Exam 2 11am

TMath 120

Winter 2017

NAME: Key

1. [5] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function, and x, y, and z be real numbers with $z \neq 0$.

   T F $\frac{1}{a} + \frac{1}{b} = \frac{2}{a+b}$ $\quad \frac{b}{b}\frac{1}{a} + \frac{1}{b}\frac{a}{a} = \frac{b+a}{ab}$

   T F The degree of $\cancel{7x^5} - 4.56x^4 \cancel{- 7x^5} + 8$ is 4

   T F $2 \cdot 3^x = 6^x$ $\quad$ let $x=2$ then $2 \cdot 3^2 = 2 \cdot 9 = 18$ but $6^2 = 36$

   T F $\log_3 7$ is approximately 1.771 $\quad \log_3(7) = \frac{\log 7}{\log 3}$

   T F 2 is a root of $f(x) = x^4 - 3x^2 - x - 2$

   $2^4 - 3(2)^2 - 2 - 2 = 16 - 12 - 4 = 0$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

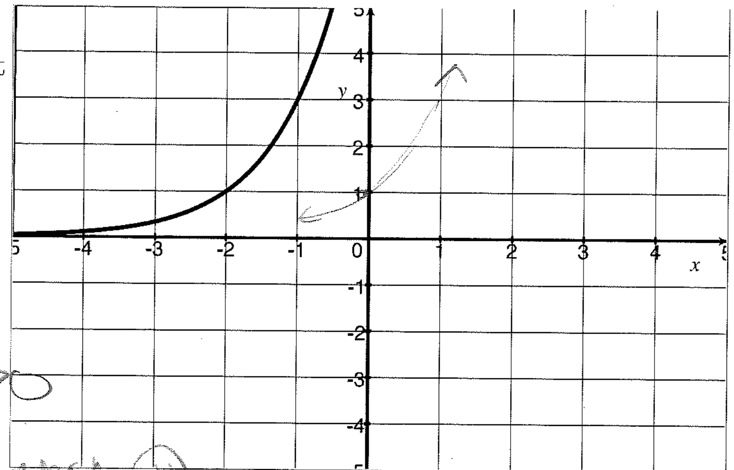
2. [3] Explain what a polynomial function is as you would to a 5th grader.

   We take numbers like 2, $\frac{1}{5}$, $\sqrt{2}$, 8 and multiply them by things like x^{15}, x^2, x, or $x^0 = 1$.
   So perhaps $2x^{15}$, $\frac{1}{5}x^2$, $\sqrt{2}x$, 8. Call each a term.
   Notice the exponents (things in the sky) are only whole numbers greater than or equal to zero.

   A polynomial is several of these terms together added or subtracted.
   So perhaps $2x^{15} - \frac{1}{5}x^2 + \sqrt{2}x + 8$
   or even just one term $\sqrt{2}x$.

start .5
true 1
def/complete 1
sense .5

3. The graph of f shown below is an exponential function that has been shifted horizontally.



(a) [1] Estimate $f(-1)$.

3

(b) [2] Estimate the range of f.

$(0, \infty)$ or $y > 0$

y value (+.5)
notation (+.5) strictly greater than (+1)

(c) [3] (quiz3 #4)
Find the algebraic rule for f.

(+.5) exponential function $y = b^x$
(+.5) Shifted LEFT two units
So $y = b^{x+2}$ (+.5)

(+.5) passes thru $(-1, 3)$ so
$3 = b^{-1+2}$
(+.5) $\Rightarrow 3 = b$
So $y = 3^{x+2}$ (+.5)

4. Simplify the following:

(a) [2] (WebHW10 #6) $2\sqrt{b}(3a^2b)^2$

$2\sqrt{b}\,(3a^2b)(3a^2b)$
$2\sqrt{b} \cdot 9(a^2)^2 b^2$
$2\sqrt{b}\; 9 \cdot a^4 b^2$
$18a^4\sqrt{b}\, b^2$

$x^2 = x^2x^2 = xxxx = x^4$

$\rightarrow 18a^4 b^{1/2} b^2$
$18a^4 b^{5/2}$

dist exp (+.5)
exp to exp (+.5)
sqrt to $1/2$ (+.5)
combine $b^{1/2}b^2$ (+.5)

(b) [2] (PracticeExam #7) $\log\left(\dfrac{10^2 \cdot 10^4}{10}\right)$

$\log\left(\dfrac{10^2 \cdot 10^4}{10}\right)$
$= \log\left(\dfrac{10^6}{10}\right)$
$= \log(10^5)$
$= 5$

$x^2x^4 = (xx)(xxxx) = x^6$

$\dfrac{x^6}{x} = \dfrac{xxxxxx}{x}$

simplify inside w/ exp (+1)
exp & logs cancelling (+1)

