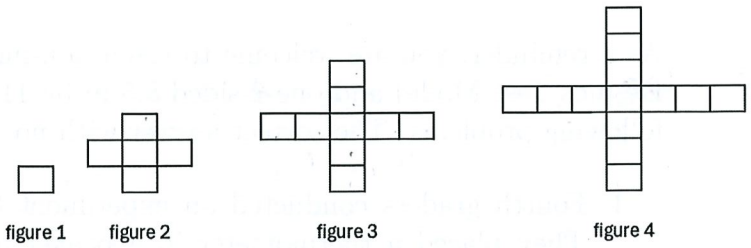
(b) [3] (§9.1 #6) Use the scales or the equation to find how many chips weigh the same as a box. Write enough that your work can be easily followed by a peer.

$2x+3=11$
$-3 \quad -3$
$2x=8$

$\frac{2x}{2}=\frac{8}{2}$
$x=4$

same to each side +1
order of ops +1
alg/get it +1

3. Ksenija is making a block pattern of windmills. The first four are shown below.



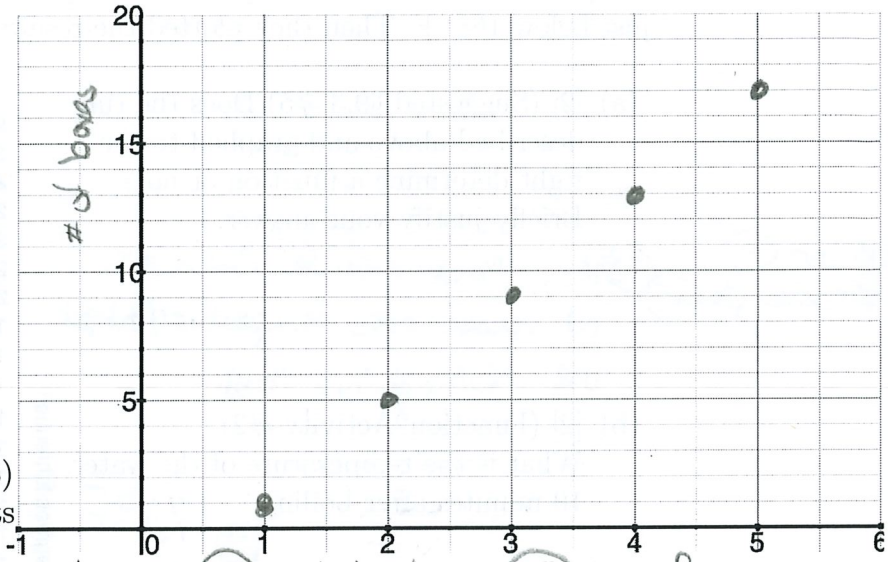
(a) [3] (§9.2 #24a) Create an input-output table with the figure number and the number of boxes in each figure for the first 5 figures.

+1.5 table/input+output
+1.5 notation/sense

figure # input	# of boxes output
1	1
2	5
3	9
4	13
5	17

+1 +1 +1 +1 +1

(b) [2] (CoordLinesActivity #3) Graph the coordinate points on the axes below.



get all of them +1.5 x/y coord +1 at least one +1.5

(c) [4] (§9.2 #24b) Write the algebraic rule for the number of boxes for the sequence.

Looks like a line: $y = mx + b$ +1

m = slope $= \frac{\Delta y}{\Delta x}$ +1.5 $= \frac{\text{rise}}{\text{run}} = \frac{4}{1}$ +1.5

so $y = 4x + b$

to find b use (1,1)

$1 = 4(1) + b$ +1

$-4 \quad -4$

$-3 = b$ +1.5

So $y = 4x - 3$ +1.5 plug in

(d) [3] (Suggested §9.1 #1) Which figure will have 93 boxes? Provide work so that your process can be followed.

