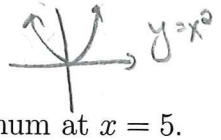
Exam 1 **TMath 120** Autumn 2018

NAME: Key

1. [5] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F.

T (F) $\frac{2}{b^2}+\frac{1}{b}=\frac{5}{b^2}$ $\frac{2}{b^2}+\frac{1}{b}\cdot\frac{b}{b}=\frac{2+b}{b^2}$

(T) F The range of $y=x^2$ is $[0,\infty)$. (y values)



T (F) The graph of $y=-\frac{3}{7}(x+5)^2-3$ has a maximum at $x=5$. (parabola opening down, vertex @ (-5,-3))

T (F) $(1-2i)(4-i)=6-9i$ $4-i-8i+2i^2=4-9i-2=2-9i$

(T) F $\frac{-1}{3-i}=\frac{-3}{10}-\frac{1}{10}i$ $\frac{-1}{3-i}\cdot\frac{(3+i)}{(3+i)}=\frac{-3-i}{9+3i-3i-i^2}=\frac{-3-i}{9+1}=\frac{-3}{10}-\frac{i}{10}$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. [3] (WebHW3 #22) Let f be the function that associates the employee number x of each employee of a c company with his or her annual salary $f(x)$ in dollars. Suppose each employee was awarded a \$800 across-the-board raise and *then* an additional 5% of his or her increased salary. Write a function that describes the new salary.

(+1) (+.5) (+.5)

$(f(x)+800)+.05(f(x)+800)$

$f(x)+800$: salary increase; $.05(f(x)+800)$: 5% raise on new salary

Start (+.5)

(+.5) using $f(x)$ as starting point (not x)

or

$f(x)+800+.05f(x)+40$

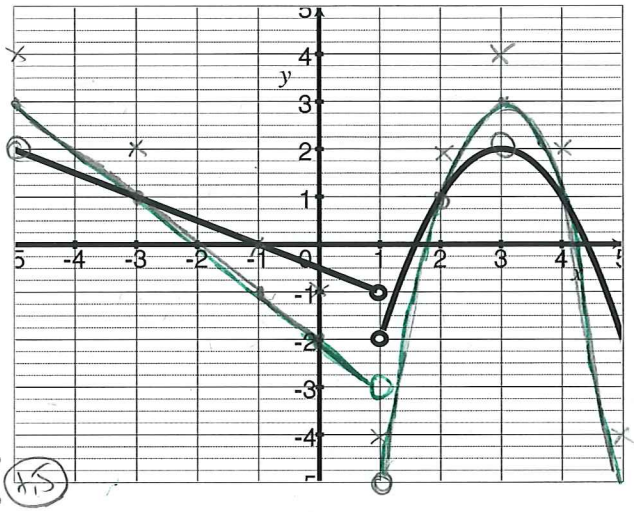
or

$f(x)+.05f(x)+840$

or

$1.05f(x)+840$

3. Let f be the piece-wise defined function comprised a line and a parabola whose graph is below.



(a) Estimate the following *if* possible:

i. [1] (Quiz1 #3) $f(4)$

1

ii. [1] (WebHW3 #1) $(f+f)(4)$

$f(4)+f(4)=1+1=2$ (+.5) (+.5)

iii. [1] (WebHW3 #3) $(f \circ f)(4)$

$f(f(4)) = f(1) =$ DNE? (+.5)

Composition (+.5)

iv. [1] (§1.1 #50) the y-intercept

$-.5$ or $-\frac{1}{2}$ (+1)

(b) [2] (Transformations Activity #5) *Estimate* all possible x such that $f(x) = 2$.

$x = -5$ and 3 (+.5) (+.5)

looking for x-values (+.5) got one (+.5)

(c) [5] (Quiz2 #3d) Find the formula for f in the indicated form:

$$f(x) = \begin{cases} (-\frac{1}{2})x - \frac{1}{2} & \text{if } x < 1 \\ -(x-3)^2+2 & \text{if } 1 < x \end{cases}$$

(+.5) line $y = mx+b$

(+1) m = slope $= \frac{\Delta y}{\Delta x} = \frac{-1}{2}$

(+.5) b = y intercept $= -\frac{1}{2}$

(+.5) parabola $y = a(x-h)^2+k$

(+.5) vertex @ $(3,2)$

(+1) $\Rightarrow y = a(x-3)^2+2$

thru $(2,1)$ so

$1 = a(2-3)^2+2$

(+1) $\Rightarrow 1 = a(1)+2$

$-2 \quad -2$

$-1 = a$

(d) [3] (WebHW3 #16) Graph $2f(x) - 1$.

