Multidimensional Finite Volume Methods

- Integral form on a rectangular grid cell
- Flux differencing form
- Scalar advection: donor cell upwind
- Corner transport upwind and transverse waves
- Wave propagation algorithms for systems
- Transverse Riemann solver

Derivation of conservation law

$$\frac{d}{dt}\iint_{\Omega} q(x,y,t)\,dx\,dy = -\int_{\partial\Omega} \vec{n}\cdot\vec{f}(q)\,ds.$$

where $\vec{f}(q) = (f(q),\ g(q))$, fluxes in x- and y-directions.

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If Ω is a rectangular grid cell $[x_{i-1/2},\ x_{i+1/2}] \times [y_{j-1/2},\ y_{j+1/2}]$

Then flux in normal directioni $\vec{n}$ is

$$\vec{n}\cdot\vec{f}(q) = \begin{cases} \mp f(q) & \text{at } x_{i\pm1/2}, \\ \mp g(q) & \text{at } y_{j\pm1/2}. \end{cases}$$

2D finite volume method for $q_t + f(q)_x + g(q)_y = 0$

Evolution of total mass due to fluxes through cell edges:

$$\begin{aligned}
\frac{d}{dt} \iint_{\mathcal{C}_{ij}} q(x,y,t)\,dx\,dy = & \int_{y_{j-1/2}}^{y_{j+1/2}} f(q(x_{i+1/2},y,t)\,dy \\
& - \int_{y_{j-1/2}}^{y_{j+1/2}} f(q(x_{i-1/2},y,t)\,dy \\
& + \int_{x_{i-1/2}}^{x_{i+1/2}} g(q(x,y_{j+1/2},t)\,dx \\
& - \int_{x_{i-1/2}}^{x_{i+1/2}} g(q(x,y_{j-1/2},t)\,dx.
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Suggests:

$$\begin{aligned}\frac{\Delta x\Delta y Q_{ij}^{n+1} - \Delta x\Delta y Q_{ij}^{n}}{\Delta t} = &-\Delta y[F^n_{i+1/2,j} - F^n_{i-1/2,j}] \\ &- \Delta x[G^n_{i,j+1/2} - G^n_{i,j-1/2}],\end{aligned}$$

2D finite volume method for $q_t + f(q)_x + g(q)_y = 0$

$$\Delta x \Delta y Q_{ij}^{n+1} = \Delta x \Delta y Q_{ij}^{n} - \Delta t \Delta y [F_{i+1/2,j}^{n} - F_{i-1/2,j}^{n}] \\ - \Delta t \Delta x [G_{i,j+1/2}^{n} - G_{i,j-1/2}^{n}],$$

Where we define numerical fluxes:

$$F_{i-1/2,j}^{n} \approx \frac{1}{\Delta t \Delta y} \int_{t_n}^{t_{n+1}} \int_{y_{j-1/2}}^{y_{j+1/2}} f(q(x_{i-1/2}, y, t))\, dy\, dt,$$

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2D finite volume method for $q_t + f(q)_x + g(q)_y = 0$

$$\begin{aligned}\Delta x \Delta y Q_{ij}^{n+1} = \Delta x \Delta y Q_{ij}^{n} &- \Delta t \Delta y [F_{i+1/2,j}^n - F_{i-1/2,j}^n] \\ &- \Delta t \Delta x [G_{i,j+1/2}^n - G_{i,j-1/2}^n],\end{aligned}$$

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Rewrite by dividing by $\Delta x \Delta y$ $\implies$ FV method in conservation form:

$$\begin{aligned}Q_{ij}^{n+1} = Q_{ij}^{n} &- \frac{\Delta t}{\Delta x}[F_{i+1/2,j}^n - F_{i-1/2,j}^n] \\ &- \frac{\Delta t}{\Delta y}[G_{i,j+1/2}^n - G_{i,j-1/2}^n].\end{aligned}$$

