Depth-averaged Models and Software for Geophysical Flows

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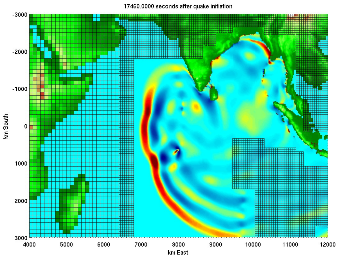

GeoClaw: http://www.clawpack.org

Collaborators

David George, University of Washington
Soon to be Mendenhall postdoctoral Fellow at the
USGS Cascades Volcano Observatory (CVO)

Marsha Berger, Courant Institute, NYU

Roger Denlinger and Dick Iverson,
USGS Cascades Volcano Observatory (CVO)

David Alexander and William Johnstone,
Spatial Vision Group, Vancouver, BC
Barbara Lence, Civil Engineering, UBC

Harry Yeh, Civil Engineering, OSU

Numerous other students and colleagues

Supported in part by NSF

Outline

- Why use depth-averaged models?
- Current and potential applications
- Why AMR is often crucial

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- Shallow water equations
- Dry states and margins
- Steady states and well-balancing

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- Depth-averaged models of complex flows
- Other applications
- Some mathematical challenges

Depth-averaged models

Two-dimensional Euler:

$$
\begin{aligned}
(\rho U)_t + (\rho U^2 + p)_x + (\rho U W)_z &= 0 \\
(\rho W)_t + (\rho U W)_x + (\rho W^2 + p)_z &= 0 \\
\rho_t + (\rho U)_x + (\rho W)_z &= 0 \\
U_x + W_z &= 0.
\end{aligned}
$$

This is generally a free-surface problem for $B(x) \leq z \leq \eta(x,t)$
$B(x) =$ bottom or bathymetry, $\eta(x,t) =$ surface.

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Assume:

$\rho(x,z,t) =$ constant (homogeneous density)
$W(x,z,t) = 0$ (vertical velocity negligible)
$p(x,z,t) = g(\eta(x,t) - z)$ (hydrostatic pressure)

Depth-averaged models

Let

$$h(x,t) = \eta(x,t) - B(x), \quad \text{(fluid depth)}$$

$$u(x,t) = \frac{1}{h(x,t)} \int_{B(x)}^{\eta(x,t)} U(x,z,t)\,dz.$$

Integrate Euler equations in z to obtain $p = \frac{1}{2}gh^2$ and ...

One-dimensional Shallow Water (St. Venant) Equations

$$h_t + (hu)_x = 0$$

$$(hu)_t + \left(hu^2 + \frac{1}{2}gh^2\right)_x = -ghB'(x)$$

Depth-averaged models

Similary, reduce three-dimensional free surface problem to...

Two-dimensional Shallow Water (St. Venant) Equations

$$h_t + (hu)_x + (hv)_y = 0$$

$$(hu)_t + \left(hu^2 + \frac{1}{2}gh^2\right)_x + (huv)_y = -ghB_x(x,y)$$

$$(hv)_t + (huv)_x + \left(hv^2 + \frac{1}{2}gh^2\right)_y = -ghB_y(x,y)$$

where (u, v) are velocities in the horizontal directions (x, y).

Depth-averaged models

Advantages:

- 2D rather than 3D
  Often critical for realistic geophysical flows
  Vastly different spatial scales, e.g. ocean to harbor
  Need Adaptive Mesh Refinement even in 2D!
- No free surface $\eta(x, y, t)$.

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Possible problems:

- When is this valid?
- What if fluid is not homogeneous, or
  shallow water assumptions don't hold?
- Often still a free boundary in te x-y domain,
  at the shoreline or at the margins of the flow.
- Small perturbations to steady state hard to capture.

Tsunamis

Generated by

- Earthquakes,
- Landslides,
- Submarine landslides,
- Volcanoes,
- Meteorite or asteroid impact

There were 97 significant tsunamis during the 1990's, causing 16,000 casualties.

There have been approximately 28 tsunamis with run-up greater than 1m on the west coast of the U.S. since 1812.

