# Chapter 1 Exercises

From: *Finite Difference Methods for Ordinary and Partial Differential Equations*
by R. J. LeVeque, SIAM, 2007. `http://www.amath.washington.edu/~rjl/fdmbook`

**Exercise 1.1** *(derivation of finite difference formula)*

Determine the interpolating polynomial $p(x)$ discussed in Example 1.3 and verify that evaluation $p'(\bar{x})$ gives equation (1.11).

**Exercise 1.2** *(use of* `fdstencil`*)*

(a) Use the method of undetermined coefficients to set up the $5 \times 5$ Vandermonde system that would determine a fourth-order accurate finite difference approximation to $u''(x)$ based on 5 equally spaced points,

$$u''(x) = c_{-2}u(x-2h) + c_{-1}u(x-h) + c_0 u(x) + c_1 u(x+h) + c_2 u(x+2h) + O(h^4).$$

(b) Compute the coefficients using the MATLAB code `fdstencil.m` available from the website, and check that they satisfy the system you determined in part (a).

(c) Test this finite difference formula to approximate $u''(1)$ for $u(x) = \sin(2x)$ with values of $h$ from the array `hvals = logspace(-1, -4, 13)`. Make a table of the error vs. $h$ for several values of $h$ and compare against the predicted error from the leading term of the expression printed by `fdstencil`. You may want to look at the m-file `chap1example1.m` for guidance on how to make such a table.

Also produce a log-log plot of the absolute value of the error vs. $h$.

You should observe the predicted accuracy for larger values of $h$. For smaller values, numerical cancellation in computing the linear combination of $u$ values impacts the accuracy observed.