Dimensional splitting vs. unsplit FV method

Hyperbolic system in 2d: $q_t + f(q)_x + g(q)_y = 0$

Split method:

$$Q^*_{ij} = Q^n_{ij} - \frac{\Delta t}{\Delta x}[F^n_{i+1/2,j} - F^n_{i-1/2,j}]$$
$$Q^{n+1}_{ij} = Q^*_{ij} - \frac{\Delta t}{\Delta y}[G^*_{i,j+1/2} - G^*_{i,j-1/2}].$$

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Unsplit method:

$$\begin{aligned} Q^{n+1}_{ij} = Q^n_{ij} &- \frac{\Delta t}{\Delta x}[F^n_{i+1/2,j} - F^n_{i-1/2,j}] \\ &- \frac{\Delta t}{\Delta y}[G^n_{i,j+1/2} - G^n_{i,j-1/2}]. \end{aligned}$$

2D finite volume method for $q_t + f(q)_x + g(q)_y = 0$

$$\begin{aligned} Q_{ij}^{n+1} = Q_{ij}^n &- \frac{\Delta t}{\Delta x}[F_{i+1/2,j}^n - F_{i-1/2,j}^n] \\ &- \frac{\Delta t}{\Delta y}[G_{i,j+1/2}^n - G_{i,j-1/2}^n]. \end{aligned}$$

Fluctuation form:

$$\begin{aligned} Q_{ij}^{n+1} = Q_{ij} &- \frac{\Delta t}{\Delta x}(\mathcal{A}^+\Delta Q_{i-1/2,j} + \mathcal{A}^-\Delta Q_{i+1/2,j}) \\ &- \frac{\Delta t}{\Delta y}(\mathcal{B}^+\Delta Q_{i,j-1/2} + \mathcal{B}^-\Delta Q_{i,j+1/2}) \\ &- \frac{\Delta t}{\Delta x}(\tilde{F}_{i+1/2,j} - \tilde{F}_{i-1/2,j}) - \frac{\Delta t}{\Delta y}(\tilde{G}_{i,j+1/2} - \tilde{G}_{i,j-1/2}). \end{aligned}$$

The $\tilde{F}$ and $\tilde{G}$ are correction fluxes to go beyond Godunov's upwind method.

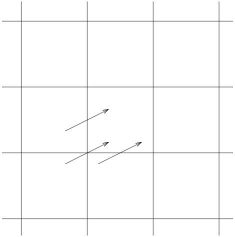
Incorporate approximations to second derivative terms in each direction (q_{xx} and q_{yy}) and mixed term q_{xy}.

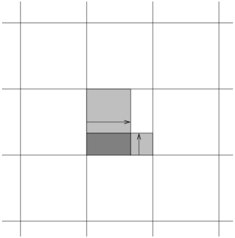
Advection: Donor Cell Upwind

With no correction fluxes, Godunov's method for advection is

Donor Cell Upwind:

$$Q_{ij}^{n+1} = Q_{ij} - \frac{\Delta t}{\Delta x}[u^+(Q_{ij} - Q_{i-1,j}) + u^-(Q_{i+1,j} - Q_{ij})]$$
$$- \frac{\Delta t}{\Delta y}[v^+(Q_{ij} - Q_{i,j-1}) + v^-(Q_{i,j+1} - Q_{ij})].$$



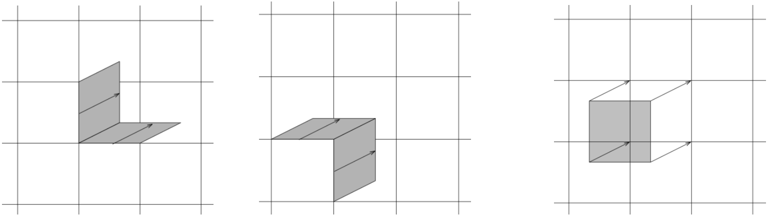


Stable only if $\left|\frac{u\Delta t}{\Delta x}\right| + \left|\frac{v\Delta t}{\Delta y}\right| \leq 1.$

Advection: Corner Transport Upwind (CTU)

Correction fluxes can be added to advect waves correctly.

