Median: 87%

Key

Notes: only pt = between things that are equal

in base 2, don't see 2's b/c regroup? ex $2_{two} = 10_{two}$

Midterm **TMath 171** Autumn 2024

As a reminder, you are welcome to use a non-internet accessing calculator (which includes Desmos Test Mode) and one 1-sided 8.5 in by 11 in sheet of notes.

1. [9] Let a, b, and c be whole numbers. Are the following statement always true, sometimes true, or never true? Briefly justify your answer.

note whole numbers includes zero (if you define it otherwise I'll follow your lead)

(a) (Add & Sub Activity #3)

$$a - 0 = a$$

Start (1.5) sense (1.5)

Always true. (1.5) (1.5) Taking away zero does not change a number. justify (+1) ex +.5

(b) (Add & Sub Activity #3)

$$a - (b + c) = (a - b) + c$$

Start (1.5) Sense (1.5)

Sometimes true. (1.5) (1.5) (If $c = 0$ then $a-(b+c) = a-(b+0) = a-b = (a-b)+0$ (1.5) true

(1.5) { If $c \neq 0$ then not true. eg $(1-(2+3))$ vs $(1-2)+3$ -4 vs 2 => False

(c) (§3.4 Suggested #21)

$$(a + b) \div c = (a \div c) + (b \div c)$$

start (1.5) sense (1.5)

Always true (+1.5) (1.5) Distribution over addition (+1) ex +.5

(when defined anyway. If $c=0$ then neither side is defined)

2. [4] (§1.1 #25) There were ships with 3 masts and ships with 4 masts at the Tall Ships Exhibition. Millie counted a total of 30 masts on the 8 ships she saw. How many of these ships had 4 masts? Provide justification but you do NOT need to explain as you would to a 3rd grader.

Start (1.5)

(+.5) Let x = # of ships with 3 masts

y = # of ships with 4 masts.

+1.5 model of ships AND masts

(+1) total # of masts = # of masts of 3 mast ships + # of masts of 4 mast ships

$30 = 3x + 4y$

(+1) total # of ships = 3 mast ships + 4 mast ships

$8 = x + y$

(+1) Algebraically we can solve

$y = 8 - x$ and sub into $30 = 3x + 4y$

$\Rightarrow 30 = 3x + 4(8 - x)$ or $30 = 3x + 32 - 4x$ or $-2 = -x \Rightarrow x = 2$

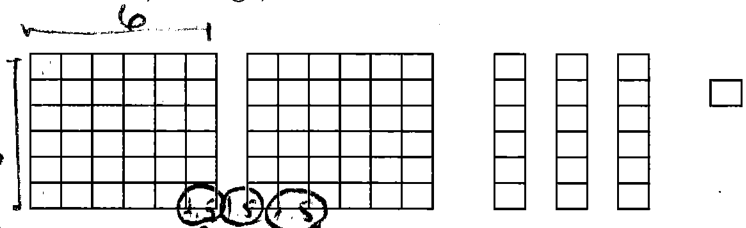
Since $y = 8 - x$ we know $y = 8 - 2$ or 6. (# of ships w/ 4 masts)

(# of ships w/ 3 masts) $x = 2$

13

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

3. Consider the number represented with 2 flats, 3 longs, and 1 unit shown below in the given base.



(a) [2] (§3.1 #11) Write the number of units in positional notation for the given base. 6

base (1.5)

2 flats + 3 longs + 1 unit

$\Rightarrow 231_{six}$

(b) [2] (Quiz3 #2) Determine the total number of units, reporting in the Hindu-Arabic number system.

2 flats + 3 longs + 1 unit $= 2(6^2) + 3 \cdot 6 + 1$ (1.5)

$= 2 \cdot 36 + 18 + 1$

$= 72 + 19 = 91$

powers of six (1)

(c) [5] (§3.3 classEx) Multiply the above number by three. Provide steps as you would for a 3rd grader. Be clear about how you communicate your answer.

start (.5)

reasonable steps (1)

correct language & idea (1)

intro algorithm model (1.5)

powers of 6/10 (1.5)

regroup (1)

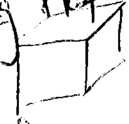
Lets use the base six version of the number so $231_{six} \times 3$. We can visualize this by stretching out 231_{six} by a factor of three or as 231_{six} added to itself three times. ~~do not love but is easier~~

Gathering the flats and longs together we have

6 flats + 9 longs + 3 units

We have enough flats we can regroup! Recall since we are in base 6, 6 flats can make 1 long flat / 1 cube

1 long flat + 9 longs + 3 units.



