Chapter 11 Exercises

From: *Finite Difference Methods for Ordinary and Partial Differential Equations*
by R. J. LeVeque, SIAM, 2007. `http://www.amath.washington.edu/~rjl/fdmbook`

Exercise 11.1 *(two-dimensional Lax-Wendroff)*

(a) Derive the two-dimensional Lax-Wendroff method from (11.6) by using standard centered approximations to u_x, u_y, u_{xx} and u_{yy} and the approximation

$$u_{xy}(x_i, y_j) \approx \frac{1}{4h^2}\left[(U_{i+1,j+1} - U_{i-1,j+1}) - (U_{i+1,j-1} - U_{i-1,j-1})\right]. \tag{E11.1a}$$

(b) Compute the leading term of the truncation error to show that this method is second order accurate.

Exercise 11.2 *(Strang splitting)*

(a) Show that the Strang splitting is second order accurate on the problem (11.18) by comparing

$$\exp\left(\frac{1}{2}Ak\right)\exp(Bk)\exp\left(\frac{1}{2}Ak\right) \tag{E11.2a}$$

with (11.22).

(b) Show that second order accuracy on (11.18) can also be achieved by alternating the splitting (11.17) in even numbered time steps with

$$\begin{aligned} U^* &= \mathcal{N}_B(U^n, k), \\ U^{n+1} &= \mathcal{N}_A(U^*, k) \end{aligned} \tag{E11.2b}$$

in odd numbered times steps.

Exercise 11.3 *(accuracy of IMEX method)*

Compute the truncation error of the method (11.26) and confirm that it is second order accurate.