AMath 574
Homework 1
Due by 11:00pm on January 17, 2023 (updated)

For submission instructions and some additional information, see:

`http://faculty.washington.edu/rjl/classes/am574w2023/homework1.html`

#1.

The gas dynamics equations (2.38) are written as conservation laws for the mass and momentum, which are conserved quantities. Often the so-called “primitive variables” $p(x,t) =$ pressure and $u(x,t) =$ velocity are more natural to use, e.g. in the acoustic equations we consider perturbations of p and u modeled by the linear system (2.5), rather than the linear system with matrix (2.46) modeling perturbations in mass and momentum.

(a) Starting from (2.38), derive the following nonlinear equations for the pressure and velocity:

$$\begin{aligned} p_t + up_x + \rho P'(\rho)u_x = 0, \\ u_t + (1/\rho)p_x + uu_x = 0. \end{aligned} \tag{1}$$

Note that these equations involve $\rho(x,t)$ which must now be determined from $p(x,t)$ using the equation of state, by inverting $p = P(\rho)$ for ρ as a function of p. (Exactly what this gives depends on the particular equation of state, so you don't need to go farther.)

Hint: Use for example $(\rho u)_t = \rho_t u + \rho u_t$ in the conservation law for momentum and then use the conservation law for ρ_t, and also note $P(\rho)_x = P'(\rho)\rho_x$, etc.

(b) The equations (1) can be written in the form

$$q_t(x,t) + A(q(x,t))q_x(x,t) = 0,$$

a non-conservative nonlinear system. Determine the matrix $A(q)$ and the eigenvalues of the matrix.

The system is said to be hyperbolic if the matrix is diagonalizable with real eigenvalues. Confirm that the resulting condition on $P'(\rho)$ (and the eigenvalues) agree with what we found from the conservative form (2.38). Also note that if we linearize (1) about

$$p \approx p_0, \quad u \approx u_0, \quad \rho P'(\rho) \approx \rho_0 P'(\rho_0) \equiv K_0,$$

then the linearized equations agree with the acoustics equations (2.50).

Note: This gives a nice derivation of the linear acoustics equations, but when solving a nonlinear gas dynamics problem it is generally necessary to use the conservative form in order to get proper modeling of shock waves and other nonlinear phenomena. Smooth solutions should agree between the two formulations.

Problem #2.7 in the book

Problem #2.8 in the book

Hint: Read Section 2.13 first and note that when linearizing about a constant specific volume V_0, the relation (2.102) between ξ and x is roughly $\xi = (x - x_0)/V_0$.

Problem #3.1(d,e,f) in the book You might want to do Problem 3.2 first.

Problem #3.2 in the book

You can use Matlab for this one, but I suggest you try writing the program in Python. A Jupyter notebook with a partial solution can be found in the class repository to help get you started.

Note that the module `numpy.linalg` contains an `eig` function similar to Matlab.

Problem #3.3 in the book

You do not need to draw the dashed lines of Figure 3.3, just the wedges with the correct wave speeds. Sketch it by hand or e.g. in a Jupyter notebook, as you please.