Corner Transport Upwind:



Stable for $\max\left(\left|\frac{u\Delta t}{\Delta x}\right|, \left|\frac{v\Delta t}{\Delta y}\right|\right) \leq 1.$

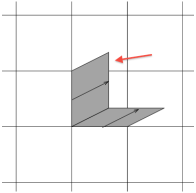
Advection: Corner Transport Upwind (CTU)

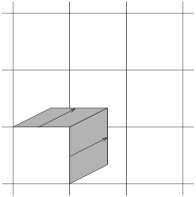
Need to transport triangular region from cell (i,j) to $(i,j+1)$:

$$\text{Area } = \frac{1}{2}(u\Delta t)(v\Delta t) \implies \left(\frac{\frac{1}{2}uv(\Delta t)^2}{\Delta x \Delta y}\right)(Q_{ij} - Q_{i-1,j}).$$

Accomplished by correction flux:

$$\tilde{G}_{i,j+1/2} = -\frac{1}{2}\frac{\Delta t}{\Delta x}\,uv(Q_{ij} - Q_{i-1,j})$$





$\frac{\Delta t}{\Delta y}(\tilde{G}_{i,j+1/2} - \tilde{G}_{i,j-1/2})$ gives approximation to $\frac{1}{2}\Delta t^2 uvq_{xy}$.

$\frac{\Delta t}{\Delta x}(\tilde{F}_{i+1/2,j} - \tilde{F}_{i-1/2,j})$ gives similar approximation.

Upwind splitting of matrix product

In 1D, the upwind method is

$$Q_i^{n+1} = Q_i^n - \frac{\Delta t}{\Delta x}[A^+(Q_i^n - Q_{i-1}^n) + A^-(Q_{i+1}^n - Q_i^n)]$$

where

$$A = R\Lambda R^{-1} = R\Lambda^+ R^{-1} + R\Lambda^- R^{-1} = A^+ \ + \ A^-$$

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In 2D the unsplit generalization uses

$$AB = (A^+ + A^-)(B^+ + B^-) = A^+B^+ \ + \ A^+B^- \ + \ A^-B^+ \ + \ B^-A^-,$$
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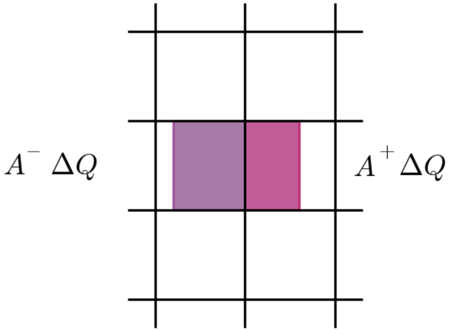
Scalar advection: only one term is nonzero in each product,
e.g. $u > 0, \ v < 0 \implies uv = vu = u^+v^-$

Wave propagation algorithm for $q_t + Aq_x + Bq_y = 0$

Decompose $A = A^+ + A^-$ and $B = B^+ + B^-$.

For $\Delta Q = Q_{ij} - Q_{i-1,j}$:



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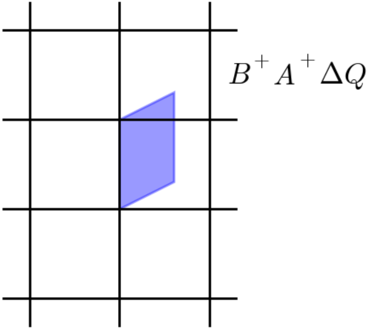

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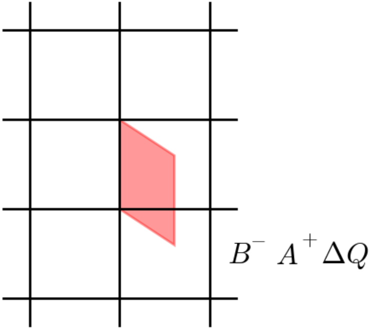