Tsunamis

- Small amplitude in ocean (< 1 meter) but can grow to 10s of meters at shore.
- Run-up along shore can inundate 100s of meters inland
- Long wavelength (as much as 200 km)
- Propagation speed $\sqrt{gh}$ (much slower near shore)
- Average depth of Pacific or Indian Ocean is 4000 m $\Longrightarrow$ average speed 200 m/s $\approx$ 450 mph

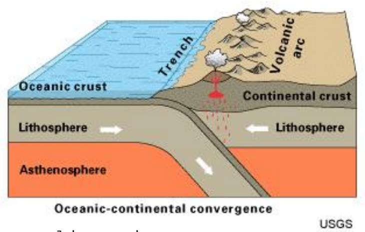
Sumatra event of December 26, 2004

Magnitude 9.1 quake near Sumatra, where Indian tectonic plate is being subducted under the Burma platelet.

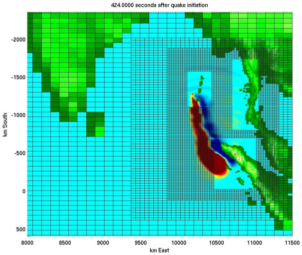
Rupture along subduction zone
$\approx$ 1200 km long, 150 km wide

Propagating at $\approx$ 2 km/sec (for $\approx$ 10 minutes)

Fault slip up to 15 m, uplift of several meters.
(Fault model from Caltech Seismolab.)

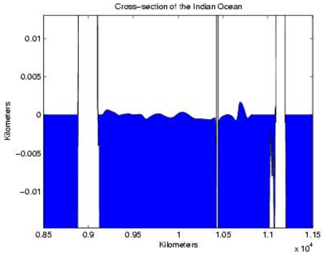


www.livescience.com

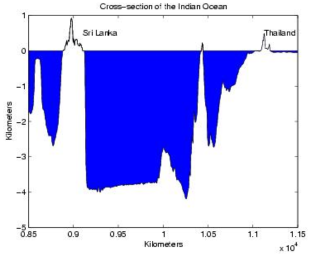


Cross section of Indian Ocean & tsunami

Surface elevation
on scale of 10 meters:



Cross-section
on scale of kilometers:



Shallow water equations with bathymetry $B(x, y)$

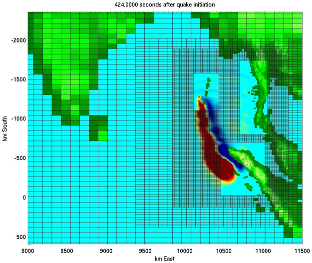
$$
\begin{aligned}
h_t + (hu)_x + (hv)_y &= 0 \\
(hu)_t + \left(hu^2 + \frac{1}{2}gh^2\right)_x + (huv)_y &= -ghB_x(x, y) \\
(hv)_t + (huv)_x + \left(hv^2 + \frac{1}{2}gh^2\right)_y &= -ghB_y(x, y)
\end{aligned}
$$

Some issues:

- Delicate balance between flux divergence and bathymetry:
  h varies on order of 4000m, rapid variations in ocean
  Waves have magnitude 1m or less.
- Cartesian grid used, with $h = 0$ in dry cells:
  Cells become wet/dry as wave advances on shore
  Robust Riemann solvers needed.
- Adaptive mesh refinement crucial
  Interaction of AMR with source terms, dry states

Tsunami simulations

- 2D shallow water + bathymetry
- Finite volume method
- Cartesian grid
- Cells can be dry ($h = 0$)
- Cells become wet/dry as wave moves on shore
- Mesh refinement on rectangular patches
- Adaptive — follows wave, more levels near shore



Local modeling near Chennai (Madras), India

Tsunami simulations

Adaptive mesh refinement is essential

Zoom on Madras harbors with 4 levels of refinement:

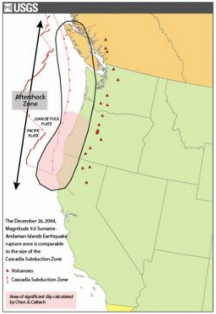
- Level 1: 1 degree resolution ($\Delta x \approx 110$ km)
- Level 2 refined by 8.
- Level 3 refined by 8: $\Delta x \approx 1.6$ km (only near coast)
- Level 4 refined by 64: $\Delta x \approx 25$ meters (only near Madras)

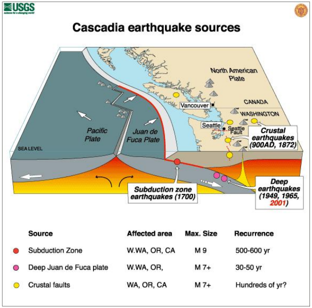
Factor 4096 refinement in x and y.

