AMath 574 Homework 3 Due by 11:00pm on February 2, 2023

For submission instructions, see:

http://faculty.washington.edu/rjl/classes/am574w2023/homework3.html

## Problem #6.1 in the book

## Problem #6.4 in the book

Note that this is for the Cauchy problem, so there are infinitely many cells, but you can assume the total variation of q(x) is finite. In the definition (6.19) referred to in the hint, "sup" stands for supremum and this just means the maximum in the case where this value might not actually be attained for any finite N (so TV(q) is the least upper bound on the sum over all choices of N points). For example, a function with infinitely many oscillations but with exponentially decaying amplitude would have finite TV, but for any finite N the sum in (6.19) would be strictly less than TV(q).

To make this problem a bit easier without changing the main point, it is fine to suppose we are just on a finite interval (or on the full real line but for functions that are identically constant outside of some finite interval). Then the hint in the problem suggests a finite number of points to consider. You might also want to first consider the case where q(x) is continuous, and choose points suggested by the mean value theorem.

Problem #6.7 in the book

Problem #8.1 in the book

Problem #8.9 in the book

Problem #7.2 in the book

## Problem #7.3 in the book

Note that you can view the Clawpack solution to this problem in the Clawpack gallery of examples from \$CLAW/apps/fvmbook, at

http://www.clawpack.org/gallery/gallery/gallery\_fvmbook.html.

Plots for this particular example can be viewed here.

The first part of the problem can be done by viewing these plots and explaining the results.

The second part requires changing the boundary condition in the Clawpack code, and for this please let me know if you are having problems installing or using Clawpack. Only one line of the setrun.py file needs to be changed, so the main point of this exercise is to get Clawpack working.

I cleaned up the code for this example a bit, so you should get the most recent version by doing a "git pull" from the am574 branch of my fork, as described in the video on installing Clawpack (Clawpack01).

Also note that Clawpack has changed since the book was written, and parameters are now set in the Python script 'setrun.py' rather than in the '.data' file mentioned in the problem. For extrapolation BCs you can set

clawdata.bc\_upper[0] = 'extrap'

Please read Section 7.2.1 about these boundary conditions.

Note also that this code is set up to use

clawdata.cfl\_desired = 1.0

so that even the upwind method is "exact" for the acoustics equations, so you are seeing a good representation of how the acoustics solution should behave with these BCs.

In addition to describing the behavior, please also include the png files from a couple frames you computed at interesting times in relation to your description. Note that if you do make .plots

in this directory then the \_plots directory contains png files of the plots at each output frame.