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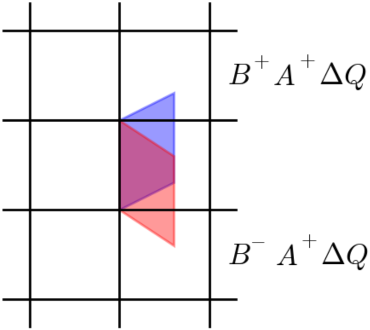

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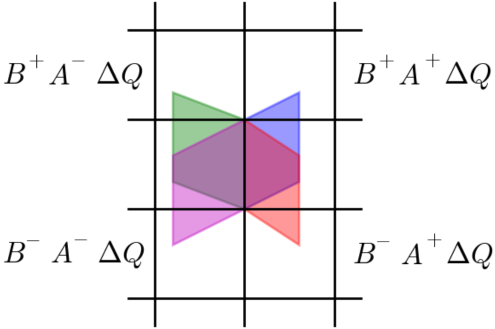
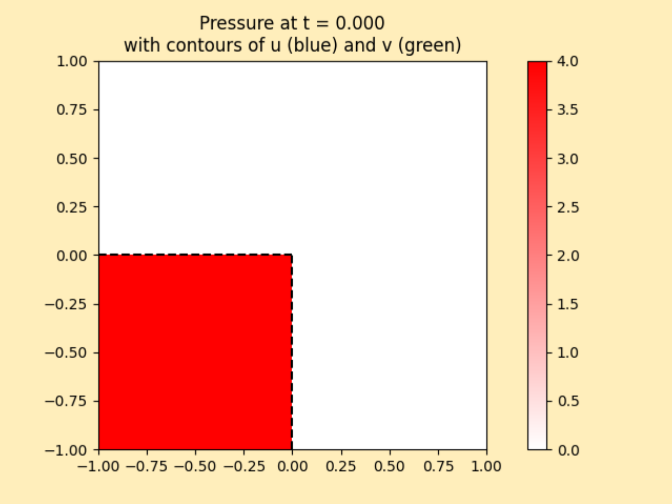


Acoustics near a cell corner

Solve 2D acoustics with $\rho = K = c = Z = 1$
$u = v \equiv 0$ and $p = 4$ in lower left quadrant, $p = 0$ elsewhere



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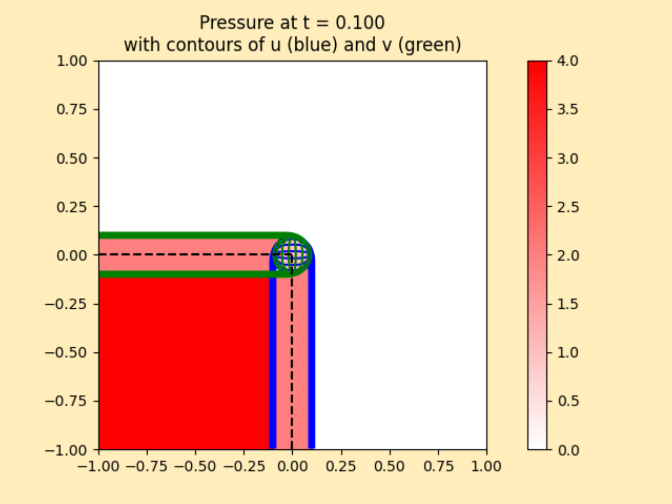

