fraction: $\frac{2+5}{2} \times 5$

simplifying $\frac{2(1+5)}{2} = 6$

Median: 63%

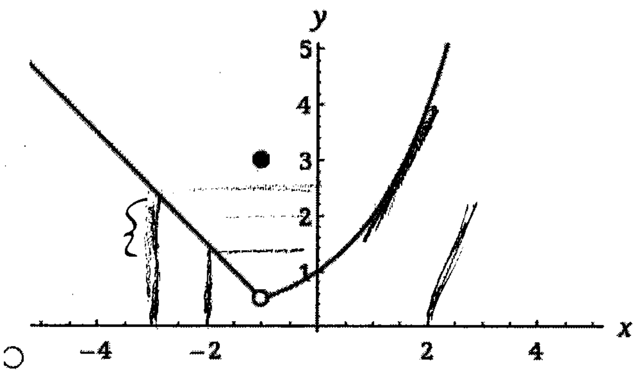
Ave: 67%

Key

EXAM 2 TMath 124 Spring 2024

Show *all* your work (numerically, algebraically, or geometrically) for the following problems. Supporting work is needed to earn credit.

1. Let $f(x) = \sin(\pi x)$.
The graph of g is given on the right. Estimate (if possible):



(a) [1] (TrigPractice#4) $f(-2)$

$f(-2) = \sin(\pi(-2)) = \sin(-2\pi)$

$= 0$

correct function (+.5)
eval (+.5)

(b) [2] (ProductActivity #1) $\frac{d}{dx}(g(x))|_{x=-2}$.

$\frac{d}{dx}(g(x))\Big|_{x=-2}$ = slope of line tangent to g @ x = -2

correct function (+.5)

$= \frac{\text{rise}}{\text{run}} = \frac{-1}{1} \approx -1$ (+.5)

graph reading (+1)

(c) [2] an x value where $g'(x) \approx 2$

correct function (+.5)
start (+.5)

input where slope of line tang. to g is about 2 (+.5)

≈ 2 (+.5)

(d) [3] (WebHW8#7) $\frac{d}{dx}(f(x)g(x))|_{x=-2}$

(+.5) product rule
(+.5) use correctly

$= f(-2)g'(-2) + f'(-2)g(-2)$

$= \underbrace{0\ (-1)}_{\text{plug in (+.5)}} + \underbrace{(\pi\cos(\pi(-2)))}_{(+.5)} \cdot \underbrace{1.5}_{(+.5)}$

(+.5) $f(x) = \sin(\pi x)$
$f'(x) = \pi\cos(\pi x)$ chain rule

(e) [3] (§3.4 #72) $(f \circ g)'(0)$

(+.5) Chain rule
(+.5) use correctly
(+.5) notation

$= f'(g(0))\, g'(0)$

$= \underbrace{f'(1)}_{(+.5)} \cdot \underbrace{1}_{(+.5)}$

$= \pi(\cos(\pi \cdot 1))$ (+.5)

$= \pi(-1) = -\pi$

11

try pluging in +.5

2. [2] (PracticeExam2#1) Find $\lim_{\theta\to 0} \frac{\sin(7\theta)}{\theta}$ (either numerically, graphically, or algebraically), if it exists.

start table +.5 (+1) both sides

(+.5) notation

algebraically:

(+.5) { Recall $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$ (+.5)

alg (+.5) So $\lim_{x\to 0} \frac{\sin 7x}{x} \cdot \frac{7}{7} = \lim_{x\to 0} \frac{7}{1} \frac{\sin 7x}{7x}$

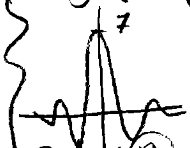
calc (+.5) $= \lim_{x\to 0} \frac{7}{1} \cdot \lim_{x\to 0} \frac{\sin 7x}{7x} = 7(1)$

or numerically

θ	$-.0001$	$.0001$	$.01$
$\frac{\sin(7\theta)}{\theta}$	6.99999	6.99999	6.99

so limit is 7 (+.5)

