TMath 120

NAME: *Key*

1. [5] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function, and x, y, and z be real numbers with $z \neq 0$.

T (F) $\frac{1}{a} + \frac{1}{b} = \frac{2}{a+b}$
$\frac{1}{a} + \frac{1}{b} = \frac{b}{b}\frac{1}{a} + \frac{1}{b}\frac{a}{a} = \frac{b+a}{ab}$

(T) F The range of $y = x^2$ is $[0, \infty)$. range = y values

T (F) The graph of $y = -\frac{3}{7}(x+5)^2 - 3$ has a maximum at $x = 5$.
maximum is @ vertex which is @ $x = -5$

T (F) $(12 - 2i) - (4 - i) = 8 - 3i$
$12 - 2i - 4 + i = 8 + i$

(T) F $\frac{-1}{3-i} = \frac{-3}{10} - \frac{1}{10}i$
$\frac{-1}{3-i} \cdot \frac{3+i}{3+i} = \frac{-3-i}{9+3i-3i-i^2} = \frac{-3-i}{9-(-1)} = \frac{-3-i}{10}$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. [3] Explain what an inverse function is as you would to a 5th grader.

Start (+.5)
true (+1)
def/example (+1)
sense (+.5)

[3] Explain what two notation $f(x)$ means as you would to a 5th grader.

Start (+.5)
true (+1)
def/example (+1)
sense (+.5)

3. Let f be the piece-wise defined function comprised a line and a parabola whose graph is below.

(a) Estimate the following *if* possible:
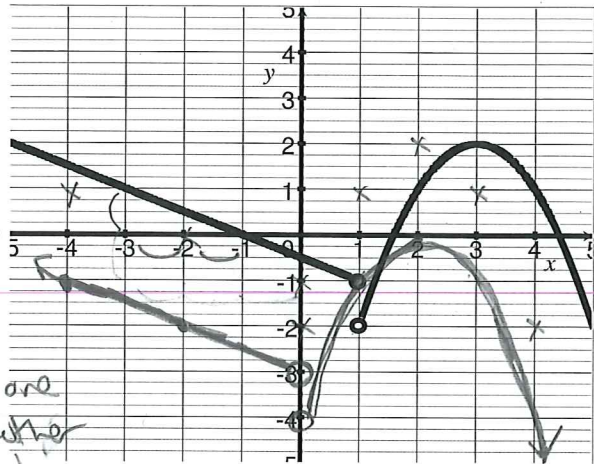
   i. [1] (§1.3 #56) $f(4)$

      1

   ii. [1] $f(1)$ Trick Question?

      Does not exist

   iii. [2] (WebHW2 #7) the x-intercept(s)

      (+1) got one
      (+.5) another
      (+.5) last one

      $-1, 1.5, 4.5$

(b) [2] (Quiz1 #3) *Estimate* all possible x such that $f(x) = 1$.

      $-3, 2, 4$

      (+1) got one
      (+.5) another
      (+.5) last one

(c) [5] (Transform2Wks #1) Find the formula for f in the indicated form:

      Start (+.5)

$$f(x) = \begin{cases} -\tfrac{1}{2}x - \tfrac{1}{2} & \text{if } x < 1 \\ -(x-3)^2 + 2 & \text{if } 1 < x \end{cases}$$

(+.5) line: $y = mx + b$

(+.5) $m = \dfrac{\text{rise}}{\text{run}} = \dfrac{-1}{2}$

(+.5) y-intercept $-\tfrac{1}{2}$

Quadratic (+.5)   form (+.5) $y = a(x-h)^2 + k$

vertex @ $(3, 2)$ $\Rightarrow$ $y = a(x-3)^2 + 2$

thru $(2,1)$ so $1 = a(2-3)^2 + 2$

$1 = a + 2$

$-1 = a$

flip (+.5) h (+1) k (+.5)

(d) [3] (WebHW5 #4) Graph $f(x+1) - 2$.

      using graph

      in 3 (+.5)

      got it (+1.0)

(+.5) horiz Shift LEFT 1

(+.5) vertical Shift DOWN 2

2

4. Let h be the *function* defined by: $h(x) = \begin{cases} 3x^2 - 1 & -1 \le x \le 1 \\ -2x + 5 & 1 < x < 4 \end{cases}$

(a) [1] (§1.1 #48)
Find the y-intercept

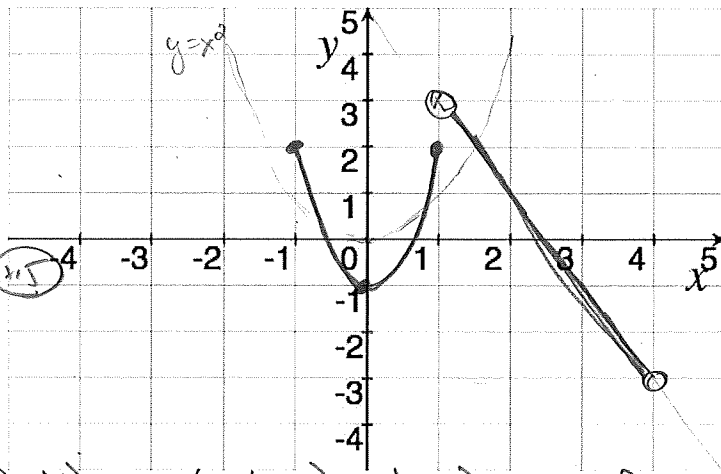*(handwritten)* ⓘ when $x = 0$ so
$3(0)^2 - 1 = \boxed{-1}$ (+.5)

