AMath 586 / ATM 581
Homework #2
Due Thursday, April 14, 2016

Homework is due to Canvas by 11:00pm PDT on the due date.

To submit, see `https://canvas.uw.edu/courses/1038268/assignments/3254815`

Problem 1

Which of the following Linear Multistep Methods are convergent? For the ones that are not, are they inconsistent, or not zero-stable, or both?

(a) $U^{n+3} = U^{n+1} + 2kf(U^n)$,

(b) $U^{n+2} = \frac{1}{2}U^{n+1} + \frac{1}{2}U^n + 2kf(U^{n+1})$,

(c) $U^{n+1} = U^n$,

(d) $U^{n+4} = U^n + \frac{4}{3}k(f(U^{n+3}) + f(U^{n+2}) + f(U^{n+1}))$,

(e) $U^{n+3} = -U^{n+2} + U^{n+1} + U^n + 2k(f(U^{n+2}) + f(U^{n+1}))$.

Problem 2

Consider the IVP

$$\begin{aligned} u_1' &= 2u_1, \\ u_2' &= 3u_1 + 2u_2, \end{aligned}$$

with initial conditions specified at time $t = 0$. Solve this problem in two different ways:

(a) Solve the first equation, which only involves u_1, and then insert this function into the second equation to obtain a nonhomogeneous linear equation for u_2. Solve this using (5.8).

(b) Write the system as $u' = Au$ and compute the matrix exponential using (D.35) to obtain the solution. (See Appendix C.3 for a discussion of the Jordan Canonical form in the defective case.)

Problem 3

(a) Determine the general solution to the linear difference equation $U^{n+2} = U^{n+1} + U^n$.

(b) Determine the solution to this difference equation with the starting values $U^0 = 1$, $U^1 = 1$. Use this to determine U^{30}? (Note, these are the *Fibonacci numbers*, which of course should all be integers.)

(c) Show that for large n the ratio of successive Fibonacci numbers U^n/U^{n-1} approaches the "golden ratio" $\phi \approx 1.618034$.

Problem 4

Perform numerical experiments to confirm the claims made in Example 7.11 on page 160, by comparing the performance of Forward Euler and the Midpoint methods for different choices of parameters. In particular, you might want to try $a = 1$ and compare $b = 0$ with $b = 1$ over $0 \leq t \leq 10$, but feel free to experiment and perhaps find other values that illustrate this better. Explore how each method behaves for different grid resolutions in each case.

Compute the exact solution to this system (e.g. using the matrix exponential) so that you can plot the true solution along with the computed solution for various cases.

Problem 5

Implement the 3rd order 3-step Adams-Bashforth method for the linearized pendulum problem from Problem 4. As starting values, try the following two choices:

(a) Use Forward Euler to compute U^1 and then Midpoint to compute U^2 from U^0 and U^1.

(b) Use the 2-stage Runge-Kutta method (5.32) to compute U^1 and also to compute U^2.

In each case, what global order of accuracy do you observe at some fixed time? (E.g. $t = 10$ with parameters $a = 1,\ b = 0.2$)? Estimate this by solving the problem with several different grid resolutions.

In each case, explain why the global order you observe agrees with what should be expected.