Exam 2

TMath 120

Autumn 2018

NAME: 12ay

1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let x and y be positive real numbers.

T F $\frac{2}{b^2} + \frac{1}{b} = \frac{5}{b^2}$ $\qquad \frac{2}{b^2} + \frac{1 \cdot b}{b \cdot b} = \frac{2+b}{b^2}$

T F $(x^3)^3 = x^6$. $\qquad (x^3)^3 = x^3x^3x^3 = (xxx)(xxx)(xxx) = x^9$

T F The range of $y = e^x - 2$ is $(-1, \infty)$. $\qquad y = e^x$ range: $(0, \infty)$ $\quad y = e^x - 2$ transform down 2 $\Rightarrow (-2, \infty)$

T F $y = (x-2)^3x(x+50)$ has 3 roots. @ 2, 0 and -50

T F $x^{-2} = x^{\frac{1}{2}}$ $\qquad x^{1/2} = \sqrt{x}$ and $x^{-2} = 1/x^2$

T F $\log(x - y) = \log(x) - \log(y)$ $\qquad \log(x) - \log(y) = \log\left(\frac{x}{y}\right)$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

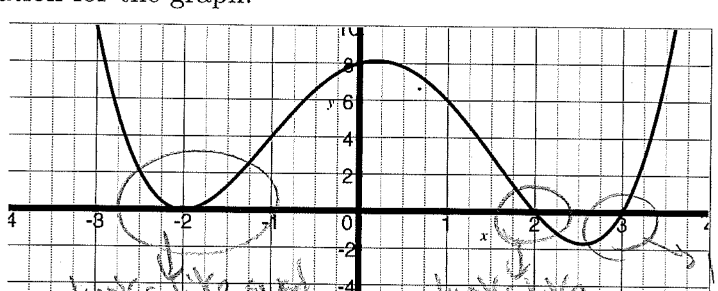
2. [3] (WebHW7 #22) How long does it take for a deposit of \$1400 to double at 3% compounded continuously? $Pe^{rt} = A$ (+1)

use ln (+.5) ordered op (+1) (+.5)

$\frac{2800}{1400} = \frac{1400e^{.03t}}{1400} \Rightarrow 2 = e^{.03t} \Rightarrow \ln 2 = .03t \Rightarrow t = \frac{\ln 2}{.03} \approx 23.1$ years

(+.5) degree 4
(+.5) polynomial

3. [4] (Quiz3 #2) Given the graph below is of a polynomial of degree four, find the algebraic rule/write an equation for the graph.



(+.5)
-2 is a root $\Rightarrow (x+2)$ is a factor
2 is a root $\Rightarrow (x-2)$ is a factor
3 is a root $\Rightarrow (x-3)$ is a factor

looks like quad (+.5) $\Rightarrow (x+2)^2$

looks like line $\Rightarrow (x-2)^1$

looks like line $\Rightarrow (x-3)^1$

So $y = a(x+2)^2(x-2)(x-3)$

passes thru $(0, 8)$ (+.5)

$\Rightarrow 8 = a(0+2)^2(0-2)(0-3)$

$\Rightarrow 8 = a \cdot 4 \cdot 6 \Rightarrow a = 1/3$ (+.5)

So $y = \frac{1}{3}(x+2)^2(x-2)(x-3)$

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4. [3] (§3.2 #56) The graph is of an exponential function $\log_b(x)$ that has been horizontally shifted. Find the algebraic rule/write an equation for the graph.

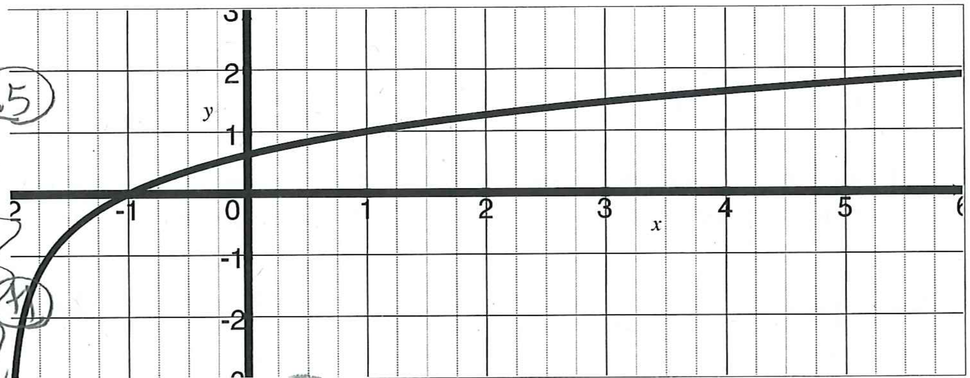
horiz. shifted

$\Rightarrow \log_b(x+h) = y$ (+.5)

usually thru (1,0) but now thru (-1,0)

$\Rightarrow$ shift LEFT by 2 (+1)

$\Rightarrow y = \log_b(x+2)$



form/log (+.5)

Thru (1,1) (+.5)

$\Rightarrow 1 = \log_b(1+2)$

$\rightarrow 1 = \log_b(3)$

$\Rightarrow b^1 = 3$ (+.5)