Less refinement needed in time since $c \approx \sqrt{gh}$.

Runs in a few hours on a laptop. Movie

Cascadia subduction fault





- 1200 km long off-shore fault stretching from northern California to southern Canada.
- Last major rupture: magnitude 9.0 earthquake on January 26, 1700.
- Tsunami recorded in Japan with run-up of up to 5 meters.
- Historically there appear to be magnitude 8 or larger quakes every 500 years on average.

Cascadia event simulations

Magnitude 9.0 earthquake similar to 1700 event.

Dave Alexander, Bill Johnstone, SpatialVision, Vancouver, BC

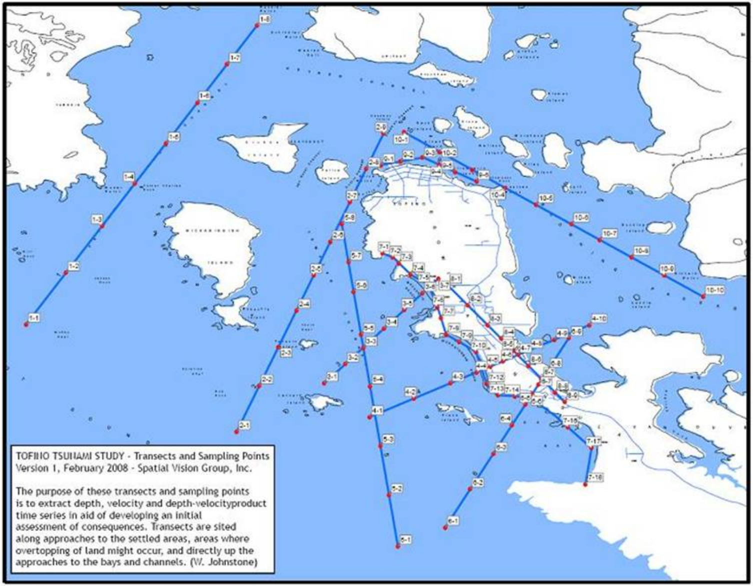
Barbara Lence, Civil Engineering, UBC

Movies:

