Key

EXAM 1 TMath 125 Spring 2026

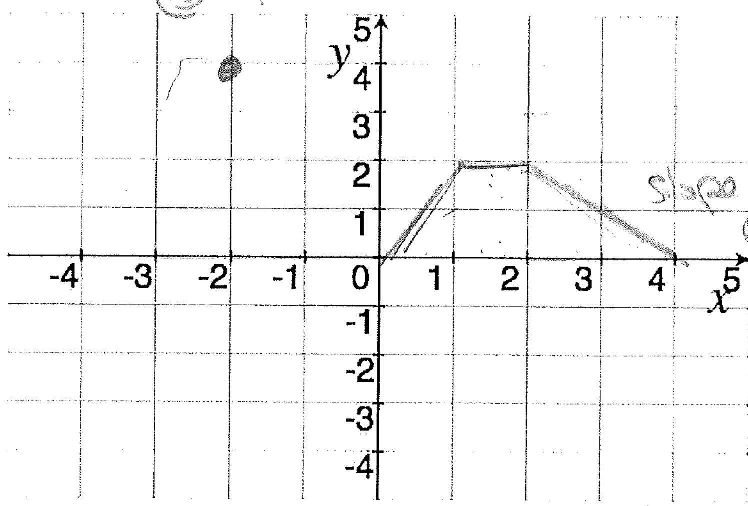
Show *all* your work. Reasonable supporting work must be shown for any partial credit.

1. [4] (Quiz0 #4) Sketch the graph of a function α that satisfies *all* of the following. (+1)

(a) $\alpha(-2) = 4$ (+1)

(b) $\alpha'(3) = -1$ (+1)

(c) $\int_0^3 \alpha(x)\,dx = 5$ (+1)



Slope of line tang @ $x=3$ is -1

2. [2] (SummationActivity #1) Expand $\sum_{i=0}^{3} \frac{(-1)^i \cdot i}{2}$

form (+.5) sum (+.5) $i=0$ (+.5) to $i=3$ (+.5)

$$\frac{(-1)^0 \cdot 0}{2} + \frac{(-1)^1 \cdot 1}{2} + \frac{(-1)^2 \cdot 2}{2} + \frac{(-1)^3 \cdot 3}{2}$$

3. [8] (WebHW5.5 #1 & PracticeExam#5) Find:

$\int 3xe^{-x^2}\,dx$ (+.5) $3\int e^u x\,dx$

$u = -x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

try sub (+.5)
did sub right (+1)
integrate e^u (+.5)
notation (+1)

$= 3\int e^u \left(-\frac{1}{2}\right) du$

$= -\frac{3}{2}\int e^u\,du = -\frac{3}{2}e^u + C$

$= -\frac{3}{2}e^{-x^2} + C$ (+.5)

$\frac{d}{dx}\left(\int_0^{3x+5} \frac{t}{1+t^3}\,dt\right)$

FTC I that derivatives & integrals undo each other (+1)

chain rule?
inside function $= 3x+5$
inside$'$ $= 3$

try chain (+.5)
did chain (+1)
$\frac{d}{dx}(3x+5)$ (+.5)

$\frac{d}{d\,\text{inside}}\left(\int_0^{\text{inside}} \frac{t}{1+t^3}\,dt\right) = \frac{(\text{inside})}{1+(\text{inside})^3}$

so chain rule

$$\Rightarrow \frac{3x+5}{1+(3x+5)^3} \cdot 3$$

start (+.5)
notation (+.5)

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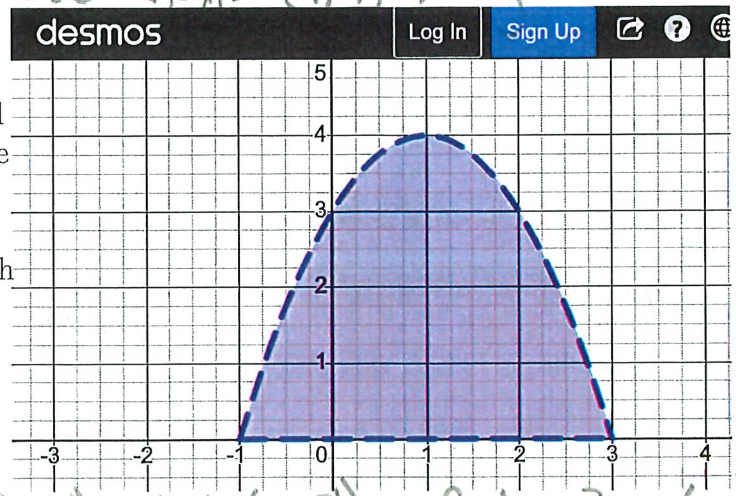
or $-(x^2-3x+x-3) = -x^2+2x+3$

note vertex @ (1,4) ⇒ $y=-(x-1)^2+4$ works too :)

4. [3] (DefiniteActivity #2)
Describe the shaded area trapped below the parabola and above the x axis as a definite integral. Make sure you write it in such a way that technology could finish the problem for you.