So $y = \log_3(x+2)$

5. For each equation below, find x.

(a) [3] (exponentActivity #3) $(x+1)^{-1} + 3 = x$

neg exp (+1.5)
order of op (+1)
solve quad (+1)

$\frac{1}{x+1} + \cancel{3} = x$

$\quad\;\; -3 \quad -3$

$\cancel{(x+1)} \frac{1}{\cancel{x+1}} = x-3\,(x+1)$

$1 = (x-3)(x+1)$

$1 = x^2 + x - 3x - 3$

$1 = x^2 - 2x - 3$

$x^2 - 2x - 4 = 0$

$x^2 - 2x + 1 - 4 = 1$

$(x-1)^2 - 4 = 1$

$(x-1)^2 = +5$

$x - 1 = \pm\sqrt{+5}$

$x = 1 \pm \sqrt{-5}$

(b) [3] (WebHW8 #21) $6^{1-x} = 5^{5x+6}$

use logs (+.5)
log prop (+1)
order of op (+1.5)

$\ln(6^{1-x}) = \ln(5^{5x+6})$

$(1-x)\ln(6) = (5x+6)\ln(5)$

$\ln(6) - x\ln(6) = x \cdot 5\ln(5) + \cancel{6\ln(5)}$

$-6\ln(5) \quad +x\ln(6) \qquad\qquad +x\ln(6) \; -6\ln(5)$

$\ln(6) - 6\ln(5) = x(5\ln(5) + \ln(6))$

$\rightarrow x = \frac{\ln(6) - 6\ln(5)}{5\ln(5) + \ln(6)}$

$\approx -.799$

(c) [4] (§3.4 #76) $\log_4 \sqrt{x+3} - \log_4 \sqrt{2x-1} = \frac{1}{4}$

log prop (+1)
use exp (+.5)
use exp right (+.5)
exponent props/sq (+1)
order of op (+1)

$\log_4 \frac{\sqrt{x+3}}{\sqrt{2x-1}} = \frac{1}{4}$

$4^{\frac{1}{4}} = \frac{\sqrt{x+3}}{\sqrt{2x-1}}$

$\left(4^{\frac{1}{4}}\right)^2 = \left(\frac{\sqrt{x+3}}{\sqrt{2x-1}}\right)^2$

$2 = \frac{x+3}{2x-1}$

$\rightarrow 2(2x-1) = x+3$

$4x - 2 = x + 3$

$-x + 2 \quad -x \;\; +2$

$\frac{\cancel{3}x}{\cancel{3}} = \frac{5}{3}$

$x = 5/3$

Check ✓

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6. [3] (PracticeExam#6) The function $f(x) = 3 \cdot 2^x + 5$ passes the horizontal line test so has an inverse. Find $f^{-1}(x)$.

Use logs (+.5)
Use notation (+.5)
order of ops (+1)

switch x's & y's

$y = 3 \cdot 2^x + 5$

(+1) $x = 3 \cdot 2^y + 5$
$-5 \qquad -5$

$\frac{x-5}{3} = \frac{3 \cdot 2^y}{3}$

$\frac{x-5}{3} = 2^y$

$\log_2\left(\frac{x-5}{3}\right) = y$

7. [2] (Quiz3 #3) Write the following logarithmic statement in exponential form:

$$2\log_b(a) = x$$

move 2 around (+1)
exp (+1)

$\log_b(a)^2 = x$

$\Rightarrow b^x = a^2$

$\log_b(a) = \frac{x}{2}$

$b^{x/2} = a$

8. [3] Write a polynomial p that satisfied the following criteria:

- as x goes to ∞, then y goes to $-\infty$
- the only roots are: -1,2 and 4

Polynomial (+.5)

$-3(x-1)(x-2)(x-4)$
(+.5) (+.5) (+.5)

Note: there is more than one right answer!!! (+1) to get y to go to $-\infty$ as $x \to \infty$

9. [4] (WebHW6 #16) Find the remainder when $x^4 - 398x^2 + 1$ is divided by $x^2 - 20x + 1$

(+1) (+1) (+.5)

$$\begin{array}{r}
x^2 + 20x + 1 \\
x^2 - 20x + 1 \,\big)\, x^4 + 0x^3 - 398x^2 + 0x + 1 \\
-(x^4 - 20x^3 + x^2) \\
\hline
20x^3 - 399x^2 + 0x + 1 \\
-(20x^3 - 400x^2 + 20x) \\
\hline
x^2 - 20x + 1 \\
-(x^2 - 20x + 1) \\
\hline
0
\end{array}$$

Zero?

set up (+.5)
algorithm (+1)

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10. [3] (logProperties #3) Let $\log_2(x) = 7$ and $\log_2(y) = 6$. Find $\log_2(8xy)$

$\log_2(8xy)$
$= \log_2(8) + \log_2(x) + \log_2(y)$ (+1)
$= 3 + 7 + 6$ (+1)
$= 16$ (+1)

OR $\log_2(x) = 7 \Rightarrow 2^7 = x$ (+.5)
$\log_2(y) = 6 \Rightarrow 2^6 = y$ (+.5)
So $\log_2(8xy) = \log_2(8 \cdot 2^7 \cdot 2^6) = 16$ (+1) (+1)

11. [4] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

4 (a) (WebHW9 #10) *pH Scales:* Let $[H^+]$ be the concentration of hydrogen ions in solution X measured in moles per liter (denoted M). Then

$$\text{pH level of solution } X = -\log[H^+].$$

Suppose that hydrogen ion concentration in a solution A is 10,000 times that in solution B. Find a function relating the pH level of solution A to the pH level of solution B.

