

Chapter 3 Exercises

From: *Finite Difference Methods for Ordinary and Partial Differential Equations*
by R. J. LeVeque, SIAM, 2007. <http://www.amath.washington.edu/~rjl/fdmbook>

Exercise 3.1 (*code for Poisson problem*)

The MATLAB script `poisson.m` solves the Poisson problem on a square $m \times m$ grid with $\Delta x = \Delta y = h$, using the 5-point Laplacian. It is set up to solve a test problem for which the exact solution is $u(x, y) = \exp(x + y/2)$, using Dirichlet boundary conditions and the right hand side $f(x, y) = 1.25 \exp(x + y/2)$.

- (a) Test this script by performing a grid refinement study to verify that it is second order accurate.
- (b) Modify the script so that it works on a rectangular domain $[a_x, b_x] \times [a_y, b_y]$, but still with $\Delta x = \Delta y = h$. Test your modified script on a non-square domain.
- (c) Further modify the code to allow $\Delta x \neq \Delta y$ and test the modified script.

Exercise 3.2 (*9-point Laplacian*)

- (a) Show that the 9-point Laplacian (3.17) has the truncation error derived in Section 3.5.
Hint: To simplify the computation, note that the 9-point Laplacian can be written as the 5-point Laplacian (with known truncation error) plus a finite difference approximation that models $\frac{1}{6}h^2 u_{xxyy} + O(h^4)$.
- (b) Modify the MATLAB script `poisson.m` to use the 9-point Laplacian (3.17) instead of the 5-point Laplacian, and to solve the linear system (3.18) where f_{ij} is given by (3.19). Perform a grid refinement study to verify that fourth order accuracy is achieved.