Chapter 1 Exercises

From: Finite Difference Methods for Ordinary and Partial Differential Equations

Exercise 1.1  (derivation of finite difference formula)

Determine the interpolating polynomial $p(x)$ discussed in Example 1.3 and verify that evaluation $p'(x)$ gives equation (1.11).

Exercise 1.2  (use of fdstencil)

(a) Use the method of undetermined coefficients to set up the $5 \times 5$ Vandermonde system that would determine a fourth-order accurate finite difference approximation to $u''(x)$ based on 5 equally spaced points,

$$u''(x) = c_{-2}u(x - 2h) + c_{-1}u(x - h) + c_0u(x) + c_1u(x + h) + c_2u(x + 2h) + O(h^4).$$

(b) Compute the coefficients using the MATLAB code fdstencil.m available from the website, and check that they satisfy the system you determined in part (a).

(c) Test this finite difference formula to approximate $u''(1)$ for $u(x) = \sin(2x)$ with values of $h$ from the array $hvals = \logspace(-1, -4, 13)$. Make a table of the error vs. $h$ for several values of $h$ and compare against the predicted error from the leading term of the expression printed by fdstencil. You may want to look at the m-file chap1example1.m for guidance on how to make such a table.

Also produce a log-log plot of the absolute value of the error vs. $h$.

You should observe the predicted accuracy for larger values of $h$. For smaller values, numerical cancellation in computing the linear combination of $u$ values impacts the accuracy observed.