Similarly we can regroup 6 longs to make a flat. So 9 longs = 1 flat + 3 longs

1 long flat + 1 flat + 3 longs + 3 units

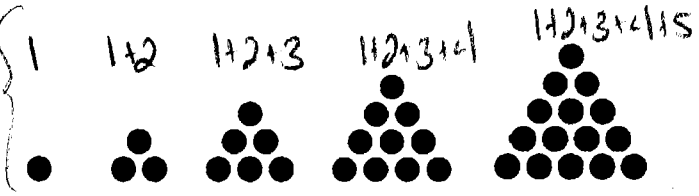
So our answer is 1133_{six}

also known as 273

got + (.5)

9

4. (PatternActivity #4) Consider the sequence of numbers illustrated below.



(a) [3] Find the next two numbers in the sequence.

start +1.5 notation +1.5

$15+6 = 21$ +1.5

$21+7 = 28$ +1.5

(b) [2] Identify if the sequence is recursive, arithmetic, geometric, or none of the above. Justify your answer.

not arithmetic +.5 not geometric +.5 +.5 Recursive. +.5 To get from the next step we add the step# to the previous entry. So $a_{n+1} = a_n + (n+1)$

(c) [2] Find the 50th number in the sequence.

see pattern +.5 start +.5 +1 $1+2+3+4+5+\ldots+48+49+50$

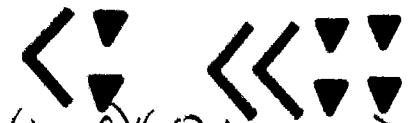
note we can pair these up

$51+51+51+\ldots+51$ (25 times)

so 51×25 or 1275

5. [4] (NumSysActivity #4) For each of the pairs of numbers below, determine which is bigger, justify your answer. (You do NOT need to explain as you would to a 3rd grader, just provide evidence.)

(a)



$(10+2)\times 60 + (20+4)\times 1$ +1

$12\times 60 + 24$

$720 + 24 = 744$

(BIGGER)

or 508_{nine} $= 5\times 9^2 + 0\times 9 + 8\times 1$ +1

$= 5\times 81 + 8$

$= 405 + 8$

$= 413$

Note that Babylonian could be interpreted as $12\times 60^2 + 24$ or $12\times 60^2 + 24(60)$ etc. in all cases, this is BIGGER

(b)

or 243_{twelve} $= 2\times 12^2 + 4\times 12 + 3$ +1

$= 2\times 144 + 48 + 3$

$= 288 + 51$

$= 339$

(BIGGER)

+1 $1+10+100\times 100+10\times 10\times 100+1\times 1$

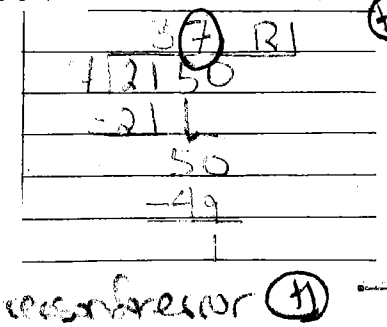
$11 + 200 + 2500 + 102$

$211 + 122$

333

6. The work below for both problems is wrong. Find the error(s) & try to detect the reason for the error.

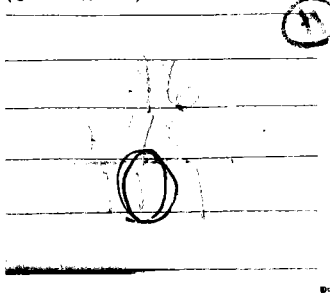
(a) [3] (§3.4 Suggested #19)



Start +.5
sense +1.5
reasonableness +1

+1 The trouble is in the tens spot of the quotient.
The number 7 does not easily divide 5 tens so we would record that with a zero in the tens spot. It looks like the student brought down the 50 (units) together & forgot to record the zero in the tens.

(b) [2] (§3.2 #22)



Start +.5
reason +.5

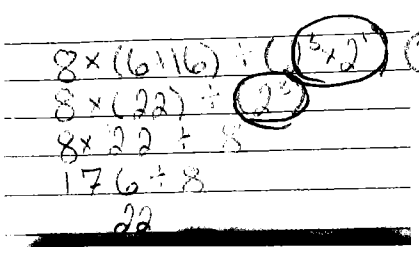
+1 The trouble is in the tens spot of the sum. It looks like when adding the units 8+6 = 14 the student forgot to regroup the 10 from the 14 and add it to the tens spot/re carry.