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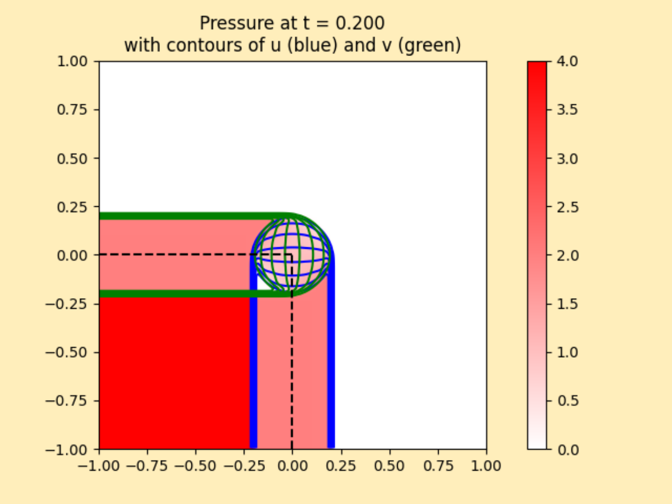

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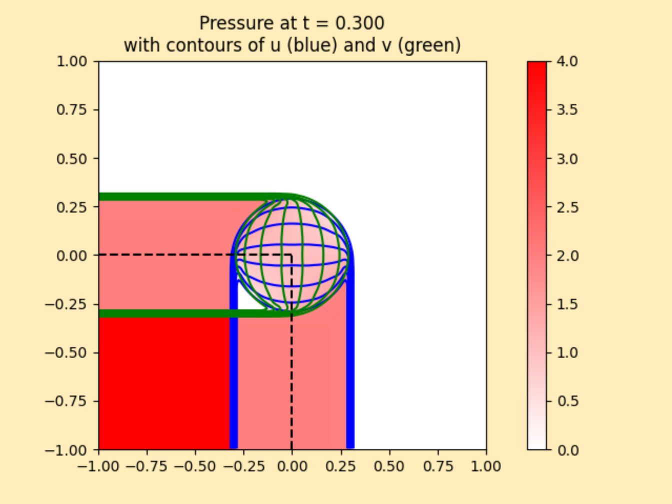

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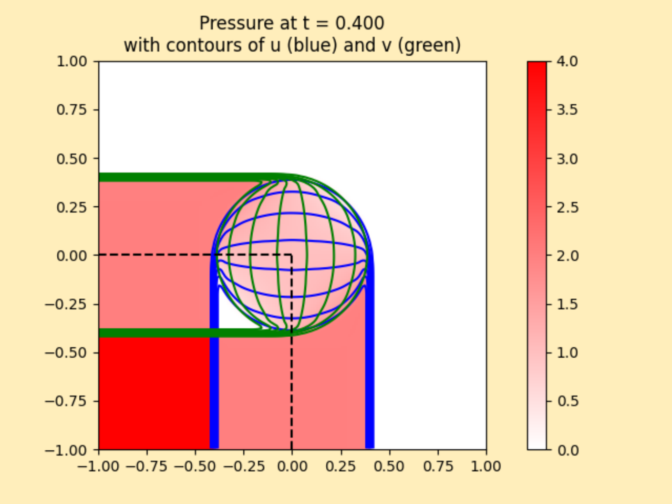

