Rossby Waves

Recall the linear, SWE "QGPV" equation, β-plane

$$\boxed{\left[(\eta_{xx}+\eta_{yy}) + \frac{1}{a^2}\eta\right]_t + \beta\eta_x = 0} \quad (*)$$

$\frac{\partial}{\partial t}$ vorticity — stretching — change of latitude

which is an approximation of:

$$\frac{D}{Dt}\left(\frac{\zeta+f}{h}\right) = 0$$

- For Poincaré waves $(\omega > f)$ changes in latitude were small and ~~so~~ $\frac{\zeta}{h}$ was conserved
- For steady flow streamlines are along $\frac{f}{h} = \text{const}$
- (*) allows us to find a new class of low frequency $(\omega < f)$ wave solutions, called "Rossby Waves"

Assume plane wave solutions of the form

$$\eta = \mathrm{Re}\left\{ \eta_0 \exp i\left(kx + ly - \omega t\right)\right\}$$

then from (*) the dispersion relation is:

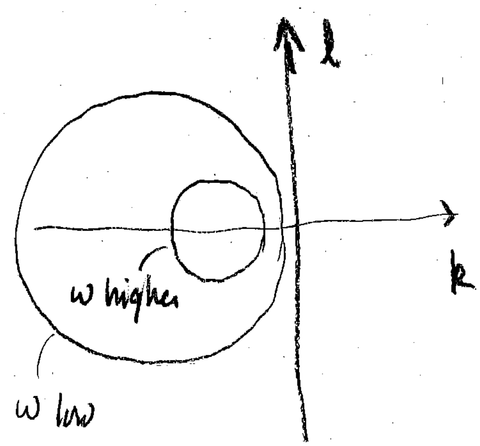
$$\omega = \frac{-\beta k}{k^2 + l^2 + \frac{1}{a^2}}$$

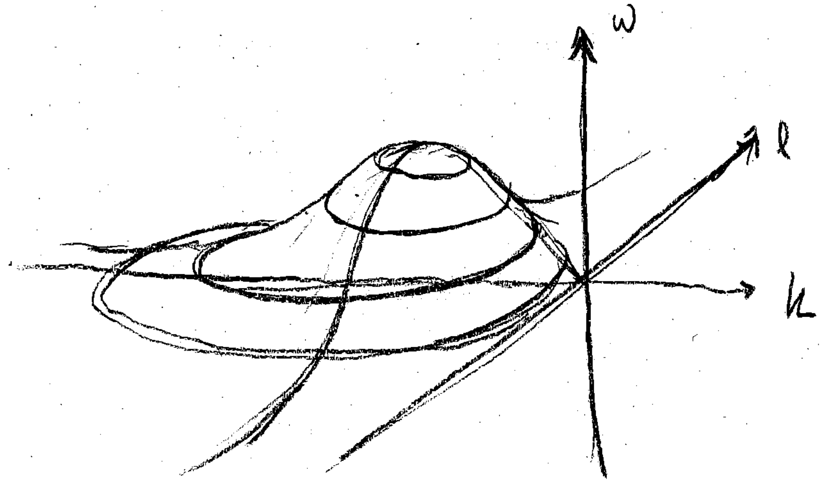
$\Rightarrow$ C_p always tends westward (k is always negative if we assume positive ω)

Two limits:

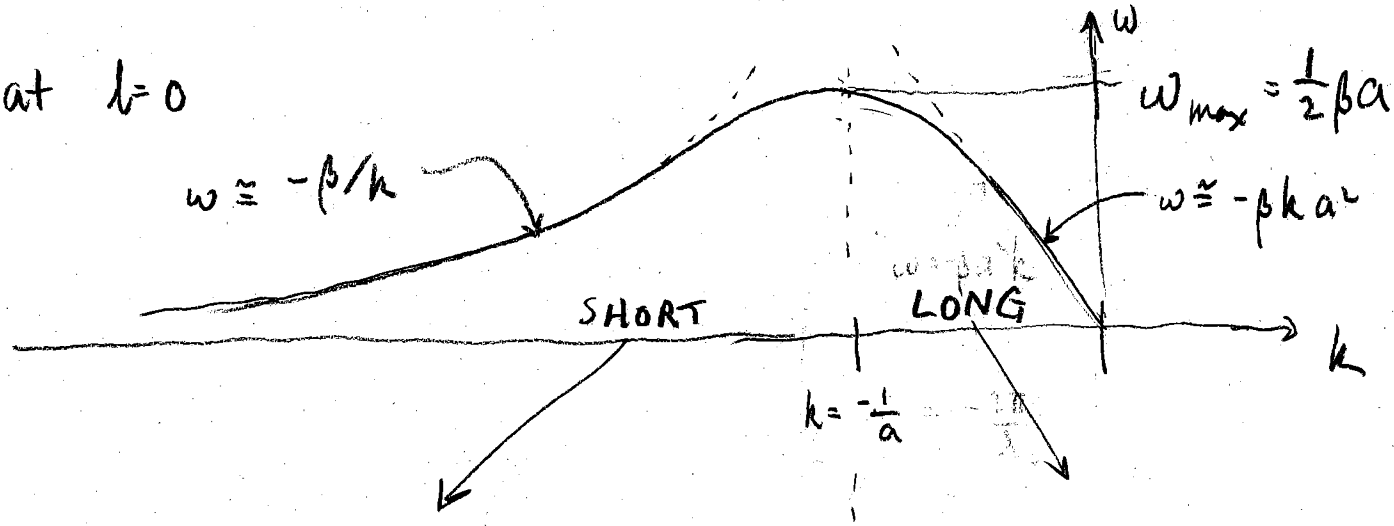
	SHORT WAVES	LONG WAVES
wave #:	$\frac{1}{a} \ll K_H$	$K_H \ll \frac{1}{a}$
frequency:	$\frac{-\beta k}{k^2+l^2}$	$-\beta k a^2$
$C_p\big\vert_{l=0} = \frac{\omega}{k}$:	$\frac{-\beta}{k^2}$ $\leftarrow$ SLOW	$-\beta a^2$ $\leftarrow$ "FAST", NON-DISPERSIVE
$C_g^x\big\vert_{l=0} = \frac{\partial\omega}{\partial k}$:	$\frac{\beta}{k^2}$ $\rightarrow$	$-\beta a^2$ $\leftarrow$