(b) [1] (WebHW2 #10)
Find $h(1)$ use 1st line (+.5)
$3(1)^2 - 1 = \boxed{2}$ (+.5)

(c) [2] (WebHW5 #11)
Find $(h \circ h)(2)$
$h(h(2)) = h\left(-2(2) + 5\right) = h(-4 + 5) = h(1) = 3(1)^2 - 1 = 2$
(+.5) use 2nd line (+.5) arithmetic (+.5) use 1st line (+.5)

(d) [4] (WebHW2 #12) Graph h on the axes above.

(handwritten) $2 \rightarrow -2(2) + 5 = 1 \rightarrow 3(1)^2 - 1 = 2$

$3x^2 - 1$
(.5) parabola
(+.5) multiply by 3
(.5) shift down 1
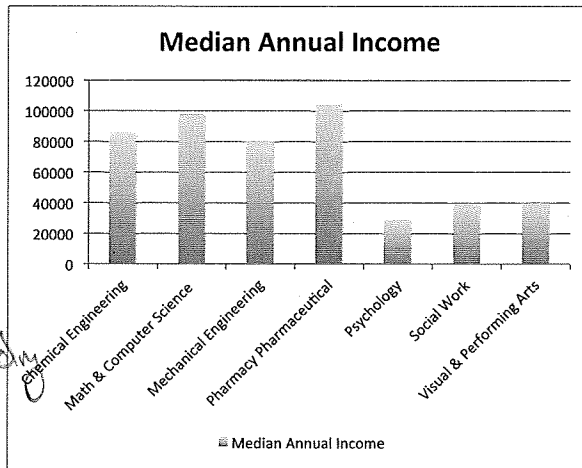
$-2x + 5$
(+.5) line
(.5) slope -2
(+.5) intercept @ 5

endpoints ④

5. (PracitceExam #9) Let the domain of f be undergraduate majors and $f(x)$ be the median annual earnings of people with the the undergraduate major x.

(a) [2] Is f a function? Why are why not?

(handwritten) (+1) Yes.
(+1) Each major has only 1 corresponding median annual earning

(Median Annual Income bar chart with categories: Chemical Engineering, Math & Computer Science, Mechanical Engineering, Pharmacy Pharmaceutical, Psychology, Social Work, Visual & Performing Arts; y-axis from 0 to 120000)

Median Annual Income

(b) [2] Some data of f is shown in the graph on the right, what is $f(\text{Psychology})$ and what does it mean?

(handwritten) $f(\text{Psychology}) = 30,000$ (+1)

Means: The median annual earnings of people with an undergraduate degree in Psychology is $30,000, ... which is low. (+1)

6. Let $\alpha(x) = \dfrac{x}{x+2}$ and $\beta(x) = 2 - \sqrt{x+1}$. Both α and β have inverses that exist.

(a) [3] (WebHW1#12) Find $\alpha(x+h) - \alpha(x)$ and simplify.

$$\frac{\boxed{}}{\boxed{}+2} - \frac{x}{x+2}$$

$$\overset{(+1)}{\frac{x+h}{x+h+2}} - \frac{x}{x+2}$$

Simplify (+1)

$$\frac{x+h}{x+h+2} \cdot \frac{x+2}{x+2} - \frac{x}{x+2} \cdot \frac{x+h}{x+h}$$

$$\frac{(x+2)(x+h) - x(x+h+2)}{(x+2)(x+h+2)}$$

$$\frac{x^2 + hx + 2x + 2h - x^2 - xh - 2x}{(x+2)(x+h+2)}$$

$$\frac{2h}{(x+2)(x+h+2)}$$

(b) [2] (§1.6 #38) $(\alpha \circ \beta)(x)$.

$$\alpha(\beta(x)) = \overset{(+.5)}{\alpha}\left(\overset{(+.5)}{\boxed{2 - \sqrt{x+1}}}\right) = \overset{(+.5)}{\boxed{\frac{2 - \sqrt{x+1}}{2 - \sqrt{x+1} + 2}}}$$

alg (+.5)

(c) [1] (§1.7 #27) $(\alpha \circ \alpha^{-1})(\pi)$.

$$\pi$$

B/c α and α^{-1} undo eachother

7. [2] (Quiz2 #1) Find the equation of a line perpendicular to the line that passes through $(-2, 1)$ and $(4, -3)$. Note, there are many right answers!