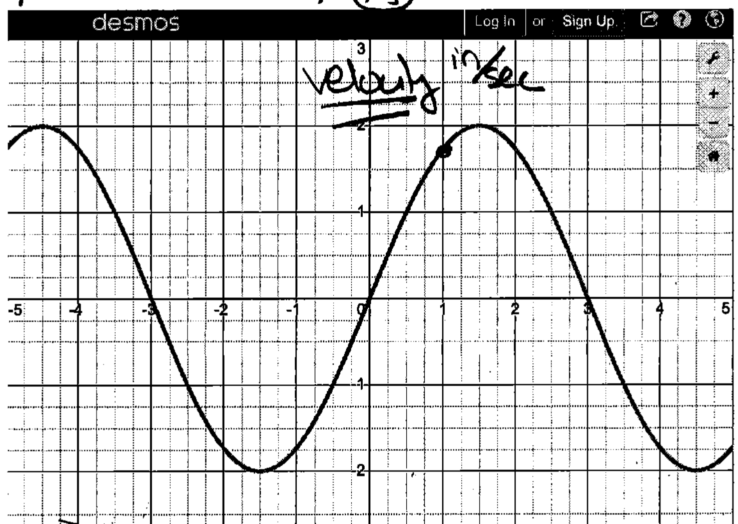
or graphically



picked (+1)

so limit is 7

3. Consider a particle that is moving with *velocity* is $v(t) = 2\sin\left(\frac{\pi t}{3}\right)$ (inches per second), graphed on the right.



(a) [1] Find the velocity when $t = 1$.

use correct function (+.5)

$v(1) = 2\sin\left(\frac{\pi \cdot 1}{3}\right)$

$= 2 \frac{\sqrt{3}}{2} = \sqrt{3}$

eval (+.5)

≈ 1.72

(b) [2] (WordProblem2#3) Find 2 times that the particle is at rest?

partial graph reading +.5

use correct function (+.5)

ie when velocity $= 0$
ie $v(t)$ crosses t-axis } (+.5)

got one (+.5)
got two (+.5)

about $-3, 0, 3$ (every 3 seconds)

(c) [3] (WordProblem2#3) Find the acceleration as a function of t.

notation (+.5)

(+.5) { acceleration $= \frac{d}{dt}(v(t))$

$= \frac{d}{dt}\left(2\sin\left(\frac{\pi t}{3}\right)\right)$

Chain rule (+.5)
did right (+.5)

$= 2\frac{d}{dt}\left(\sin\left(\frac{\pi t}{3}\right)\right)$

(+.5) (+.5)

$= 2\left(\cos\left(\frac{\pi t}{3}\right)\right) \cdot \frac{\pi}{3}$

$= \frac{2\pi}{3}\cos\left(\frac{\pi t}{3}\right)$

(d) [3] (§3.3 #40) Find a time when the acceleration is -1(inch per second squared).

only need one

start (+.5)
got it (+.5)

(+1) { acceleration $= -1$

$= \frac{d}{dt}(v(t))$

$\Rightarrow$ from (c)

$\frac{2\pi}{3}\cos\left(\frac{\pi t}{3}\right) = -1$

OR

algebraically:

(+1) { $\frac{2\pi}{3}\cos\left(\frac{\pi t}{3}\right) = -1$

$\cos\left(\frac{\pi t}{3}\right) = \frac{-3}{2\pi}$

$\frac{\pi t}{3} = \arccos\left(\frac{-3}{2\pi}\right)$

$t = \left[\arccos\left(\frac{-3}{2\pi}\right)\right]\frac{3}{\pi}$

$= 1.975$ works

(+1) { graphically $\approx t = -1.975$
1.975
4.025 work

4. [3] (WebHW9#7 or WebHW8#1) Find $\frac{dy}{dx}$ of *ONE* of the listed functions below. Doing both does not earn extra credit and only one will be marked so clearly indicate what you want marked!