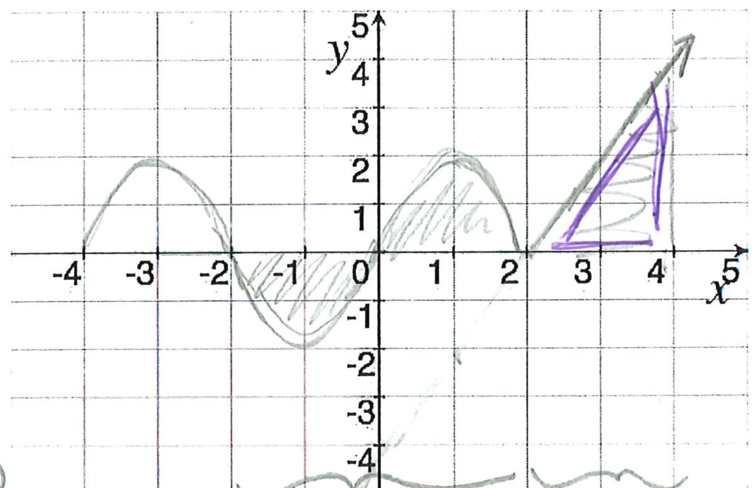
$\int_{-1}^{3}$ parabola dx (+1) (+.5)

$\int_{-1}^{3} -(x+1)(x-3)\,dx$

roots −1 ⇒ (x − −1) is factor
roots 3 ⇒ (x−3) is a factor
⇒ (x+1)(x−3) (+1)
open down −1 (+.5)

5. Let $f(x)$ be defined below.

$$f(t) = \begin{cases} 2\sin\left(\frac{\pi}{2}t\right) & \text{if } t < 2 \\ 2t-4; & \text{if } 2 < t \end{cases}$$



(a) [3] (PracticeExam1 #3)
Graph f on the axis.

bounds (+.5) line (+1) trig (+1.5)

freq $\frac{2\pi}{\pi/2} = 4$

(b) [3] (§5.2 #60)
Find $\int_{-2}^{4} f(t)\,dt$.

notation (+.5) area shaded (+.5)

(+1) cancel each other out

$\frac{1}{2}\cdot$base$\cdot$height (+1)

$\frac{1}{2}\cdot 2\cdot 4 = 4$

or

$\int_{-2}^{2} 2\sin\left(\frac{\pi}{2}t\right)dt + \int_{2}^{4} 2t-4\,dt$ (Desmos) ⇒ 4

(c) [2] (WebHW5.4&5.3 #12) Let $g(x) = \int_{2}^{x} f(t)\,dt$, find $g(4)$.

$g(4) = \int_{2}^{4} f(t)\,dt = \boxed{\text{area}} = \frac{1}{2}\cdot\text{base}\cdot\text{height} = \frac{1}{2}(2)(4) = 4$

(+.5) (+.5) (+1)

(d) [2] (Quiz2 #4) Find $g'(4)$.

$g'(4) = \frac{d}{dx}\left(\int_{2}^{x} f(t)\,dt\right)\Big|_{x=4} = f(4) = 4$

(+.5) (+1) (+.5)

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6. [4] (Exam1 '23#7 & Quiz2) #1) Each of the following is wrong. Find the step with the error and explain why it was wrong.

(a) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin(x)}{\cos^2(x)}\, dx \overset{+.5}{=} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{-1}{u^2}\, du = u^{-1}]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \left(\frac{\pi}{3}\right)^{-1} - \left(\frac{\pi}{6}\right)^{-1} = \frac{3}{\pi} - \frac{6}{\pi} = \frac{-3}{\pi}$

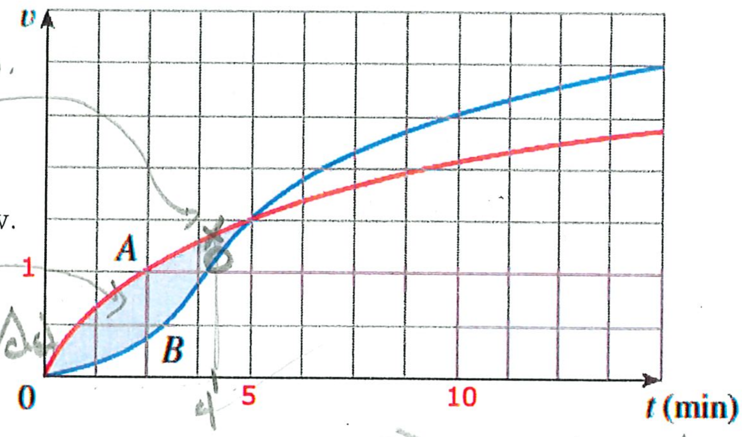
sense +.5 if set $u=\cos(x)$

+1 and we replace dx with du we need to either change the bounds to be in terms of u - or do the general antider. elsewhere & come back.