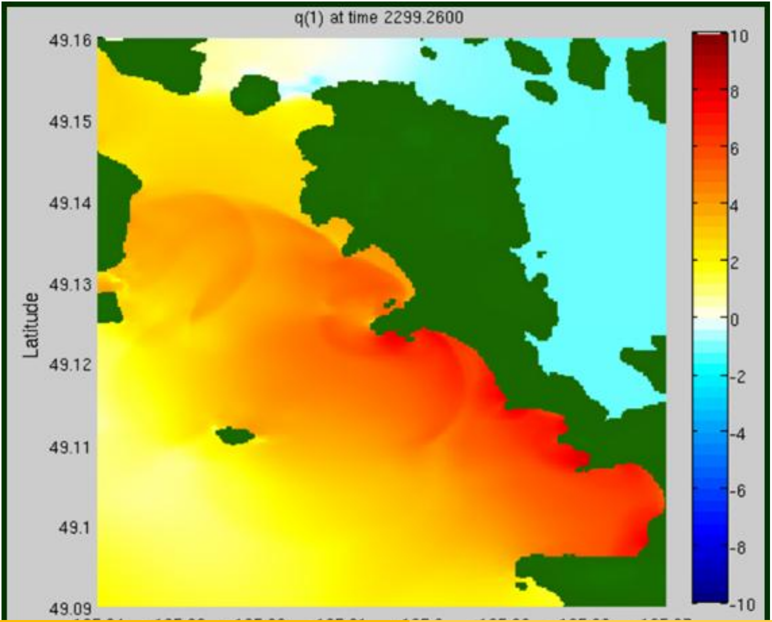
Vancouver Island and Olympic Penninsula

Ucluelet

Hazard Study for Tofino, BC



Hazard Study for Tofino, BC



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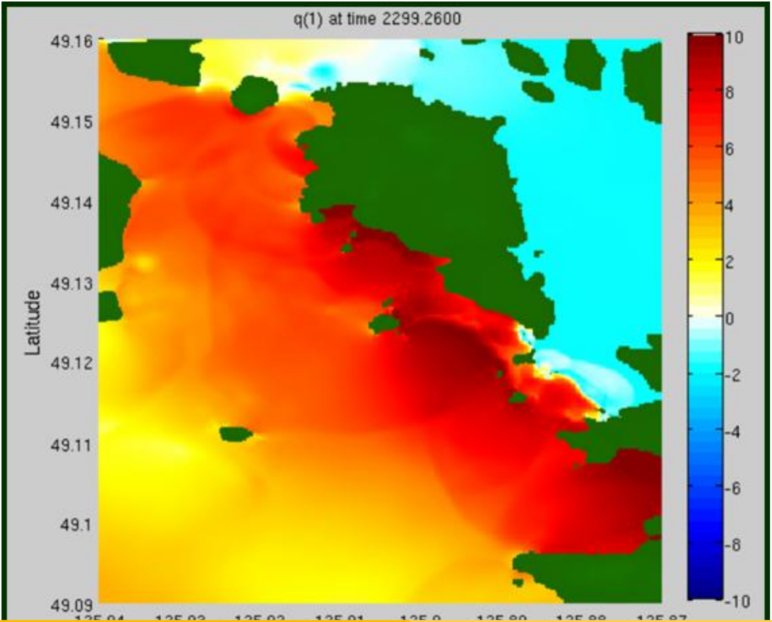

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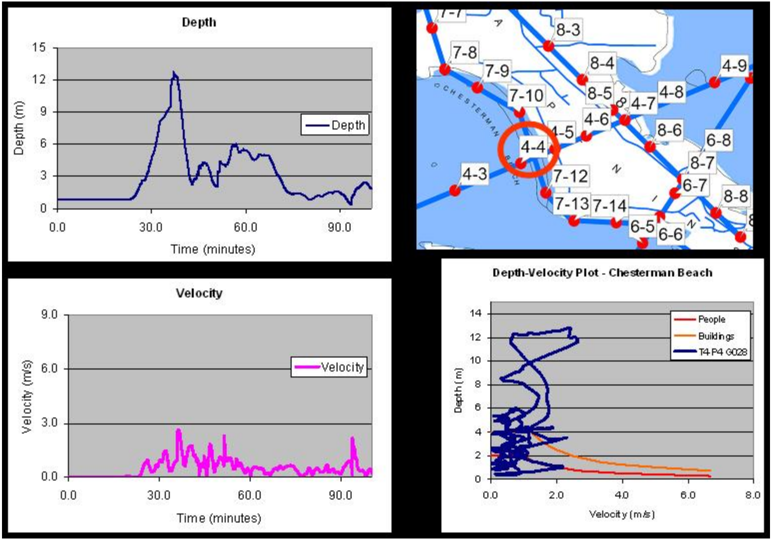
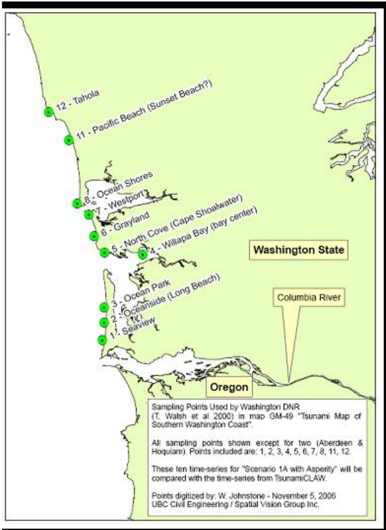
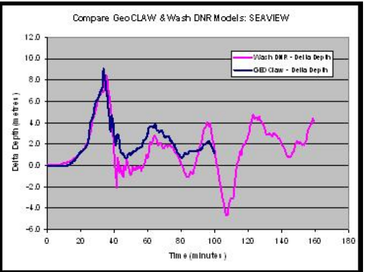
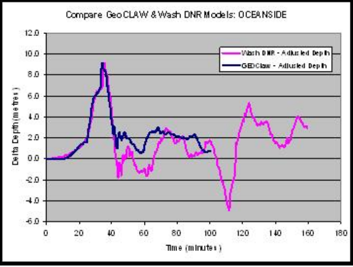


Comparison to NOAA model

Thanks to Tim Walsh (WA State DNR)







GeoClaw Software

TsunamiClaw: Dave George's code based on Clawpack.

Recently made more general as GeoClaw.