lines of constant ω are circles on k, l plane





at $l = 0$



$(\eta_{xx} + \eta_{yy})_t + \beta \eta_x \cong 0$	$-\frac{1}{a^2}\eta_t + \beta \eta_x \cong 0$
$\frac{D}{Dt}(\zeta + f) \cong 0$	$\frac{D}{Dt}\left(\frac{f}{h}\right) \cong 0$
change of ζ balances change of f	stretching balances change of f
Energy mostly KE	Energy mostly PE

Some scales:

- For disturbances to the ocean thermocline

  $c \sim 3 \text{ m s}^{-1}$ and $a \simeq 30$ km

  then $\omega_{max} \cong \dfrac{2\pi}{300 \text{ days}}$ and $c_{p\,max} = -1.4 \text{ cm s}^{-1}$
  (long waves)

- For ~~mid~~-latitude synoptic disturbances of the atm.
  the short wave limit is more appropriate

  and for $\lambda = 6000$ km $\Rightarrow k \cong 10^{-6} \text{ m}^{-1} \cong l$

  and $C_p = \dfrac{-\beta}{k^2+l^2} = \dfrac{-1.6 \cdot 10^{-11}}{\text{m s}} \dfrac{\text{m}^2}{2 \cdot 10^{-12}} = -18 \text{ m s}^{-1}$

  ($c \sim 8 \text{ m s}^{-1}$ for $k = l$)

so weather patterns move East slower than the mean zonal wind (Holton 7.7)

[See Appendix for a physical explanation of C_p]

Appendix

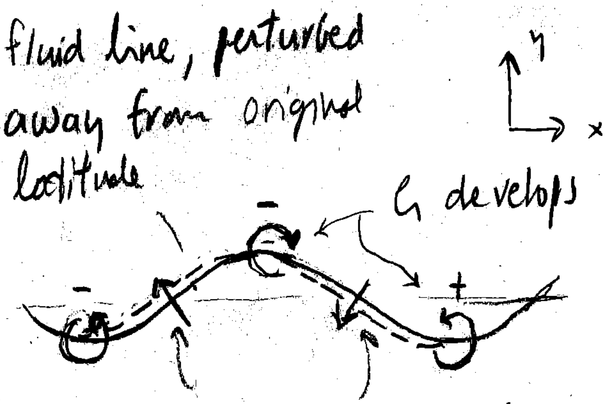
Why do Rossby waves have Westward phase propagation (negative k)?

- The velocity pattern is mainly geostrophic, and hence steady.
- Have to look to smaller ageostrophic velocity to predict movement:

SHORT WAVES

stretching negligible,
ζ dominates response to Δf

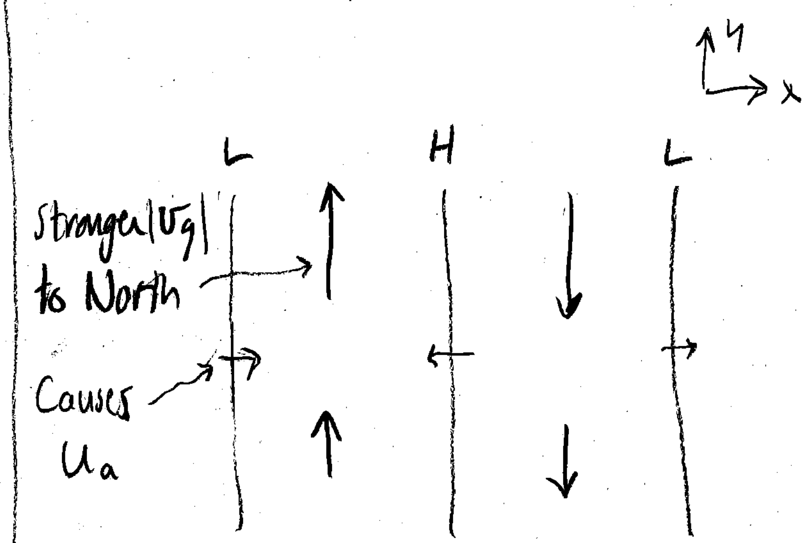
- fluid line, perturbed away from original latitude



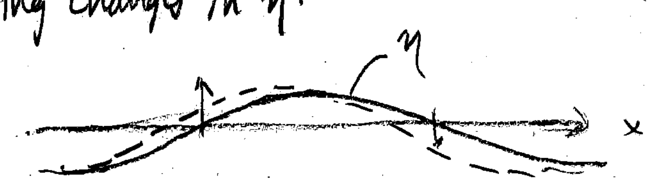
causes self advection, and moving pattern to West ---

LONG WAVES

ζ negligible,
stretching dominates response to Δf



Causing changes in η:



that move pattern to the West.