notes: it is important to pay attention to the function (there are alot around) #1, #3

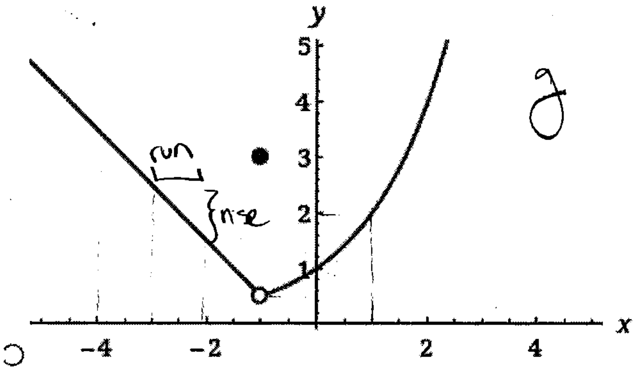
Median 66% Key

EXAM 1 **TMath 124** Spring 2024 ~~75pts~~

note: fraction $+, -, \div, \times$ is worth reviewing

Show *all* your work (numerically, algebraically, or geometrically) for the following problems. Supporting work is needed to earn credit.

1. Let $f(x) = \frac{5-2x}{x-2}$.
   The graph of g is given on the right. Estimate:



   (a) [1] (LimitActivity#1) $f(-1)$.

   computation (+.5)
   plug in f (+.5)

   $f(-1) = \frac{5-2(-1)}{(-1)-2} = \frac{5+2}{-3} = \frac{-7}{3}$

   (b) [2] (Quiz1#1) $g(-1) + \lim_{x\to -1} g(x)$

   evaluate individually (+.5)

   $3 + \frac{1}{2} = 3.5$ (+.5) (+1)

   (c) [2] (WebHW4#10) $\lim_{x\to\infty} f(x)$

   inf limit def (+1)
   got it (+.5) notation (+.5)

   $\lim_{x\to\infty} \frac{5-2x}{x-2}$

x	100	10000
$f(x)$	-1.9898	-1.9999

   -2

   (d) [3] (§2.3#2) $\lim_{x\to 1}(f(x)g(x)) = \lim_{x\to 1} f(x) \cdot \lim_{x\to 1} g(x)$ (+.5)

   $\left(\frac{5-2(1)}{1-2}\right) \cdot 2 = \frac{3}{-1} \cdot 2 = -6$ (+1) (+1) notation (+.5)

   (e) [1] (§2.5 #20) Where f is *not* continuous.

   looking at graph: pick up pencil @ $x = 2$ (+.5) (+.5)

   (f) [2] (Quiz2#1) $g'(-3)$

   $=$ Slope of line tang. to g when $x = -3$ (+.5)

   $= \frac{\text{rise}}{\text{run}} \approx \frac{-1}{1} = -1$ (+.5) (+.5) graph reading (+.5)

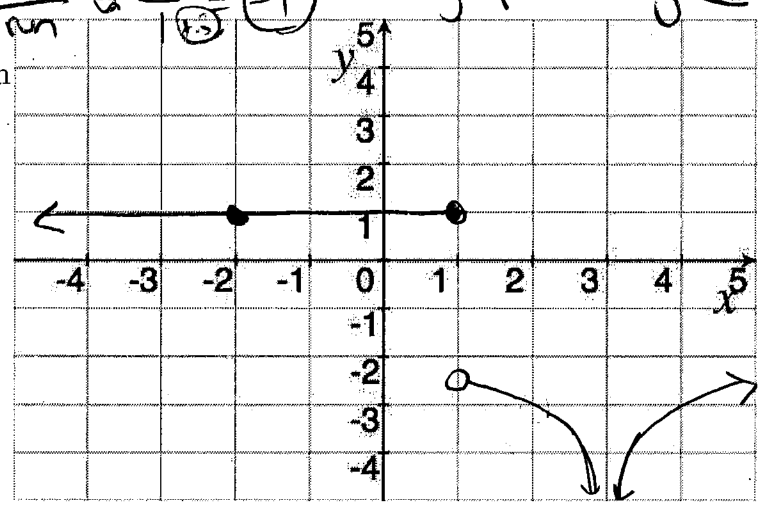
2. [5] (Quiz1#2) Draw one graph for a function $\alpha(x)$, that satisfies all of the following: (+1)

   (+1) (a) $\lim_{x\to 3} \alpha(x) = -\infty$,

   (+1) (b) α is not continuous when $x = 1$,

   (+1) (c) $\alpha(-2) = 1$, and ✓

   (+1) (d) $\lim_{x\to 2^+} \alpha(x) = -3$. ✓



note: there are LOTS of correct answers

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3. [4] (Practice Exam#8) Let $f(x) = x^2 - 5$. Find the limit (either numerically, graphically, or algebraically), if it exists, of $\lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$

notation (+.5)