Currently includes:

- 2d library for depth-averaged flows over topography.
- Well-balanced Riemann solvers that handle dry cells.
- General tools for dealing with multiple data sets at different resolutions.
- Tools for specifying regions where refinement is desired.
- Graphics routines (Matlab currently, moving to Python).
- Output of time series at gauge locations or on fixed grids.

GeoClaw

Future:

- Other depth-averaged flow rheologies
- Two-dimensional vertical slices,
  full 3D models for flow on topography
- Subsurface flows, seismology, etc.

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Test version recently released as part of Clawpack 5.0
see www.clawpack.org.

Some other new features:

- Python interface tools and open source graphics
- EagleClaw (Easy Access Graphical Laboratory for Exploring Conservation Laws)
- Will include other generalizations such as David Ketcheson's WENOCLAW (high order methods).

Some other applications

Shallow water equations:

- Storm surges, hurricanes
- River flooding
- Dam breaks

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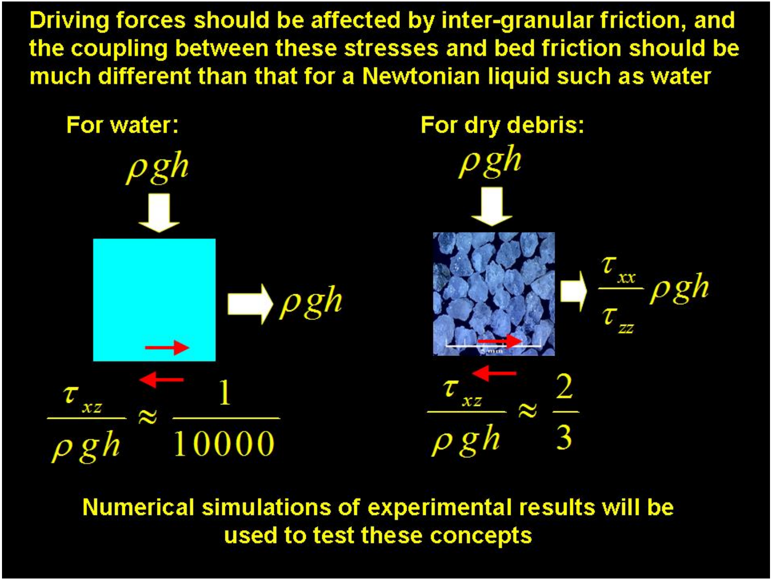
More complex flows:

- Flow on steep terrain
- Debris flows and lahars
- Lava flows
- Pyroclastic flows and surges
- Landslides and avalanches
- Underwater landslides / tsunami generation
- Multi-layer, internal waves

Debris flows

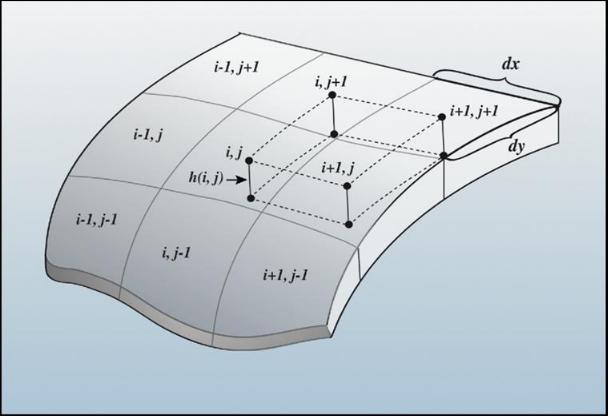
movie

Debris flow


Driving forces should be affected by inter-granular friction, and the coupling between these stresses and bed friction should be much different than that for a Newtonian liquid such as water
For water:
ρgh
ρgh
$\frac{\tau_{xz}}{\rho g h} \approx \frac{1}{10000}$
For dry debris:
ρgh
$\frac{\tau_{xx}}{\tau_{zz}} \rho g h$
$\frac{\tau_{xz}}{\rho g h} \approx \frac{2}{3}$
Numerical simulations of experimental results will be used to test these concepts

