TMath 120

NAME:

1. [5] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function, and x, y, and z be real numbers with $z \neq 0$.

   T   F $\dfrac{1}{a} + \dfrac{1}{b} = \dfrac{2}{a+b}$

   T   F   The degree of $7x^5 - 4.56x^4 - 7x^5 + 8$ is 4
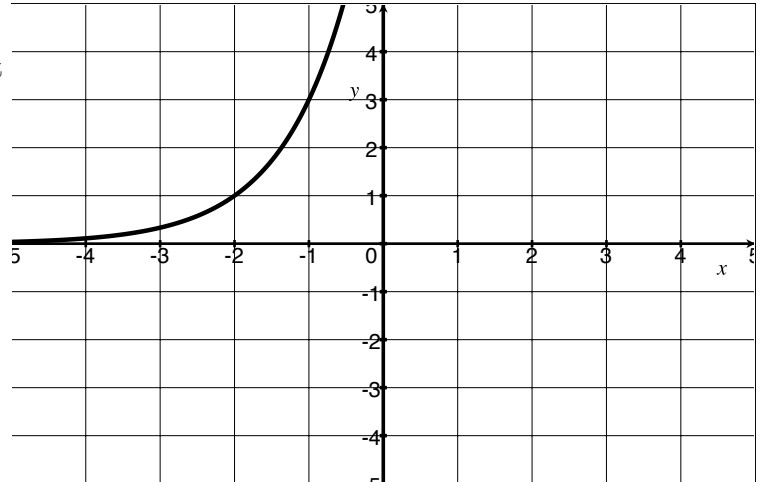
   T   F $2 \cdot 3^x = 6^x$

   T   F $\log_3 7$ is approximately 1.771

   T   F   2 is a root of $f(x) = x^4 - 3x^2 - x - 2$

   Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. [3] Explain what a polynomial function is as you would to a 5th grader.

3. The graph of f shown below is an exponential function that has been shifted horizontally.

   (a) [1] Estimate $f(-1)$.

   (b) [2] Estimate the range of f.

   (c) [3] (quiz3 #4)
   Find the algebraic rule for f.

4. Simplify the following:

   (a) [2] (WebHW10 #6) $2\sqrt{b}(3a^2b)^2$

   (b) [2] (PracticeExam #7) $\log\left(\dfrac{10^2 \cdot 10^4}{10}\right)$

5. [6] Find all real or complex x values in each of the following:

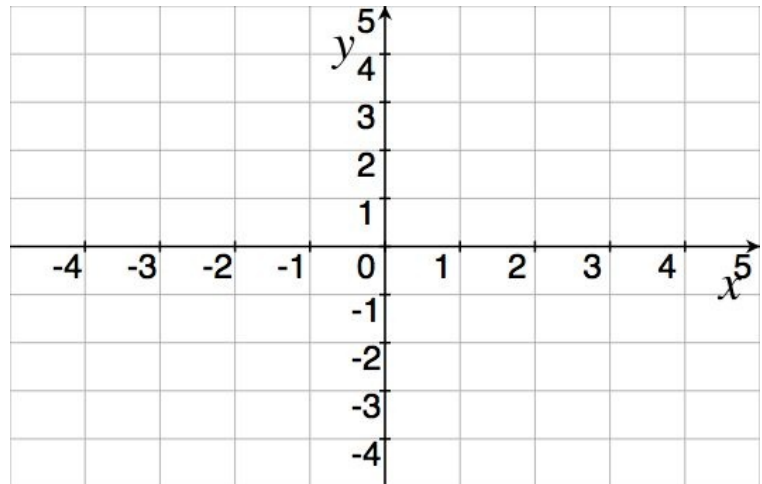   (a) (exponent wks #3) $3x^{-4} + 5 = 4$

   (b) (WebHW13 #7) $\dfrac{4}{2^{x-1} + 4} = 15$

6. Let h be the *function* defined by: $h(x) = \begin{cases} \frac{1}{2}(x+2)(x-1)^2 & x < 2 \\ -x + 4 & 2 < x \end{cases}$

   (a) [1] Find $h(0)$ if possible.

   (b) [1] Find $h(3)$ if possible.

   (c) [1] Find $h(2)$ if possible.

   (d) [4] (PracticeExam2 #9) Graph h on the axes above.

3

7. [2] (§3.3 #14) Let $3 = \log_2(x)$ and $8 = \log_2(y)$. Find $\log_2\left(\frac{y}{x}\right)$.

8. The area of a rectangle is $5x^4 - 15x^3 + 22x^2 - 6x + 8$. It's length can be computed by $x^2 - 3x + 4$.

   (a) [2] If the length of the rectangle is 4, what is x?

   (b) [3] Find the polynomial function that outputs the width of a rectangle as a function of x.

9. [2] (WebHW14 #1) Suppose that $ 2,500 is invested in an account that pays interest compounded continuously. Find the amount of time that it would take for this account to grow to $4,500 at 5.25 %.

10. Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.

No, doing both questions will not earn you extra credit.

(a) Given a mortgage M we can compute the regular payments P. Let r be the annual interest rate, t be the number of years, and n be the number of payments per year. Then we can find:

$$P = \left[\frac{rM}{1 - (1 + \frac{r}{n})^{-nt}} \right] \div n$$

  i. (§3.3 #95) [2] What is the monthly payment on a mortgage of $120,000 with a 6% interest rate for 20 years?

  ii. (§3.3 #97) [3] The First National Bank offers Andy an 8.5% interest rate on a 30-year mortgage to be paid back in monthly payments. The most Andy can afford to pay in monthly payments is $850.00. What mortgage amount can Andy afford?

(b) (WordProblem2 #3) Entropy S is a function of the number of possible states W, that are accessible to a system with a given amount of energy. We can explicitly compute entropy by

$$S = k \ln(W)$$

where k is Boltzmann's constant and equal to $1.38064852 \times 10^{-23} \mathrm{m^2 kg s^{-2} K^{-1}}$.

  i. [2] Find the entropy S of flipping one coin where the states are counting what side is up.

  ii. [3] If liquid A has $100,000$ times more possible states than liquid B, which liquid has a higher entropy and what is the difference?