$\lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$ (+.5) (+.5)

$= \lim_{h\to 0} \frac{[(1+h)^2-5]-[1^2-5]}{h}$ dist (+.5)

$= \lim_{h\to 0} \frac{\cancel{1}+2h+h^2-\cancel{5}+\cancel{4}}{h}$

algebra (+.5)

$= \lim_{h\to 0} \frac{h^2+2h}{h}$ (+.5) fact/facts

$= \lim_{h\to 0} \frac{h(h+2)}{h}$

$= \lim_{h\to 0} h+2$

$= 0+2$ (+1)

OR

$\lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = f'(1)$ (+1)

power rule (+.5)

(+.5) $\Rightarrow f'(x) = 2x + 0$

$\Rightarrow f'(1) = 2(1) = 2$ (+1)

notation (1)

4. [3] (WebHW4#9) Let $f(x) = x^2 \left(1 - \cos\left(\frac{1}{x}\right)\right)$. Find the limit (either numerically, graphically, or algebraically), if it exists, of $\lim_{x\to 0} f(x)$

$\lim_{x\to 0} x^2\left(1-\cos\left(\frac{1}{x}\right)\right)$

(+.5) $=$ ~~$0\left(1-\cos\left(\frac{1}{0}\right)\right)$~~ ERROR

(+1) (+1)

x	-.01	-.001	.001	.01
f(x)	.0000138	[illegible]	.000000437	.0000138

so zero (+.5)

OR Note $-1 \le \cos(1/x) \le 1$ multiply by -1

$\Rightarrow 1 \ge -\cos(1/x) \ge -1$ add 1

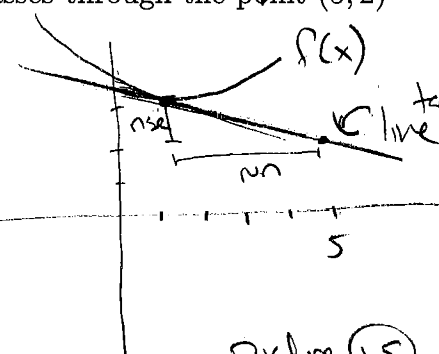
or $2 \ge 1-\cos(1/x) \ge 0$ since $x^2 \ge 0$

we can preserve the inequalities & mult by x^2

$2x^2 \ge x^2(1-\cos(1/x)) \ge 0x^2$

Note $\lim_{x\to 0} 2x^2 = 0 = \lim_{x\to 0} 0x^2$ so by Squeeze (3)

5. [3] (§2.7 #28) If the tangent line to $y = f(x)$ at $(1, 3)$ passes through the point $(5, 2)$ find the following.



(a) $f(1) = 3$ (+.5)

(b) $f'(1) =$ slope of line tang to f @ x=1 (+.5)

$= \frac{\text{rise}}{\text{run}}$ (+.5)

$= \frac{3-2}{1-5} = \frac{1}{-4}$ (+.5) (+.5)

picture (+.5)

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Typo on 6(d) - no hash mark please!

(+1)

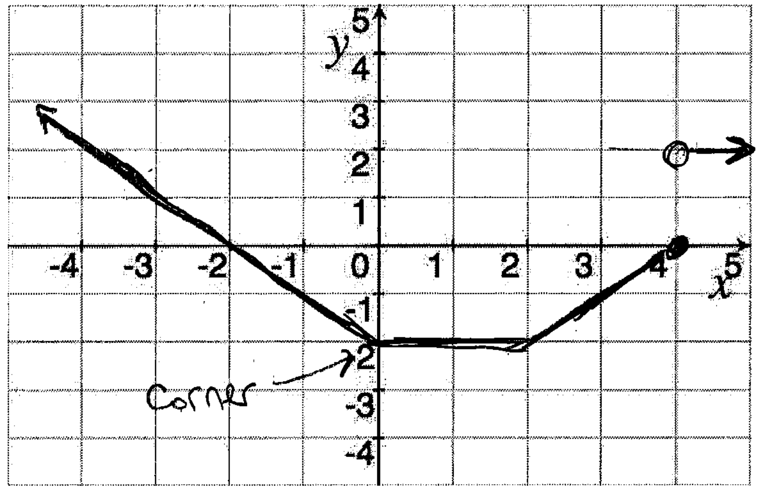
6. [5] (WebHW5#8) Draw one graph for a function $\beta(x)$, that satisfies all of the following:

   (+1) (a) $\lim_{x\to\infty} \beta(x) = 2$, ✓

   (+1) (b) β is continuous on the interval $[-4, 4]$, ✓

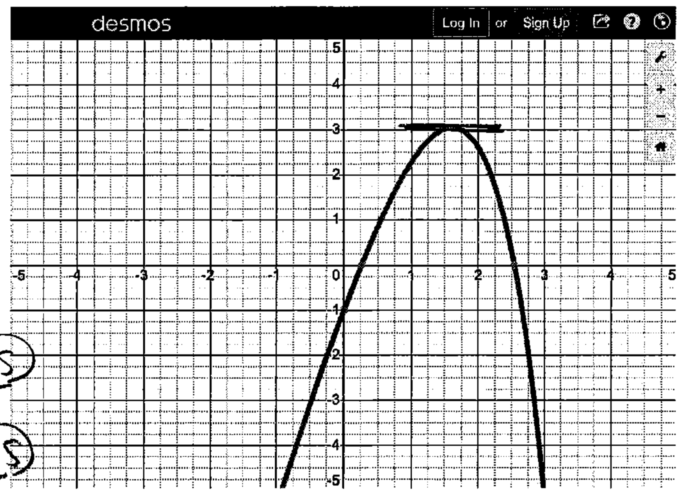
   (+1) (c) $\beta'(0)$ does not exist, and ✓

   (+1) (d) $\frac{d}{dx}\beta|_3 = 1$. ✓



Note: there are LOTS of correct answers!

7. Consider $f(x) = -e^x + 5x$ graphed to the right.



   (a) [3] (WebHW7#9) Find $\frac{df}{dx}$

$$\frac{df}{dx} = \frac{d}{dx}(-e^x + 5x)$$
$$= \frac{d}{dx}(-e^x) + \frac{d}{dx}(5x) \quad (+.5)$$
$$= -\frac{d}{dx}(e^x) + 5\frac{d}{dx}(x) \quad (+.5)$$
$$= -e^x + 5(1)x^0 = -e^x + 5$$
(+.5) (+1)

notation (+.5)

note we could do this algebraically
$\frac{df}{dx} = 0$
$-e^x + 5 = 0$
$\Rightarrow x = \ln(5)$

   (b) [1] (DerivativeActivity#5) Estimate when $f'(x) = 0$ ≈ 1.6 (+.5)

draw horiz line (+.5)

   (c) [4] (ExpActivity#4) Find the equation of the line tangent to f that is parallel to the line $y = \cancel{6}x + 7$ 4

Looking for $y - y_1 = m(x - x_1)$ (+.5)

m = slope of line tang. to f that is parallel to $y = 6x + 7$ $= 4$ (+.5)

need to find the point 's so

(+1) $f'(x_1) = 4$

$-e^{x_1} + 5 = 4$ $\Rightarrow$ $-e^{x_1} = -1$
$\quad -5 \quad -5$

$\rightarrow e^{x_1} = +1$

$x_1 = \ln(+1)$

$x_1 = 0$

solve for x_1 (+1) alg. is plug in pt. to f +.5

so then $(0, f(0)) = (0, -1)$ (+1)

So $y - -1 = 4(x - 0)$ or $y = 4x - 1$ 13

8. (StoryProblems #6) A rock thrown upwards on planet Mars with velocity $15\frac{\text{m}}{\text{s}}$ has a height
$h(t) = 15t - 1.86t^2$ meters t seconds later.

(a) [2] Find a velocity function that describes the velocity of the rock at t seconds.

(b) [2] Recall gravity is the constant acceleration experienced by an object from the planet. Find the gravity on Mars.

(c) [2] *When* does the rock reach its maximum height? Provide evidence.

a) velocity $= \frac{d}{dt}(\text{position})$ (+.5)

$= \frac{d}{dt}(15t - 1.86t^2)$ (+.5) (use h)

$= \frac{d}{dt}(15t) - \frac{d}{dt}(1.86t^2)$ derivative (+1)

$= 15\frac{d}{dt}(t) - 1.86\frac{d}{dt}(t^2) = 15 - 1.86 \cdot 2t = 15 - 3.72t$

b) gravity = acceleration $= \frac{d}{dt}(\text{velocity})$ (+.5)

$= \frac{d}{dt}(15 - 3.72t)$ (+.5) (use part (a))

$= \frac{d}{dt}(15) - \frac{d}{dt}(3.72t)$

$= 0 - 3.72\frac{d}{dt}(t) = -3.72$ derivative (+1)

c) When does the rock reach max height?

graphically: Desmos $f(x) = 15x - 1.86x^2$

max @ 4.032 sec

start (+.5)

sense (+.5)

alg/graph (+.5) precalc method: vertex @ $\frac{-b}{2a} = \frac{-15}{2(-1.86)} = 4.032$ sec

notation (+.5)

↳ max value of height function.

Calculus method: Find when the inst. velocity = 0

from (a) ⇒ $15 - 3.72t = 0$

$+3.72t \quad +3.72t$

$\frac{15}{3.72} = \frac{3.72t}{3.72}$

⇒ $t = 4.032$ sec

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