+.5 shape

(+.5) mult the y coord by 2

(+.5) then

(+.5) move down 1 unit.

end points (+.5)

(+.5)

(+.5)

4. Let h be the *function* defined by: $h(x) = \begin{cases} \frac{1}{2}x - 1 & -4 \le x \le 2 \\ -2x + 4 & 2 < x < 3 \end{cases}$

(a) [1] (FunctionActivity#1a)
Find $h(1)$

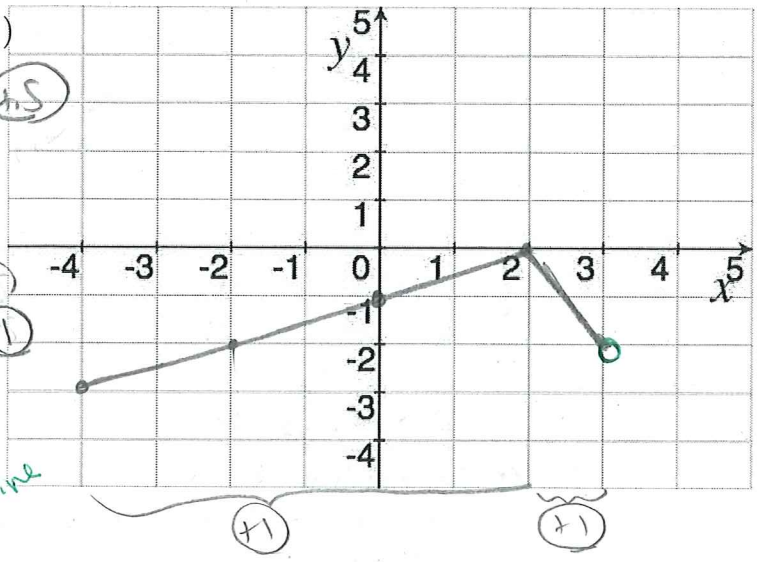
$-4 \le 1 \le 2$ so 1st line (+.5)
$\frac{1}{2}(1) - 1 = -\frac{1}{2}$ (+.5)

(b) [1] (WebHW1 #13)
What is the domain of h?

$-4 \le x < 3$ # (+1)
or $[-4, 3)$ ends (+1)

(c) [4] (WebHW2 #12)
Graph h on the axes.

domains (+1)



+.5 / line
(+1)
(+1)

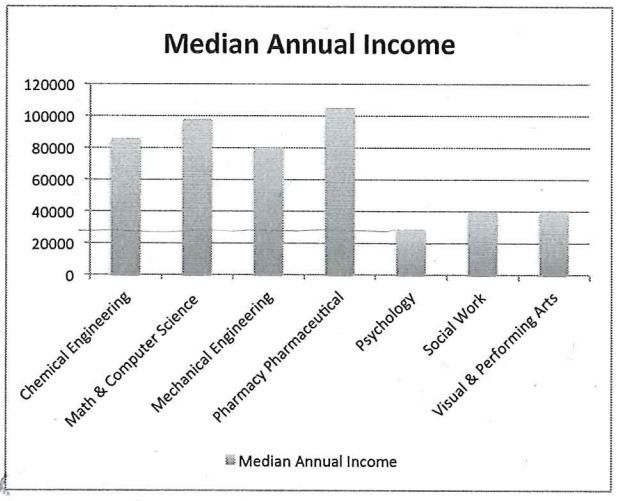
(d) [2] (LineActivity #13)
What angle is made by the graph at $x = 2$? Justify your answer.

(+1) 90° (the axis above is stretched here. so it's hard to tell)

(+1) note the slopes of the 2 lines are opposite reciprocals of each other

+.5 reasonable

5. (PracitceExam #9) Let the domain of f be undergraduate majors and $f(x)$ be the median annual earnings of people with the the undergraduate major x.



(a) [2] Is f a function? Why or why not?

yes. (+1)
(+1) Each major produces only one median annual earnings of people with undergrad major x

(b) [2] Some data of f is shown in the graph on the right, what is f(Psychology) and what does it mean?

(+1) f(Psychology) $\approx$ 30,000
The median annual earnings of people with a Psychology undergrade major is $30,000. (very low?)

6. Let $\alpha(x) = \dfrac{1}{x-1}$ ~~and $\beta(x) = 2x + 5$~~. Both α and β have inverses that exist.

(a) [3] (§1.3 #32) Find $\alpha(x+h) - \alpha(x)$ and simplify.

$\alpha(x+h) - \alpha(x) = \frac{1}{(x+h)-1} - \frac{1}{x-1}$ (+1) $= \frac{1}{x+h-1} \cdot \frac{(x-1)}{(x-1)} - \frac{1}{x-1} \cdot \frac{(x+h-1)}{(x+h-1)}$ (+1)

fractions subtraction (+1) simplify (+.5)

$= \frac{x-1-(x+h-1)}{(x-1)(x+h-1)} = \frac{x-1-x-h+1}{(x-1)(x+h-1)} = \frac{-h}{(x-1)(x+h-1)}$

(b) [2] (InverseActivity #2) Identify a point (*any* point will do!) on the graph of $\alpha^{-1}(x)$ and explain how you know.

