

## FINAL

## TMath 125

## PRACTICE

Note: This is a practice exam and is intended only for study purposes. The actual exam will contain different questions and may have a different layout.

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

1. Each of the following is wrong. Explain why.

(a) 
$$\int_a^b f(x)g(x) dx = \int_a^b f(x) dx * \int_a^b g(x) dx$$

(b) 
$$\int_1^4 \frac{3 + \sqrt{x} + x}{x} dx = \int_1^4 \frac{3}{x} + x^{-\frac{1}{2}} + 1 dx = 3 \ln(x) + 2x^{\frac{1}{2}} + x + c$$

(c) One problem required the substitution of  $x = 3 \csc(\theta)$ . Then  $\cos(\theta) = \frac{\sqrt{9-x^2}}{3}$ .

(d) The volume whose base is bounded by  $y = 0$ ,  $x = 0$ ,  $x = 1$  and  $f(x) = \frac{1}{x^2 + 3x + 2}$  and has square cross sections perpendicular to the  $x$ -axis has volume equal to 
$$\int_0^1 \pi \left( \frac{1}{x^2 + 3x + 2} \right)^2 dx.$$

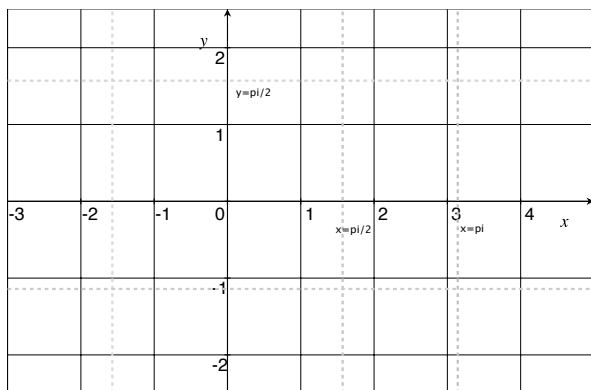
2. Explain the second Fundamental Theorem of Calculus.

3. Find  $\frac{d}{dx} \int_0^{x^2+3x} e^{t^2} dt$

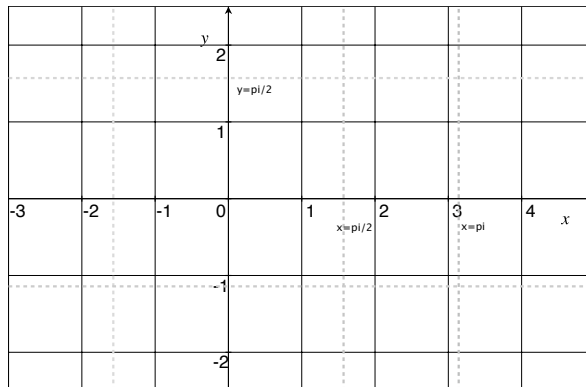
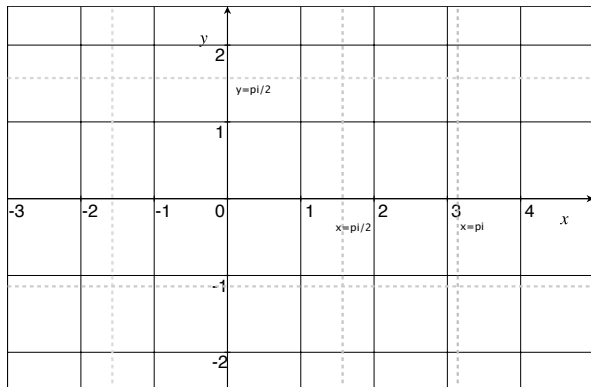
4. Let  $v$  be the function that records the velocity of a particle which is well approximated by the following formula.

(a) Carefully graph  $v(t)$  on the set of axis.

$$v(t) = \begin{cases} -2 & t \leq -1 \\ 2t & \text{if } -1 \leq x \leq 0 \\ \sin t & \text{if } 0 < t \end{cases}$$



(b) Give a rough sketch of the function recording the acceleration of the particle on the set of axis on the left.

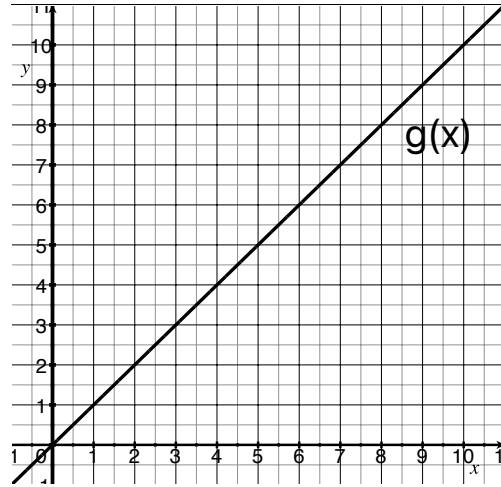


(c) Give a rough sketch of the graph  $\int_{-3}^x v(t) dt$  on the set of axis on the right.