note: logarithmic differentiation work well on each of these too?

$$y = \left(\frac{1}{x^3} + 3\right) e^x \qquad \text{OR} \qquad y = \frac{x + 7\sqrt{x}}{\cos(x)}$$

product (+.5) correct product (+.5)

$$y' = \left(\frac{1}{x^3}+3\right)(e^x)' + \left(\frac{1}{x^3}+3\right)' e^x$$

$$= \left(\frac{1}{x^3}+3\right)e^x + \left(x^{-3}+3\right)' e^x$$

(+.5) (+.5) (+1)

$$= \left(\frac{1}{x^3}+3\right)e^x + \left(-3x^{-4}+0\right)e^x$$

quotient (+.5) correct quotient (+.5)

(+.5)

$$y' = \frac{\cos(x)\left[x+7x^{1/2}\right]' - (x+7\sqrt{x})\left[\cos x\right]'}{(\cos(x))^2}$$

(+1) (+.5)

$$= \frac{\cos(x)\left[1 + \tfrac{7}{2}x^{-1/2}\right] + (x+7\sqrt{x})\sin x}{(\cos(x))^2}$$

5. The differentiable functions f and g are defined for all real numbers. Values for f, f', g, and g' for various x values are given in the table.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	4	2	6
2	1	5	8	7
3	7	7	2	9

(a) [1] Find $g(2)$. 8

(b) [3] (WebHW11#10) Find the linearization of g at $x = 2$.

start (+.5)

(+.5) find eq of a line $\quad y - y_1 = m(x - x_1)$ $\quad$ plug in (+.5)

(+.5) m = slope of line tang. to g @ x=2 $= g'(2)$

(+.5) $= 7$

thru $(2, g(2))$, $(2, 8)$ (+.5)

$y - 8 = 7(x-2)$

or

$y = 7x - 14 + 8$
$= 7x - 6$

(c) [1] (§3.10#52a) Use the linearization of g to approximate $g(2.05)$.

plug 2.05 in for x in (b) (+.5) $\quad y - 8 = 7(2.05 - 2) \rightsquigarrow y = 8.35$ end (+.5)

(d) [1] (ImplicitDifActivity#5) Given that $h(x) = [f(x)]^{g(x)}$, find $h(1)$.

$h(1) = [f(1)]^{g(1)}$ (+.5)

$= [3]^2$ (+.5)

$= 9$

(e) [4] (ImplicitDifActivity#5) Given that $h(x) = [f(x)]^{g(x)}$, find $h'(1)$.

partial if try +.5
if try +.5
if try +.5

$h'(x) = \left([f(x)]^{g(x)}\right)'$ cannot use power rule, use exp. rule

⇒ use logarithmic d.c (+.5)

(+.5) $[\ln h(x) = \ln\left([f(x)]^{g(x)}\right)$

properties (+.5) $[\ln h(x) = g(x)\ln[f(x)]$

now we can differentiate

(+1) (+1)

$$\frac{1}{h(x)} \cdot h'(x) = g(x)\frac{1}{f(x)}f'(x) + g'(x)\ln(f(x))$$

$$\Rightarrow h'(x) = h(x)\left[g(x)\frac{1}{f(x)}f'(x) + g'(x)\ln(f(x))\right]$$

plug in (+.5) $\quad 9\left[2 \cdot \frac{1}{3} \cdot 4 + 6\ln(3)\right]$ B

6. [5] (WebHW10#8 or PracticeExam2#4) Find $\frac{dy}{dx}$ of *ONE* of the listed functions below. Doing both does not earn extra credit and only one will be marked so clearly indicate what you want marked!