$\frac{-4}{15} - 4$

$\frac{4}{15} - \frac{60}{15} = \frac{-56}{15}$

5. [6] Find all real or complex x values in each of the following:

(a) (exponent wks #3) $3x^{-4} + 5 = 4$

$\frac{3}{x^4} + 5 = 4$
$-5 \quad -5$

$x^4 \cdot \frac{3}{x^4} = -1 \cdot x^4$

$3 = -x^4$

$-3 = x^4$

$\sqrt[4]{-3} = x$ or $\pm\sqrt{i}\sqrt[4]{3}$

negative exp (+.5)
did neg. exp correctly (+.5)
algebra/answer (+1)
notation (+.5)

(b) (WebHW13 #7) $\frac{4}{2^{x-1}+4} = 15$

$\cancel{(2^{x-1}+4)} \frac{4}{\cancel{2^{x-1}+4}} = 15\,(2^{x-1}+4)$

$4 = 15(2^{x-1}+4)$

$\frac{4}{15} = \frac{\cancel{15}(2^{x-1}+4)}{\cancel{15}}$

$\frac{4}{15} = 2^{x-1} + 4$
$-4 \quad -4$

$\frac{-56}{15} = 2^{x-1}$

$\log_2\left(\frac{-56}{15}\right) = x - 1$
$+1 \quad +1$

$x = 1 + \cancel{\log_2\left(\frac{-56}{15}\right)}$

ERROR

no solution?

notation (+.5)
order of op (+1)
use log (+.5)
use log correctly (+.5)
interpret solution (+.5)
finish alg after log (+.5)

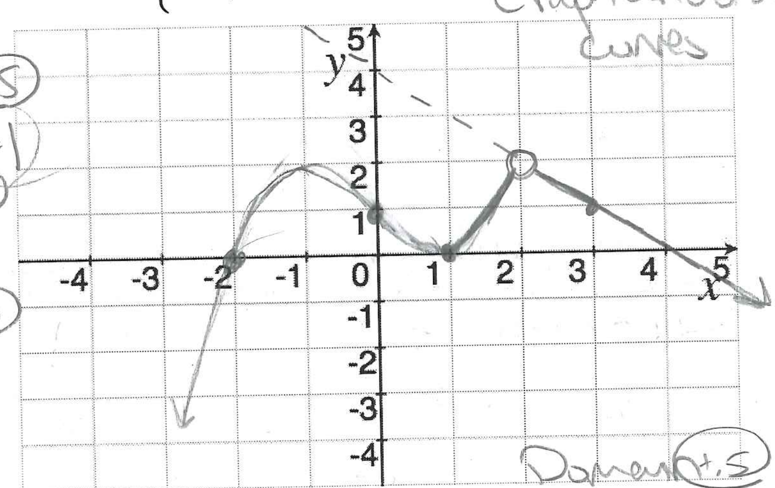
6. Let h be the *function* defined by: $h(x) = \begin{cases} \frac{1}{2}(x+2)(x-1)^2 & x < 2 \\ -x+4 & 2 < x \end{cases}$

Graph consists of solid curves

(a) [1] Find $h(0)$ if possible.

$0 < 2$ so 1st line (+.5)

$\frac{1}{2}(0+2)(0-1)^2 = \frac{1}{2}\cdot 2 \cdot 1 = 1$ (+.5)



(b) [1] Find $h(3)$ if possible.

$2 < 3$ so 2nd line (+.5)

$-3 + 4 = 1$

(c) [1] Find $h(2)$ if possible.

not in the domain of h?

Domain (+.5)
line (+1)

(d) [4] (PracticeExam2 #9) ~~Graph~~ Sketch h on the axes above.

$\frac{1}{2}(x+2)(x-1)^2 \to$ root @ -2, root @ 1 (+1)

$\Rightarrow$ degree 3 so (+.5)

$\to$ crosses @ -2, touches @ 1 (+.5)

polynomial shape (+.5)

$-x+4$ line w/ y-intercept at 4 & slope -1

7. [2] (§3.3 #14) Let $3 = \log_2(x)$ and $8 = \log_2(y)$. Find $\log_2\left(\frac{y}{x}\right)$.

$\log_2(y/x) = \log(y) - \log(x)$ or $3 = \log_2(x)$ & $8 = \log_2 y$ Property (+.5)

$= 8 - 3$ $\Rightarrow 2^3 = x$ $\Rightarrow 2^8 = y$ used (+.5)

$= 5$ So plugged $\log_2(x) = 3$ in eq (+.5)

$\log_2\left(\frac{2^8}{2^3}\right) = \log_2(2^5) = 5$ got it (+.5)

8. The area of a rectangle is $5x^4 - 15x^3 + 22x^2 - 6x + 8$. It's length can be computed by $x^2 - 3x + 4$.

   (a) [2] If the length of the rectangle is 4, what is x?

