AMath 574
Midterm Review

The midterm is Wednesday February 15, 2023 in class.
Open book and notes (ebook ok) but no calculators, internet, etc.
Here are some topics you might want to review and feel comfortable with...

General concepts:

- Definition of hyperbolicity for first-order PDEs
- Derivation of conservation laws, idea of fluxes
- Conservation form for finite volume methods, why it's important for shocks
- Linearizing simple nonlinear systems (e.g. deriving acoustics)

Linear systems:

- Characteristics and characteristic variables
- Meaning of eigenvalues, eigenvectors
- Computing eigendecomposition for 2×2 matrices
- General solution via tracing characteristics
- Solution to the Riemann problem
- Phase plane for 2 equations

Methods for advection and linear systems

- Upwind, Lax-Friedrichs, Lax-Wendroff methods
- Local truncation error and modified equations
- Courant number, CFL condition vs. stability bound
- REA (reconstruct-evolve-average) viewpoint
- Slope limiters, relation to flux and wave limiters
- Sweby region
- Wave propagation form, fluctuations plus corrections
- Extension of wave propagation method to linear systems
- Ghost cells for BCs: periodic, extrapolation, solid wall for acoustics

Scalar nonlinear problems (with convex flux)

- Weak solutions (integral form and integration against test function)
- Behavior of solutions to Burgers', LWR traffic flow examples
- Rankine-Hugoniot condition, derivation of shock speed
- Centered rarefaction waves – computing the similarity solution
- Lax Entropy Condition, vanishing viscosity solution
- Entropy functions and how to derive the entropy flux

FV methods for nonlinear scalar problems

- Godunov's method, numerical flux function
- Upwind method plus "entropy fix"
- Ideas of TV-stability, L_1-contracting, monotonicity preserving, but no detailed proofs
- Statement of Lax-Wendroff theorem, definition of consistency