start (+.5)

try something (+.5)

$$y = \frac{4^x \cdot \log_4(x)}{x^2 - 4x} \qquad\qquad y + x8^y = \ln(x)$$

either product + chain rule in quotient or logarithmic df. I'll do quotient

↳ on extra page

quotient (+1)

$$y' = \frac{(x^2-4x)(4^x \cdot \log_4(x))' - 4^x \log_4(x)[x^2-4x]'}{[x^2-4x]^2}$$

product (+1)

$$= \frac{(x^2-4x)\left[4^x[\log_4(x)]' + [4^x]'\log_4(x)\right] - 4^x\log_4(x)[2x-4]}{[x^2-4x]^2} \quad (+1)$$

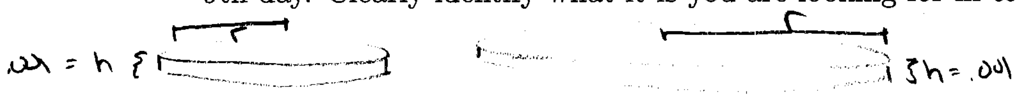
$$= \frac{(x^2-4x)\left[4^x \cdot \frac{1}{x\ln(4)} + 4^x\ln 4 \cdot \log_4(x)\right] - 4^x\log_4(x)[2x-4]}{[x^2-4x]^2}$$

(+.5) (+.5)

7. Suppose there is an oil spills that is spreading in a cylindrical pattern that has uniform thickness of .001meters. On day 9 the area of the spill area was 13,000km^2 and the radius of the spill was increasing by about 10 meters a day.

(a) [2] (WebHW11 #1)Find a function for the *rate the volume* is changing as a function of the radius r, and the rate of change of r.

(b) [3] (RelatedRatesActivity#2) Find the rate that the volume was changing on the 9th day. Clearly identify what it is you are looking for in calculus notation.



a) Volume $= h \cdot \pi r^2$ (+.5)

$= .001\pi r^2$ (+.5)

$\Rightarrow$ rate volume is changing $= \frac{dV}{dt} = \frac{d}{dt}(.001\pi r^2) = .001\pi \cdot 2r \frac{dr}{dt}$ (+.5) (+.5)

b) looking for $\left.\frac{dV}{dt}\right|_{9\text{th day}}$ (+.5)

start (+.5)

have $\frac{dr}{dt} = 10 \text{ m/day}$ (+.5)

need to find r on day 9 (+.5)

Area $= 13000 \text{ Km}^2$

$\pi r^2 = 13000 \text{ Km}^2$

$r = \sqrt{\frac{13000 \text{ Km}^2}{\pi}}$

$= 64 \text{ Km}$

$= 64000 \text{ m}$ (+.5)

$= .001\pi \cdot 2r \cdot 10 \text{ m/day}$

$= .001\text{m}\,\pi \cdot 2(64000)\text{m} \cdot 10 \text{ m/d}$

$\approx 4021 \text{ m}^3/\text{day}$

plug in (+.5)

10

(6) find dy/dx given $y + x\,8^y = \ln(x)$

We'll need to use implicit dif. b/c y is not solved for

$$\frac{d}{dx}\left(y + x\,8^y = \ln(x)\right)$$

$$\overset{+.5}{\frac{dy}{dx}} + \overbrace{x\,(8^y)' + (x)'\cdot 8^y}^{+1 \text{ product}} = \overset{+.5}{\frac{1}{x}}$$

start (+.5)

try something (+.5)

$$\frac{dy}{dx} + x\,\underbrace{8^y \ln 8 \left(\frac{dy}{dx}\right)}_{+.5} + \underbrace{1}_{+.5}\cdot 8^y = \frac{1}{x}$$

$$\frac{dy}{dx}\left[1 + x\,8^y \ln 8\right] = \frac{1}{x} - 8^y$$

$$\frac{dy}{dx} = \frac{\frac{1}{x} - 8^y}{1 + x\,8^y \ln 8}$$

solved for dy/dx (+1)

22
23
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total 45