ie find x when $y = 93$ +1

$4x - 3 = 93$

$+3 \quad +3$

$\frac{4x}{4} = \frac{96}{4}$

$x = 24$

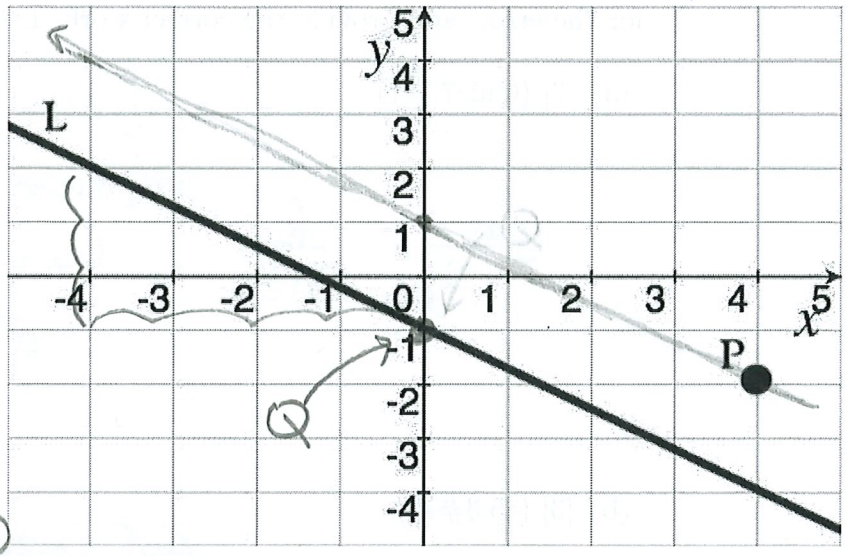
something to each side +1

order of operations +1.5

notation +1.5

12

4. Consider the coordinate axes below.



(a) [2] (CoordLineActivity #1) Write down the Cartesian coordinates for P.

(4, -2)

(b) [2] (CoordLineActivity #1) Label the point Q with coordinates $(0, -1)$ on the Cartesian plane.

(c) [2] (§9.2 #14b) Find the slope of the line L.

$m = \frac{\text{rise}}{\text{run}} = \frac{-3}{4}$

(d) [2] (§5.2 #20a) Find another fraction equivalent to your answer in part (c).

$\frac{-6}{8}$, $\frac{9}{-12}$, etc

(e) [2] (Suggested §9.2 #15a) Sketch a line on the graph that is parallel to L and passes through point P.

5. [6] Let a and b be non-zero whole numbers. Are the following statement always true, sometimes true, or never true? Briefly justify your answer (consider giving examples!).

(a) (§4.2 #10a & Suggested #11) The GCF(a, b) is a prime number.

Sometimes true

eg GCF(12,9) = 3 (which is prime)

12 = 2 · 2 · 3, 9 = 3 · 3

bc GCF(20,70) = 10 (which is not prime)

20 = 4 · 5 = 2 · 2 · 5, 70 = 2 · 5 · 7

(b) (FractionActivity #5) A rational number $\frac{a}{b}$ is in reduced form if LCM$(a, b) = 1$.

Never true?

Note that $\frac{1}{2}$ is in reduced form but LCM(1,2) = 2 ≠ 1

In fact the only way LCM(a,b) = 1 is if a = 1 = b so we'd be looking at $\frac{1}{1}$ which is not reduced b/c $\frac{1}{1} = 1$

I bet the mix up was $\frac{a}{b}$ is reduced if LCM(a,b) = ab.

16

24
26
50

6. The work for the problems below are wrong. Find the error(s), try to detect the reason for the error, and provide the correct work. Let a be a non-zero whole number.

(a) [3] (Quiz7 #1)

$$\frac{6}{9+3a} = \frac{2}{3+3a}$$

+1 [Error: need to factor 3 at of entire denominator
+.5 [Reason: Only looked at 1st term of den.

Correct work:

$$\frac{6}{9+3a} = \frac{3\cdot 2}{3\cdot 3+3a} = \frac{3\cdot 2}{3(3+a)} = \frac{2}{3+a}$$

+1 +.5

(b) [3] (§5.3#18)

$$\frac{7}{a} + \frac{1}{4} = \frac{8}{a+4}$$

+1 [Error: added straight across
+.5 [Reason: did not understand why common denominators are needed

Correct work:

$$\frac{7}{a} + \frac{1}{4} = \frac{7(4)}{a(4)} + \frac{(a)\cdot 1}{(a)4} = \frac{28+a}{4a}$$

+.5

Commanden +1

(c) [3] (FracAddMultActivity #3)

$$\frac{2}{5a^2} \div \frac{4}{a} = \frac{2}{5aa} \times \frac{a}{4} = \frac{2a}{5aa\cdot 4} = \frac{2}{5a\cdot 2} = 5a\cdot 2 = 10a$$

(keep change flip)

+1 [Error: brought denominator up to the numerator
+.5 Reason: likely forgot 1 remains in numerator

Correct work:

$$\frac{2}{5a\cdot 2\cdot a} = \frac{1}{5\cdot a\cdot 2} = \frac{1}{10a}$$

+1

Notation +.5

7. [1] What topic or concept did you study and prepare for, but not see on the exam?

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