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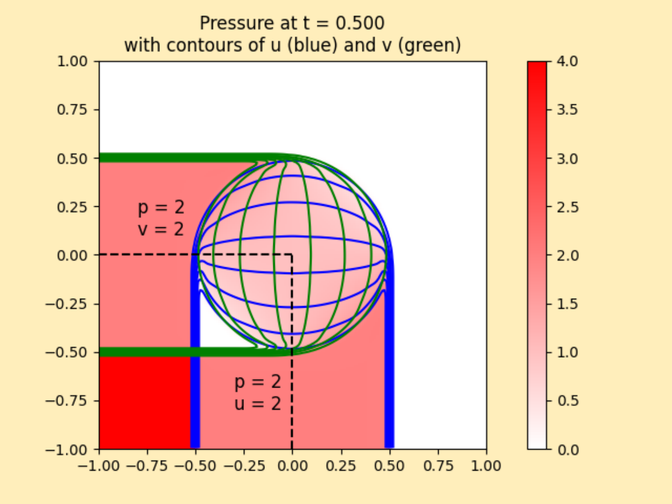
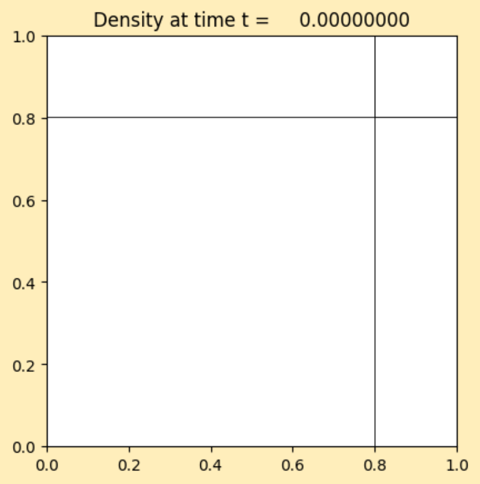


2D Riemann problem for Euler

Values in 4 quadrants chosen to give single shock between each



Clawpack Gallery: euler_2d_quadrants

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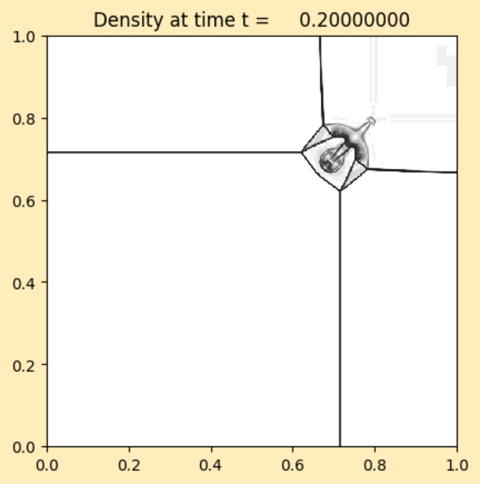

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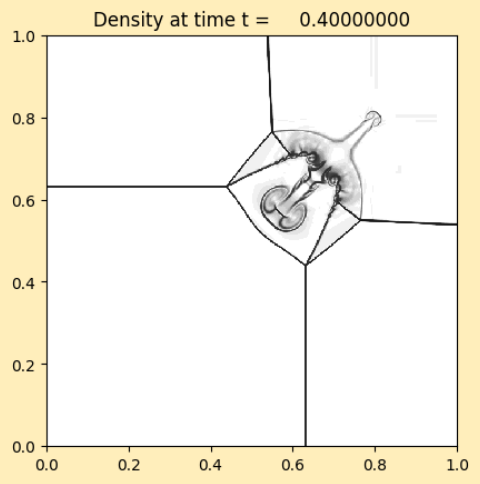

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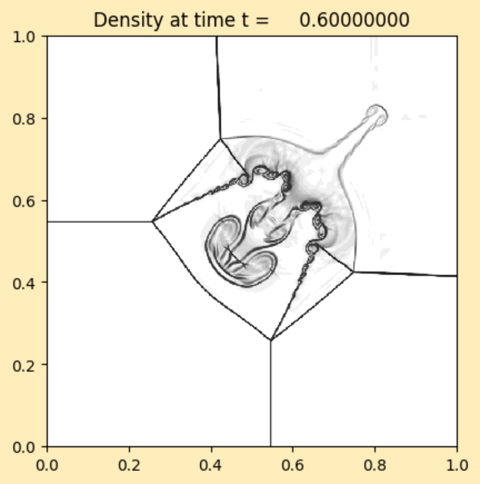