   (b) [3] Find the polynomial function that outputs the width of a rectangle as a function of x.

(a) length $= x^2 - 3x + 4$

(+.5) $4 = x^2 - 3x + 4$

Start (+.5) $-4 \quad -4$

Solve quadratic (+1)

$0 = x^2 - 3x$

$0 = x(x-3)$

$\Rightarrow x = 0$ or $x - 3 = 0$

$x = 0$ or $x = 3$

(b) length · width = area (+.5) $\Rightarrow$ width $= \frac{\text{area}}{\text{length}}$

width $= \dfrac{5x^4 - 15x^3 + 22x^2 - 6x + 8}{x^2 - 3x + 4}$ (+1) (+.5) long division

$5x^2 + 2$ (+.5)

$x^2 - 3x + 4 \,\overline{)\, 5x^4 - 15x^3 + 22x^2 - 6x + 8}$

$-(5x^4 - 15x^3 + 20x^2)$

$2x^2 - 6x + 8$

$-(2x^2 - 6x + 8)$

R 0

width $= 5x^2 + 2$ got it (+.5)

9. [2] (WebHW14 #1) Suppose that \$ 2,500 is invested in an account that pays interest compounded continuously. Find the amount of time that it would take for this account to grow to \$4,500 at 5.25 %.

$\rightarrow Pe^{rt} = A$ (+.5)

$4500 = 2{,}500e^{.0525t}$ (+.5)

order of op (+.5)

used ln's (+.5)

$\frac{45}{25} = e^{.0525t}$

$\ln(9/5) = .0525t$

$\frac{\ln(9/5)}{.0525} = t \approx 11.2$ years

10. Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

(a) Given a mortgage M we can compute the regular payments P. Let r be the annual interest rate, t be the number of years, and n be the number of payments per year. Then we can find:

$$P = \left[\frac{rM}{1-(1+\frac{r}{n})^{-nt}}\right] \div n$$

i. (§3.3 #95) [2] What is the monthly payment on a mortgage of \$120,000 with a 6% interest rate for 20 years?

ii. (§3.3 #97) [3] The First National Bank offers Andy an 8.5% interest rate on a 30-year mortgage to be paid back in monthly payments. The most Andy can afford to pay in monthly payments is \$850.00. What mortgage amount can Andy afford?

(b) (WordProblem2 #3) Entropy S is a function of the number of possible states W, that are accessible to a system with a given amount of energy. We can explicitly compute entropy by

$$S = k\ln(W)$$

where k is Boltzmann's constant and equal to $1.38064852 \times 10^{-23}\text{m}^2\text{kgs}^{-2}\text{K}^{-1}$.

i. [2] Find the entropy S of flipping one coin where the states are counting what side is up.

ii. [3] If liquid A has 100,000 times more possible states than liquid B, which liquid has a higher entropy and what is the difference?

(a) i)

Start (+.5)
use .06 (+.5)
plug in correctly (+1)

$$\left[\frac{.06 \cdot 120{,}000}{1-(1+\frac{.06}{12})^{-12(20)}}\right] \div 12$$

$\approx \$850$

ii)

use .085 (+.5)
plug in correctly (+.5)
algebra to solve for m (+1.5)

$$850 = \left[\frac{.085 \cdot m}{1-(1+\frac{.085}{12})^{-12\cdot 30}}\right] \div 12$$

$$10200 = \frac{.085m}{1-(1.00708\overline{3})^{-360}}$$

$$\frac{10{,}200\,(1-(1.00708\overline{3})^{-360})}{.085} = \frac{\cancel{.085}m}{\cancel{.085}}$$

$m = \$110{,}545.59$

(b) i) there are 2 states: heads up or tails up

Start (+.5)
2 states (+1)
plug in (+.5)

so

$= (1.38064852 \cdot 10^{-23})\ln(2)$

or $= 9.5697263 \cdot 10^{-24}$

ii) W_A = possible states of A
W_B = possible states of B (+.5)
$W_A = 100{,}000W_B$
notation (+.5)

we want to compare S_A to S_B
we want to find difference or $S_A - S_B$ (+.5)

$S_A - S_B = K\ln W_A - K\ln W_B$ (+1)
$= K\ln 100{,}000W_B - K\ln W_B$
$= K(\ln 100{,}000 + \ln W_B) - K\ln W_B$
$= K\ln 100{,}000 + \cancel{K\ln W_B - K\ln W_B}$
$\approx 1.5895304 \cdot 10^{-22}$

A has larger entropy (+.5)
by