(b) $\int_1^8 \frac{3}{\sqrt[3]{x}}\, dx = \int_1^8 3x^{\frac{-1}{3}}\, dx \overset{+.5}{=} 3\left(\frac{-1}{3}\right) x^{\frac{-4}{3}}|_1^8 = -1 \cdot 8^{\frac{-4}{3}} - (-1 \cdot (1)^{\frac{-4}{3}}) = -9.375$ looks like extra error here too? .9375

sense +.5 +1 looks like we took the derivative instead of an antiderivative

7. (WebHW6-1#9) Two cars A and B start side by side and accelerate from rest. The figure shows the graphs of their velocity functions (km per min).



start +.5 (a) [2] Which car is going faster at $t = 4$? Explain how you know.

+1 velocity is on the vert. axis. Car A has a higher y-coord @ x=4

+.5 (b) [2] Which car is in the lead when $t = 4$? Explain how you know.

sense +.5 +1 Dist. is recorded by area trapped below graph.

+.5 Car A has more area/dist

(c) [2] Which car is accelerating more when $t = 4$? Explain how you know.

+.5 Acceleration is the derivative of velocity

+.5 => the slope of line tang. to curves @ x=4.

+.5 notice the slope +.5 of car B is steeper than car A @ x=4

(d) [3] What is the meaning of the shaded region in the context of this problem?

sense +.5 start +.5 The shaded region is the distance car A is +.5 ahead of car B at x=5 minutes. +1 +.5

(e) [2] Consider the distance between the cars at 5 minutes. Is it greater than 6km or less? Provide reasoning. +.5

+.5 units that are shaded ≈ 3 squares.

+1 each square is about $\frac{1}{2}\left(\frac{\text{km}}{\text{min}}\right) \cdot \left(\frac{5}{4}\right) \text{min} = \frac{5}{8}$ km

so 3 squares => $3 \cdot \frac{5}{8}$ km $= \frac{15}{8} < 2$ km.

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$\frac{27}{23}$
$\overline{50}$

8. [3] (WebHW5.1 #4) Oil leaked from a tank at a rate of $r(t)$ liters per hour. The rate decreased as time passed and values of the rate at variable time intervals are shown in the table. Estimate a worst case scenario by overestimating total oil leaked over the 10 hours.

sketch (+.5)

t (hours)	0	2	3	6	7	10
$r(t)$ (Liters/hour)	8.7	7.6	6.8	6.2	5.7	5.3

(+1) [We'll use the left end points to overestimate (bigger than right side)

$2 \cdot 8.7 + 1 \cdot 7.6 + 3 \cdot 6.8 + 1 \cdot 6.2 + 3 \cdot 5.7 = 68.7$ (+.5)

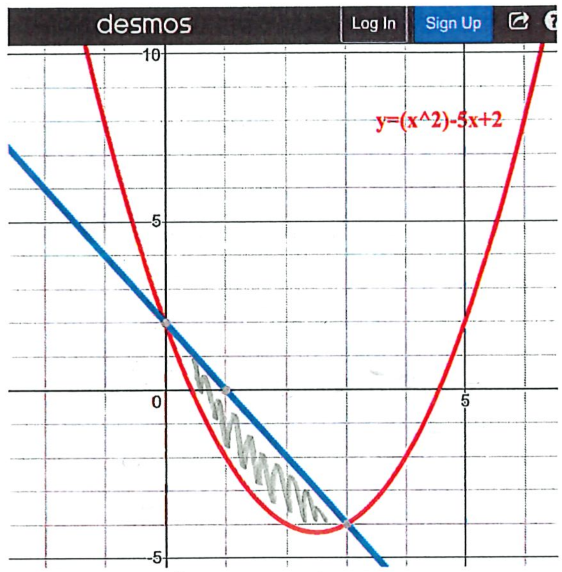
(+.5)

note the Δtime changes (+.5)

9. (WordProblem1 #13) Consider the parabola $y = x^2 - 5x + 2$ & the line graphed below.

(a) [1] Shade the area trapped by the parabola and the line.

(b) [4] Set up the definite integral (but do *not* compute!) that will find the area of the region trapped by the line and the parabola.
Make sure you write it in such a way that technology could finish the problem for you.



$$\int_0^3 \text{line} - \text{parabola}\, dx = \int_0^3 (-2x+2) - (x^2-5x+2)\, dx$$

(+1) (+.5) (+.5)

finding equation of line

want $y = mx + b$ (+.5)

$m = \frac{\text{rise}}{\text{run}} = \frac{-2}{1} = -2$ (+.5)

$b =$ y intercept @ $y = 2$ (+.5)

So $y = -2x + 2$ (+.5)

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