(b) (§3.2 #115) *Benford's Law:* The probability that the first decimal-digit of a raw data sample (from 1 to 9) m is is given by $P_m = \log(m+1) - \log(m)$.
What percentage of data would financial investigators expect the numbers in accounting books to have 3 as a first digit?

(a) $[H_A^+]$ = concentration of hyd. ions of Sol. A
$[H_B^+]$ = concentration of hyd ions of Sl B (+.5) (+1)
pH_A = pH level of sol A
pH_B = pH level of sol B
start (+.5) def variables

Given $[H_A^+] = 10{,}000[H_B^+]$ } (+1) interpreting (+.5)
want relation between pH_A + pH_B } (+.5)

$pH_A = -\log[H_A^+]$
$= -\log(10000[H_B^+])$
$= -(\log 10{,}000 + \log[H_B^+])$
$= -\log(10{,}000) - \log[H_B^+]$ (+1)
$= -4 + (-\log[H_B^+])$
$= -4 + pH_B$ So $\boxed{pH_A = pH_B - 4}$

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(b) probability that the first decimal digit of a raw data sample is 3 is given by
$P_3 = \log(3+1) - \log(3)$ (+1)
$= \log(4) - \log(3)$
$= \log(4/3)$
$\approx .1249$ so $\approx$ 12.5%
complete (+1)

Start (+1)
interpret correctly? (+1)

12. [5] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

(a) (expActivity #5) *Newton's Law of Cooling:* If D is the initial temperature difference between an object and its surroundings, and if its surroundings have a temperature T, then the temperature of the object A ~~and~~ at time t is modeled by:

$$A = T + De^{-kt}$$

where k is a positive constant that depends on the type of object.
Initially coffee has a temperature of 200°F in a room that is 70°. After ten minutes the temperature is 150°. What will the temperature of the coffee be after an additional ten minutes passes?

(b) (§3.3 #108) *Amortization formula*: If r is the annual interest rate, M is the mortgage amount, t is the number of years, and n is the number of payments a year, then the payment P on a mortgage is given by:

$$P = \frac{r \cdot M}{1 - \left(1 + \frac{r}{n}\right)^{-nt}} \div n$$

The Garcia family wanted to take out a mortgage for \$150,000 with 8% with monthly payments. The family can afford monthly payments of \$1200. How long would they have to make payments to pay off their mortgage and how much interest would they be paying?

(a) start (+.5)

$A = T + De^{-kt}$

T: 70 (+.5); D: (200-70) (+1); t: ?

(+.5) we want A when t=20 but we need to find k first?

(+.5) Note (10,150)

$\Rightarrow 150 = 70 + 130e^{-k(10)}$

$\Rightarrow 80 = 130e^{-k10}$

$\Rightarrow 8/13 = e^{-10k}$

$\Rightarrow \ln(8/13) = -10k$

$\Rightarrow -\frac{1}{10}\ln(8/13) = k \approx .0486$

alg (+1.5) logs +.5

So $A = 70 + 130e^{-.0486t}$

$\Rightarrow 70 + 130e^{-.0486 \cdot 20} \approx \boxed{119.2°F}$

plug in 20 (+.5)

(b) start (+.5)

$P = \frac{r \cdot m}{1 - (1+\frac{r}{n})^{-nt}} \div n$

(+1.5) 1200, .08, \$150,000, 12 b/c monthly

(+.5) want to find t and $\underbrace{t n P - m}_{\text{interest paid}}$

$1200 = \frac{.08 \cdot 150000}{1-(1+\frac{.08}{12})^{-12t}} \div 12$

$\Rightarrow 14400 = \frac{12000}{1-(1+\frac{.08}{12})^{-12t}}$

$\Rightarrow 1-(1+\frac{.08}{12})^{-12t} = \frac{12000}{14400} = \frac{5}{6}$

$\Rightarrow (1+\frac{.08}{12})^{-12t} = 1 - 5/6 = 1/6$

$\Rightarrow -12t \ln(1+\frac{.08}{12}) = \ln(1/6)$

$\Rightarrow t = -\frac{\ln(1/6)}{12\ln(1+\frac{.08}{12})} \approx 22.5$ years

alg (+1.5) logs +.5

alg & end (+.5) So $t \cdot n \cdot P - M = 22.5 \cdot 12 \cdot 1200 - 150{,}000 \approx 174{,}000$

More than the house? (+1)

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$$\begin{array}{r} {}^{2}\\ 26 \\ 19 \\ 5 \\ \hline 50 \end{array}$$