(d) Describe the physical meaning of  $\int_{-3}^x v(t) dt$ .

5. Let  $g$  be the line graphed below on the right. Let  $f$  be a function that is continuous and twice differentiable to continuous functions. Assume that we also have the following values for  $f$  and  $f'$ .

$x$	$f(x)$	$f'(x)$
0	2	3
4	7	5



(a) Find  $f(4)$ .

(b) Find  $g'(4)$ .

(c) Evaluate  $\int_3^5 g(x) dx$

(d) Find the rule for  $\frac{d}{dx} \left( \int_0^x g(t) dt \right)$  for  $x > 0$ .

(e) Evaluate  $\int_0^4 f'(x) + 1 dx$

(f) Given  $\int_0^2 f'(x) dx = 1$ , find  $\int_2^4 f'(x) dx$

(g) Evaluate  $\int_0^4 g(x) f''(x) dx$

6. For each of the following outline the method(s) you would use to find the general antiderivative. For example, if you think trigonometric substitution would work write "trigonometric substitution" and identify what substitution ( $u$ ) you would use.

$$\int_0^{\frac{\pi}{4}} \sec^4 x \tan^4 x \, dx$$

$$\int x \cos^2 x \, dx$$

$$\int_1^{\infty} \frac{1}{x^2} \, dx$$

$$\int \frac{1}{x^2 \sqrt{x^2 + 4}} \, dx$$

$$\int_0^3 \frac{1}{x-1} \, dx$$

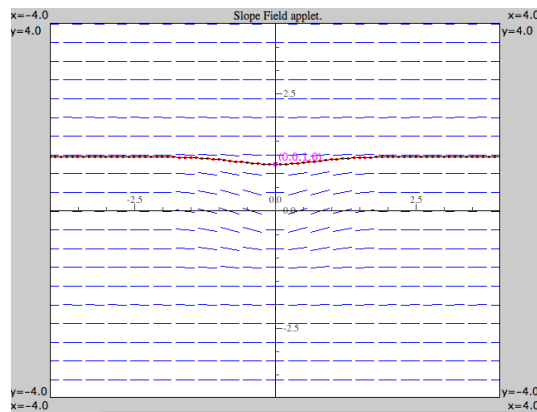
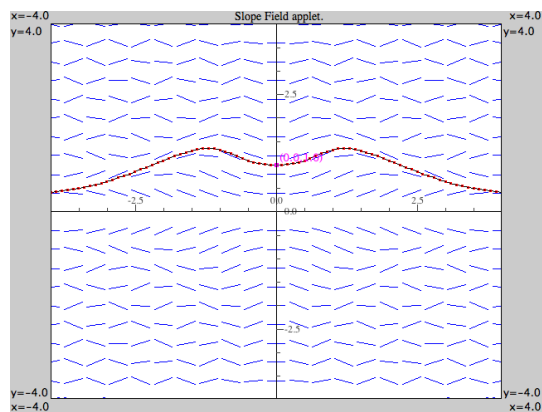
$$\int \frac{17x-1}{2x^2+3x-2} \, dx$$

7. Match the differential equations with the solutions graphs.

*Justify* your choice.

(a)  $y' = xe^{-x^2-y^2}$

(b)  $y' = \sin(xy) \cos(xy)$



[h]

8. Write the following in sigma notation  $-\frac{1}{3} + \frac{3}{7} - \frac{1}{2} + \frac{5}{9} - \frac{3}{5} + \frac{7}{11}$

9. Expand  $\sum_{i=-3}^1 \frac{i \ln(x)}{(-1)^i(5+i)}$

10. Find the area of the region bounded by  $y = x^2$ , the tangent line to this parabola at  $(1, 1)$ , and the  $x$ -axis.

11. Consider the region trapped between  $f(x) = \frac{1}{x}$ , the  $x$ -axis, and from  $x = 0$  to  $x = 1$ .

(a) If this region was revolved about the  $y$ -axis, what would the resulting volume be?

(b) What would its volume be if it was revolved about the  $x$ -axis?

12. Spirit accidentally steered off a 7250m cliff on Mars. Thankfully, Spirit was attached to a base station at the top of the cliff by a 1000 meter cable that is .6 kg per meter. How much work does it take to haul Spirit back up to the top of the cliff. Gravity on Mars is  $3.69 \frac{\text{m}}{\text{s}^2}$  and Spirit has 185kg of mass.

13. The University of Washington, Tacoma has 3000 students. On Monday two students noticed police outside Dr. Card's office and heard Dr. Card was dead. A rumor began to spread and by Tuesday 200 students had heard it. It is reasonable to assume that rate of the spread of the rumor is proportional to the number of possible encounters between students who have heard the rumor and those who have not. Let  $y = y(t)$  be the number of students who have heard the rumor after  $t$  days. Write a differential equation that describes the above model and solve the differential equation for  $P(t)$ .