(c) [2] On one division problem, a large number M was divided by 36. The student ended up with a quotient q and a remainder of 40. The student is certain they are correct since $M = 36q + 40$.

Start +.5
remainder +1

The error is in the division algorithm
$M = 36q + 40 = 36q + 36 + 4 = 36(q+1) + 4$
+1.5 We could increase the quotient by one - as the remainder should be less than 36.

(d) [2] (§3.4 Suggested #35)



Start +.5
+1

The error is with the exponents.
Since $2^3 \times 2^1 = (2\times2\times2)\times2 = 2^4$
but this work wrote 2^3 on the next line.

9

Note there are LOTS of right answers

7. [3] Find a number that:

(+1) • is not written in base 10, ✓
(+1) • has 4 digits, and ✓
(+1) • is made of less than 50 units. ✓

$1000_{two} = 1\times2^3 + 0\times2^2 + 0\times2^1 + 0$
$= 8$ < 50 ✓

8. Show work and compute:

(a) [2] (§3.2 #8) $43_{five} - 14_{five}$ $(4\times5+3) - (1\times5+4) = 23-9 = 14$

placement system (+1) regrouping (+1) got it (+3)

— OR —

(4 longs + 3 units) − (1 long + 4 units) = 3 longs + 3 units − 4 units
= 3 longs − 1 unit = 2 longs + 5 − 1 units = 24_{five}

(b) [2] (§3.2 Suggested #7) $43_{five} + 14_{five}$ (regroup)

placement system (+1) regrouping (+1) got it (+3)

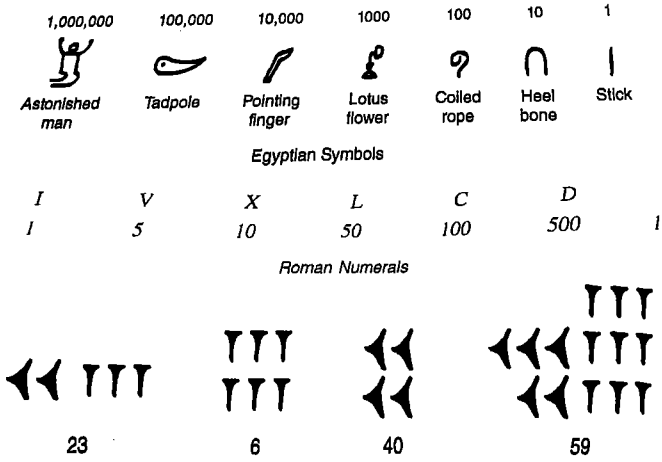
$43_{five} + 14_{five} = (4\times5+3) + (1\times5+4) = 23+9 = 32$

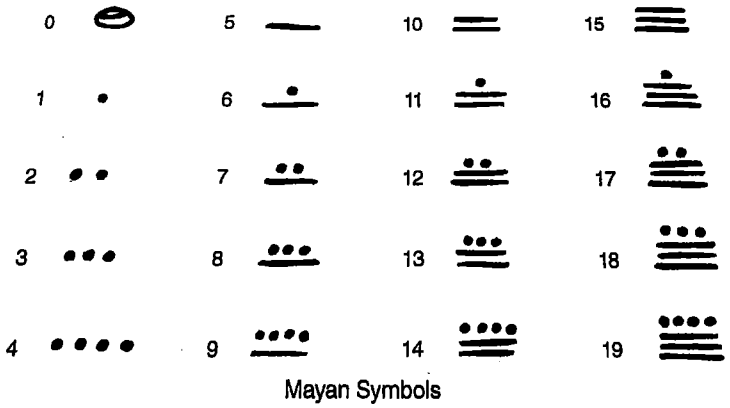
— OR —

(4 longs + 3 units) + (1 long + 4 units) = 5 longs + 7 units
= 1 flat + 7 units = 1 flat + 1 long + 2 units = 112_{five}

9. [1] What topic or concept did you study and prepare for, but not see on the exam?

name a topic (+1)





8

Tue 10/29 Class/ Mellen (Ch 1+3)
Rewrite HW 4 Dc

Thur 10/31 Class/ 4.2

Fri 11/1 no Written HW Dc
TQ Ch 3 Dc

Tue 11/5 Class/
Rewrite [illegible] Dc

22
20
8
50