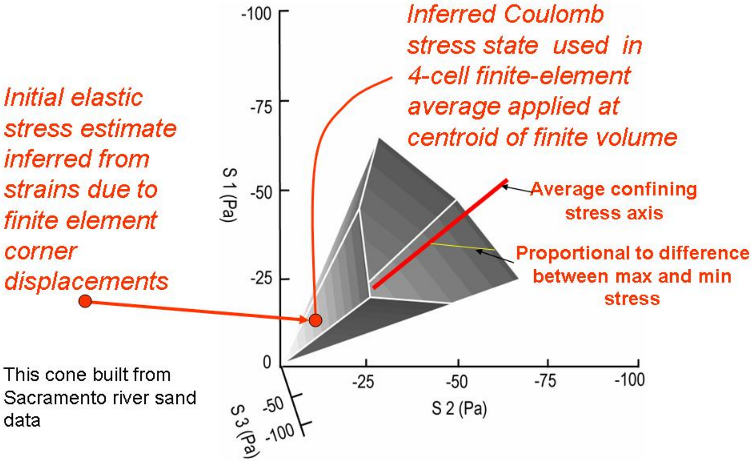
Debris flow

Stress Update: At the end of each time step, the new solution for *U* is used to estimate stresses in an offset finite element brick. Stresses from four finite elements contribute to each finite volume cell.



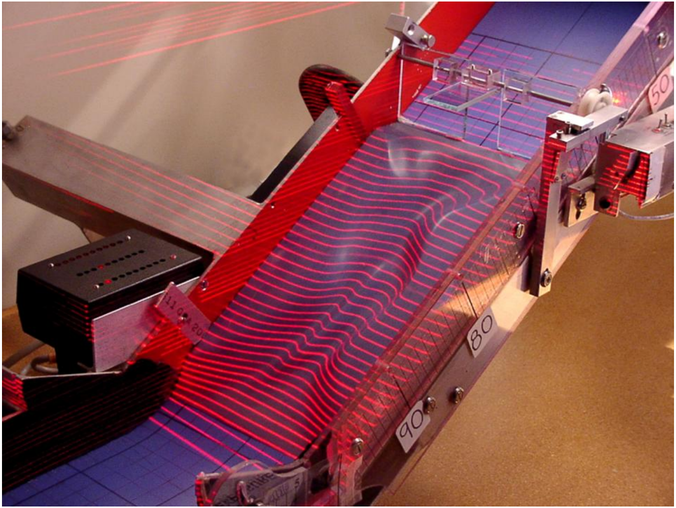
Debris flow

Inside each finite element, the elastic stress estimate is corrected back onto a Coulomb yield surface for granular debris



This cone built from Sacramento river sand data

Table top sand flume



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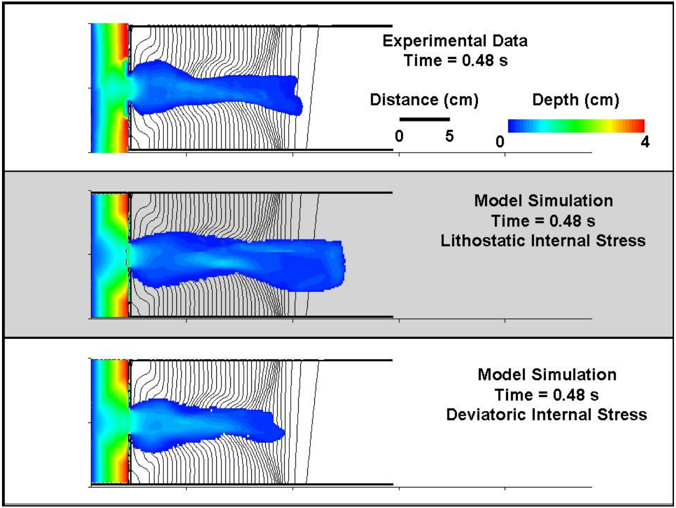

Flume

initiation   runout

Challenges for applied mathematicians

Mathematical modeling and analysis:

- Proper models for depth averaging complex rheologies
- Understanding mathematical structure, e.g.
  - Well posedness of equations
  - Loss of hyperbolicity in multi-layer equations
  - Proper interpretation of products of distributions

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Algorithm development:

- Robust methods for dry states, well-balancing
- AMR error estimation, adjoint methods?
- Nonlinear nonconservative products

Challenges for applied mathematicians

Working with real topography/bathymetry data:

- Interpolation of scattered nonsmooth noisy data
- Automatic smoothing of mismatches between topo/bathy

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Some challenging applications:

- Erosion and sedimentation,
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- Debris flows, land slides, avalanches
- Lava flows
- Many more — American Geophysical Union annual
  meeting is a good source of problems!

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Collaborate with earth scientists for maximal impact.