$$\text{Slope} = \frac{-3 - 1}{4 - -2}$$

$$= \frac{-4}{6}$$

$$= -2/3 \quad (+1)$$

$$\overset{(+.5)}{\perp} \text{ slope} = 3/2$$

$$\overset{(+.5)}{\text{equation of a line:}} \quad 3/2 \, x = y$$

$$3/2 \, x + 1 = y$$

$$3/2 \, x + \sqrt{2} = y$$

8. [3] Find the real or complex solutions to $2(x+3)^2 + \frac{3}{2} = 0$.

algebra (+1.5)

both sol (+1)

notice i (+.5)

$$2(x+3)^2 + 3/2 = 0$$

$$-3/2 \qquad -3/2$$

$$\frac{2(x+3)^2}{2} = \frac{-3/2}{2}$$

$$(x+3)^2 = -3/4$$

$$x + 3 = \pm\sqrt{-3/4}$$

$$x = -3 \pm \sqrt{-3/4}$$

$$x = -3 \pm i\sqrt{3/4}$$

OC

$$2(x+3)^2 + 3/2 = 0$$

$$2(x+3)(x+3) + 3/2 = 0$$

$$2(x^2 + 6x + 9) + 3/2 = 0$$

$$2x^2 + 12x + 18 + 3/2 = 0$$

$$2x^2 + 12x + 39/2 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-12 \pm \sqrt{144 - 4(2)\left(\frac{39}{2}\right)}}{2 \cdot 2}$$

quad for (+.5)
correctly used (+.5)
both sol (+.5)
algebra (+1.5)

$$= \frac{-12 \pm \sqrt{144 - 156}}{4} = -3 \pm \frac{\sqrt{-12}}{4}$$

$$= -3 \pm \frac{2\sqrt{3}}{4} = \frac{-3 \pm i\sqrt{3}}{2}$$

4

9. Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
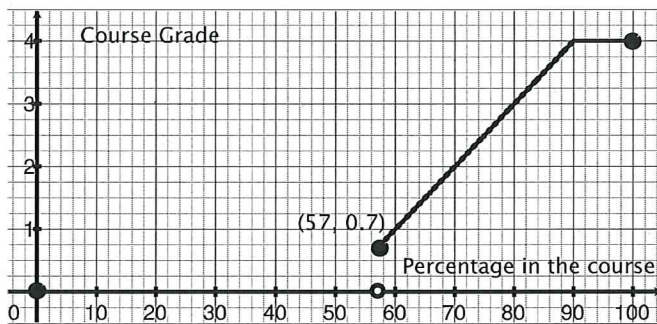   No, doing both questions will not earn you extra credit.

   (a) You have 8 oz of mocha that is 20% espresso sitting in a 16 oz cup.

      i. [3] Write a rational expression in x whose values give the percentage (in decimal form) of espresso in the cup when x oz of espresso are added to it.

      ii. [2] Find the domain of the function in part i.

   (b) (WordWks #9) Princess Leia is in this course and curious about her marks now that she's taken two exams. She has looked at the gradebook on MyMathLab and has computed the averages listed below. The weights specified in the syllabus and the graph of the function f that takes your class percentage x and returns your score on a 4. scale are also provided.

      Assume Leia's work does not drastically change in the remaining 3 weeks and her averages remain about the same.

      i. [3] Find a function that provides her overall course percentage as a function of her final exam score.

      ii. [2] What minimum grade does she need to get on the final to receive a 4.0 in the course?

	weight	Leia's ave
Mini-Quizzes	5%	95%
WebAssign	10%	100%
WrittenHW	15%	100%
Quizzes	15%	83%
2 Exams	30%	95%
Final	25%	

Course Grade graph with point (57, 0.7), Percentage in the course on x-axis (0 to 100), Course Grade 0 to 4 on y-axis.

(a) i. $\text{percentage of espresso} = \dfrac{\text{total in cup}}{\text{espresso}}$ (+.5)

$= \dfrac{\text{original espresso in cup} + \text{new esp.}}{\text{original mix} + \text{added liquid}}$

something (+.5)

start (+.5)
variables (+.5)

$= \dfrac{.2 \cdot 8 + x}{8 + x}$ (+.5) (+.5)

$= \dfrac{1.6 + x}{8 + x}$

(ii) Domain Conversion $\Rightarrow x + 8 \neq 0$
$\Rightarrow x \neq -8$

partial (+1)

(+1) { In Reality we can't add negative esp
        so $x \geq 0$

(+1) { Also once the cup is full we have to
        stop so $x \leq 8$ so $[0, 8]$

(b) y = overall course %
start (+.5)
variables (+.5)

i) x = final exam score
right function (+.5)

$y = 95 \cdot .05 + 100 \cdot .10 + 100 \cdot .15 + 83 \cdot .15$
$\quad + 95 \cdot .30 + \underline{x \cdot .25}$ or
                              (+1)

$y = 4.75 + 10 + 15 + 12.45$
$\quad + 28.5 + .25x$ or
                        (+.5) correct wts
$\boxed{y = 70.7 + .25x}$ (+.5 added together)

ii) $90 = 70.7 + .25x$
(+1) b/c 90% corr to 4.0 above

$19.3 = .25x$ algebra (+1)

$\boxed{77.2\% = x}$