$\alpha(0) = \frac{1}{0-1} = -1$ (+1)

So $(0,-1)$ is on the graph of α

$\Rightarrow$ (+1) $(-1,0)$ is on the graph of α^{-1} since α^{-1} swaps the x's & y's of α

(c) [3] (§1.7 #78) Find the algebraic rule/expression for $\alpha^{-1}(x)$.

$x = \frac{1}{y-1}$ (+1)

$x(y-1) = 1$

$xy - x = 1$

$xy = 1 + x$

$y = \frac{1+x}{x}$

order of ops (+1) fractions (+.5) get it (+.5)

7. [3] Find the real or complex solutions to $\frac{2}{3}(x-1)^2 + \frac{5}{4} = 0$.

$\frac{2}{3}(x-1)^2 + \frac{5}{4} = 0$

$-\frac{5}{4} \quad -\frac{5}{4}$

$\frac{3}{2}\cdot\frac{2}{3}(x-1)^2 = -\frac{5}{4}\cdot\frac{3}{2}$

$(x-1)^2 = \frac{-15}{8}$

$x - 1 = \pm\sqrt{\frac{-15}{8}}$

$x = 1 \pm \sqrt{\frac{-15}{8}}$ or $1 \pm i\sqrt{\frac{15}{8}}$

order of ops (+1) both solutions (+1) algebra (+1)

or

$\frac{2}{3}(x-1)^2 + \frac{5}{4} = 0$

$\frac{2}{3}(x-1)(x-1) + \frac{5}{4} = 0$

$\frac{2}{3}(x^2 - 2x + 1) + \frac{5}{4} = 0$

$\frac{2}{3}x^2 - \frac{4}{3}x + \frac{2}{3} + \frac{5}{4} = 0$

$\frac{2}{3}x^2 - \frac{4}{3}x + \frac{8+15}{12} = 0$

$x = \frac{\frac{4}{3} \pm \sqrt{\left(\frac{4}{3}\right)^2 - 4\left(\frac{2}{3}\right)\left(\frac{23}{12}\right)}}{2\left(\frac{2}{3}\right)}$

$=$

8. Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

(a) You have 8 oz of mocha that is 10% espresso sitting in a 16 oz cup.

i. [3] Write a rational expression in x whose values give the percentage (in decimal form) of espresso in the cup when x oz of espresso are added to it.

ii. [2] Find the domain of the function in part i.

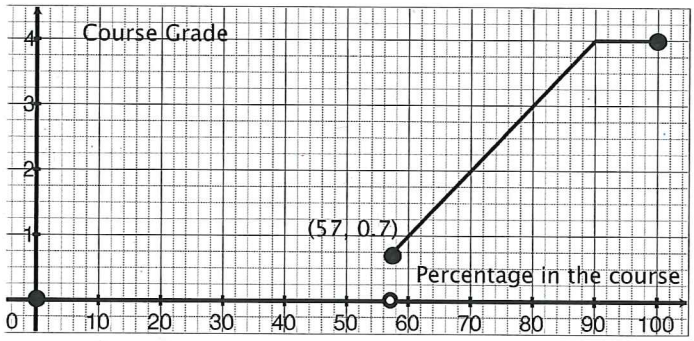
(b) (WordWks #11) Princess Leia is in this course and curious about her marks now that she's taken two exams. She has looked at the gradebook on MyMathLab and has computed the averages listed below. The weights specified in the syllabus and the graph of the function f that takes your class percentage x and returns your score on a 4. scale are also provided.

Assume Leia's work does not drastically change in the remaining 3 weeks and her averages remain about the same.

i. [3] Find a function that provides her overall course percentage as a function of her final exam score.

ii. [2] What minimum grade does she need to get on the final to receive a 4.0 in the course?

	weight	Leia's ave
Mini-Quizzes	5%	95%
WebAssign	10%	100%
WrittenHW	15%	100%
Quizzes	15%	83%
2 Exams	30%	95%
Final	25%	



a) i) 

% of esp = espresso amount / mocha amount (+.5)

= original esp + new esp (+.5) / original liquid + new liquid (+.5)

= $\frac{1.8 + x}{8 + x}$ (+.5)

= $\frac{.8 + x}{8 + x}$ got it (+.5)

Start (+.5)

ii) (+1) $0 \le x \le 8$ (+1)

Can't take away espresso b/c in mix of Mocha => $0 \le x$

can only fit 8 more oz in the cup => $x \le 8$

b) Let x = final exam score

Course % = .0595 + .1·100 + .15·100 + .15·83 + .395 + .25x (+.5)

Course % = 70.7 + .25x got it (+.5)

Start (+.5) mult term (+.5) add (+.5) percentages vs dec

ii) (+1) to get a 4.0 she needs 90%

90 = 70.7 + .25x alg (+1)

−70.7 −70.7

$\frac{19.3}{.25} = \frac{.25x}{.25}$

77.2 = x

She needs 77.2% on her final

$$\frac{\frac{1}{2}\times\frac{1}{4}\times\frac{1}{8}}{\frac{1}{50}}$$