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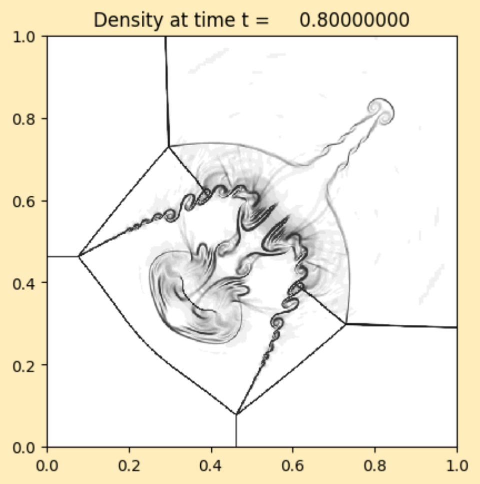

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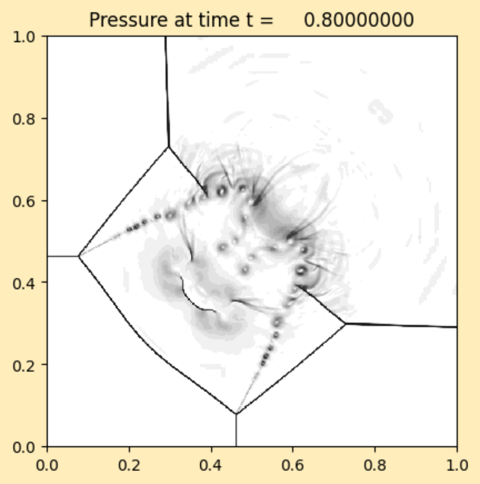

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Wave propagation algorithms in 2D

Clawpack requires:

Normal Riemann solver `rpn2.f`
Solves 1d Riemann problem $q_t + Aq_x = 0$
Decomposes $\Delta Q = Q_{ij} - Q_{i-1,j}$ into $\mathcal{A}^+\Delta Q$ and $\mathcal{A}^-\Delta Q$.
For $q_t + Aq_x + Bq_y = 0$, split using eigenvalues, vectors:

$$A = R\Lambda R^{-1} \implies A^- = R\Lambda^- R^{-1}, A^+ = R\Lambda^+ R^{-1}$$

Input parameter `ixy` determines if it's in x or y direction.
In latter case splitting is done using B instead of A.
This is all that's required for dimensional splitting.

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This is all that's required for dimensional splitting.

Transverse Riemann solver `rpt2.f`
Decomposes $\mathcal{A}^+\Delta Q$ into $\mathcal{B}^-\mathcal{A}^+\Delta Q$ and $\mathcal{B}^+\mathcal{A}^+\Delta Q$ by splitting this vector into eigenvectors of B.

(Or splits vector into eigenvectors of A if `ixy=2`.)

Transverse Riemann solver in Clawpack

`rpt2` takes vector `asdq` and returns `bmasdq` and `bpasdq` where

`asdq` $= \mathcal{A}^*\Delta Q$ represents either
$\mathcal{A}^-\Delta Q$ if `imp = 1`, or
$\mathcal{A}^+\Delta Q$ if `imp = 2`.

Returns $\mathcal{B}^-\mathcal{A}^*\Delta Q$ and $\mathcal{B}^+\mathcal{A}^*\Delta Q$.

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Returns $\mathcal{B}^-\mathcal{A}^*\Delta Q$ and $\mathcal{B}^+\mathcal{A}^*\Delta Q$.

Note: there is also a parameter `ixy`:

`ixy = 1` means normal solve was in x-direction,

`ixy = 2` means normal solve was in y-direction,
In this case `asdq` represents $\mathcal{B}^-\Delta Q$ or $\mathcal{B}^+\Delta Q$ and the routine must return $\mathcal{A}^-\mathcal{B}^*\Delta Q$ and $\mathcal{A}^+\mathcal{B}^*\Delta Q$.

