Quiz 3

Key

Show *all* your work. Reasonable supporting work must be shown to earn credit. There are *two* sides to this quiz.

1. (§2.3 #90) The volume of a rectangular solid is $x^4 + 3x^3 + 5x^2 + x + 3$ cubic inches. Its length and width are $(x+3)$ and $(x+1)$ inches, respectively.

   (a) [1] What is the volume when the width is 3 inches?

   length $= (x+3)$
   width $= x+1$ $\Rightarrow$ $3 = x+1$ $\Rightarrow$ $x = 2$ (+.5)

   So volume is $2^4 + 3 \cdot 2^3 + 5 \cdot 2^2 - 2 + 2 = 60$ (+.5)
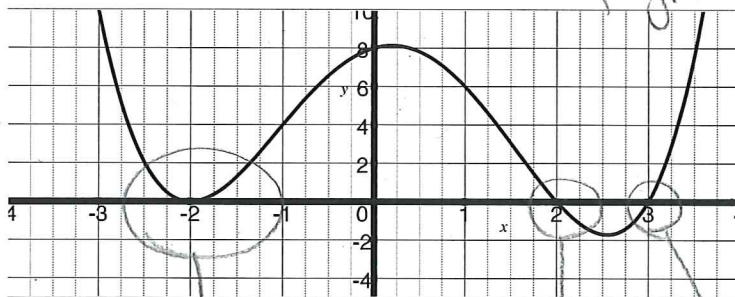
   OR
   $2^4 + 3 \cdot 2^3 + 2 + 3$
   (+.5) = 45

   (b) [2] Find its height (as a function of x).

   Vol $=$ length $\cdot$ width $\cdot$ height
   Vol $= (x+3)(x+1) \cdot$ height

   $\Rightarrow$ height $= \dfrac{\text{Vol}}{(x+3)(x+1)} = \dfrac{\text{Vol}}{x^2+4x+3}$ (+.5)

   long division?

   $$\begin{array}{r}
   x^2 - x + 1 \\
   x^2+4x+3 \overline{\smash{\big)}\, x^4+3x^3+0x^2+x+3} \\
   -(x^4+4x^3+3x^2) \\
   \hline
   -x^3 - 3x^2 + x + 3 \\
   -(-x^3 - 4x^2 - 3x) \\
   \hline
   x^2 + 4x + 3
   \end{array}$$

2. [3] (Polynomial Activity #4) Given the graph below is of a polynomial of degree four, find the algebraic rule/write an equation for the graph.

   polynomial (+.5)

   (#1) -2 is a root $\Rightarrow$ $(x-2)$ is a factor
   2 is a root $\Rightarrow$ $(x-2)$ is a factor
   3 is a root $\Rightarrow$ $(x-3)$ is a factor

   (+.5) bounces $\Rightarrow (x-2)^2$ or even power

   thru $\Rightarrow (x-2)^1$ or odd power

   thru $\Rightarrow (x-3)^1$ or odd power

   So $y = a(x+2)^2(x-2)(x-3)$ (+.5)
   has degree 4

   Graph is thru $(0,9)$ (+.5)
   $\Rightarrow 9 = a(0+2)^2(0-2)(0-3)$
   $\Rightarrow 9 = a \cdot 4(-2)(-3)$
   $\Rightarrow 9 = +24a$ $\Rightarrow a = \frac{3}{8}$

   $\boxed{y = \frac{1}{3}(x+2)^2(x-2)(x-3)}$

3. [1] (§3.2 #29) Write the following logarithmic statement in exponential form.
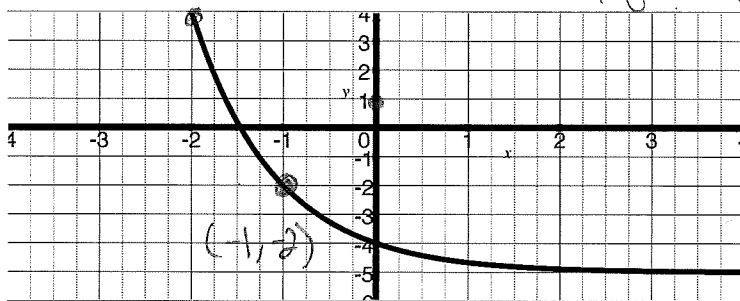
$$3\log_8(2) = 1$$

$$\frac{3\log_8(2)}{3} = \frac{1}{3}$$

$$\log_8(2) = \frac{1}{3}$$

$$\Rightarrow 8^{1/3} = 2$$

$$\log_8\left((2)^3\right) = 1$$

$$\Rightarrow 8^1 = 2^3$$

4. [3] (WebHW7 #8) The graph below is of an exponential function a^x that has been vertically shifted. Find the algebraic rule/write an equation for the graph. exp graph (+.5)

$(-1, -2)$

$y = a^x$ usually passes thru $(0,1)$

(+1) this graph passes thru $(0,-4)$

$\Rightarrow$ vert shift down 5

$\Rightarrow y = a^x - 5$

OR

$y = a^x + b$ thru $(0,-4)$ (+.5)

$\Rightarrow -4 = a^0 + b$

$\Rightarrow -4 = 1 + b$

$\Rightarrow b = 5$ (+.5)

So have $y = a^x - 5$

passes thru $(-1,-2)$ (+.5)

$\Rightarrow -2 = a^{-1} - 5$

$\Rightarrow 3 = a^{-1}$

$\Rightarrow \frac{1}{3} = a$ (+1)

So $y = \left(\frac{1}{3}\right)^x - 5$

OR

So have $y = a^x - 5$

passes thru $(-2,4)$ (+.5)

$\Rightarrow 4 = a^{-2} - 5$

$\Rightarrow 9 = a^{-2}$

$\Rightarrow 9 = \frac{1}{a^2}$

$\Rightarrow \frac{1}{9} = a^2$

$\Rightarrow \frac{1}{3} = a$ (+1)

2