Gas dynamics in 2D

$\rho(x, y, t) =$ mass density
$\rho(x, y, t)u(x, y, t) = x$-momentum density
$\rho(x, y, t)v(x, y, t) = y$-momentum density

If pressure $= P(\rho)$, e.g. isothermal or isentropic:

$$\begin{aligned}\rho_t + (\rho u)_x + (\rho v)_y &= 0\\(\rho u)_t + (\rho u^2 + p)_x + (\rho uv)_y &= 0\\(\rho v)_t + (\rho uv)_x + (\rho v^2 + p)_y &= 0\end{aligned}$$

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\end{aligned}$$

1D equation in x: $q_t + f(q)_x = 0$ is:

$$\begin{aligned}
\rho_t + (\rho u)_x &= 0 \\
(\rho u)_t + (\rho u^2 + p)_x &= 0 \\
(\rho v)_t + (\rho uv)_x &= 0 \implies v_t + uv_x = 0
\end{aligned}$$

These are just 1D equations for $(\rho,\ \rho u)$
along with an advected quantity v

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$\rho(x, y, t)v(x, y, t) = y$-momentum density

If pressure $= P(\rho)$, e.g. isothermal or isentropic:

$$\begin{aligned} \rho_t + (\rho u)_x + (\rho v)_y &= 0 \\ (\rho u)_t + (\rho u^2 + p)_x + (\rho uv)_y &= 0 \\ (\rho v)_t + (\rho uv)_x + (\rho v^2 + p)_y &= 0 \end{aligned}$$

1D equation in y: $q_t + g(q)_y = 0$ is:

$$\begin{aligned} \rho_t + (\rho v)_y &= 0 \\ (\rho u)_t + (\rho uv)_y &= 0 \implies u_t + v u_y = 0 \\ (\rho v)_t + (\rho v^2 + p)_y &= 0 \end{aligned}$$

These are just 1D equations for $(\rho,\ \rho v)$
along with an advected quantity u

Gas dynamics in 2D – transverse solver

If Roe solver is used for normal Riemann problems:

Eigenvectors of $\hat{A} \approx f'(q)$ are used for splitting in x,

$$\hat{\rho} = \frac{1}{2}(\rho_{i-1,j} + \rho_{i,j}), \qquad \hat{u} = \frac{\sqrt{\rho_{i-1,j}}u_{i-1,j} + \sqrt{\rho_{i,j}}u_{i,j}}{\sqrt{\rho_{i-1,j}} + \sqrt{\rho_{i,j}}}$$

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Use the same Roe averages for this interface to also define
$\hat{B} \approx g'(q)$ near this interface.

Split $\mathcal{A}^*\Delta Q$ into eigenvectors of $\hat{B}$ to define
$\mathcal{B}^-\mathcal{A}^*\Delta Q$ and $\mathcal{B}^+\mathcal{A}^*\Delta Q$

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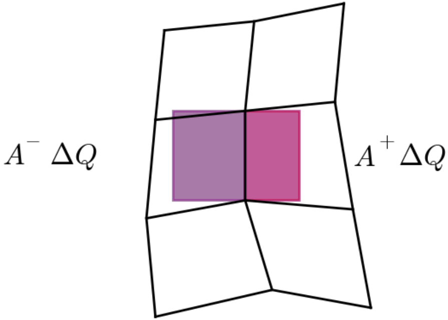
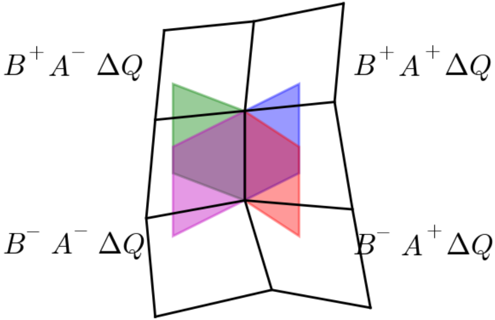
Many normal and transverse solvers available in **$CLAW/riemann/src**

Wave propagation algorithm on a quadrilateral grid



Example: **$CLAW/amrclaw/examples/advection_2d_annulus**

Wave propagation algorithm on a quadrilateral grid



Example: **$CLAW/amrclaw/examples/advection_2d_annulus**