4.6 IGW: Group Velocity & Flow over Topography

Recall: IGW <u>dispersion relation</u>: (orient coordinate system so $l=0$ $\Rightarrow K_H = k$)

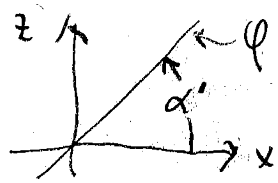
$$\omega^2 = \frac{f^2 m^2 + N^2 k^2}{m^2 + k^2} \quad (*)$$

k^2 ← for hydrostatic flow, we drop this term

$\boxed{z \text{ mom}}$ $\cancel{w'_t} = -\frac{1}{\rho_0} p'_z + b$ in derivation

neglect

angle of phase surfaces



$\tan \alpha' \equiv \alpha = \frac{k}{m}$; $\alpha^2 = \frac{\omega^2 - f^2}{N^2 - \omega^2}$ from (*)

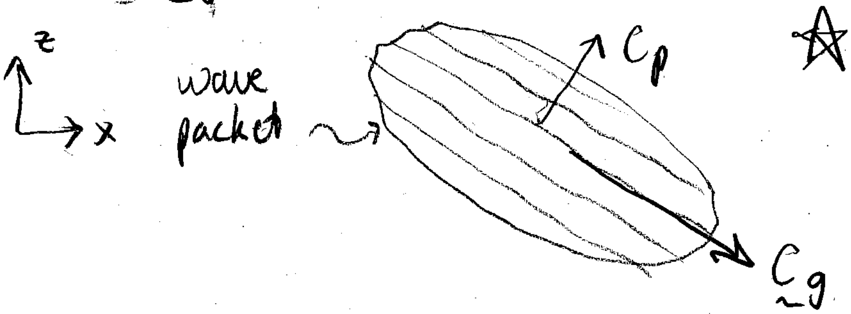
<u>Gill 8.4 defines 3 regimes</u>

1. ω close to N — Non-hydrostatic, Non-rotating — $\alpha \sim 1$
2. $f \ll \omega \ll N$ — Hydrostatic, Non-rotating — $\alpha \sim .1$
3. ω close to f — Hydrostatic, Rotating — $\alpha \sim .01$

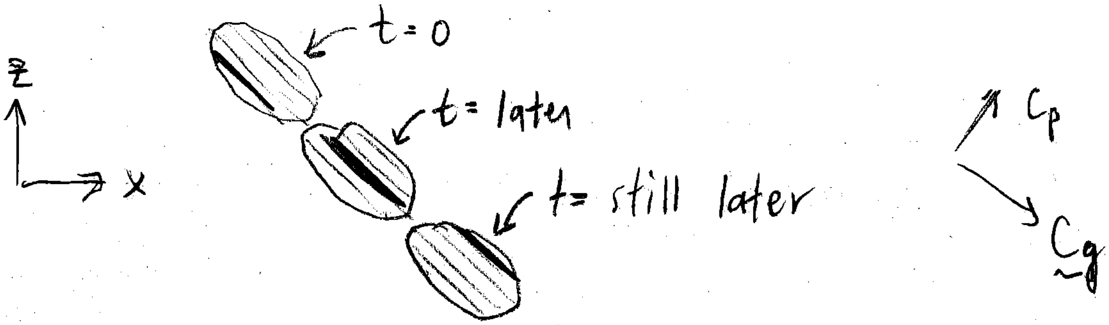
<u>Energy</u> - like <u>wave packets</u> travels with the <u>group velocity</u>

$\underset{\sim}{C_g} \equiv \left(\frac{\partial \omega}{\partial k}, \frac{\partial \omega}{\partial l}, \frac{\partial \omega}{\partial m} \right)$ and from (*) we can show $\underset{\sim}{C_g} \perp \underline{k}$

so $\underset{\sim}{C_g}$ <u>is normal to phase surfaces</u>



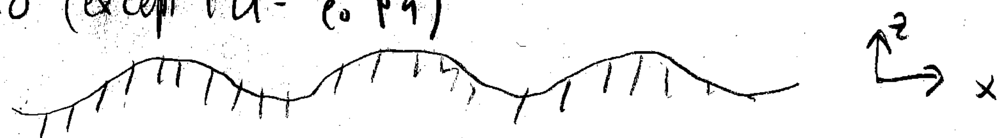
☆ <u>upward</u> phase propagation has <u>downward</u> energy propagation!



Consider IGW's forced by flow over sinusoidal hills:

- steady
- $\frac{\partial}{\partial y} = 0$ (except $f\mathcal{U} = \frac{-1}{\rho_0}\bar{p}_y$)

$\Leftarrow$ Wind: $-\mathcal{U}$ ($\mathcal{U}$ = positive, const.)

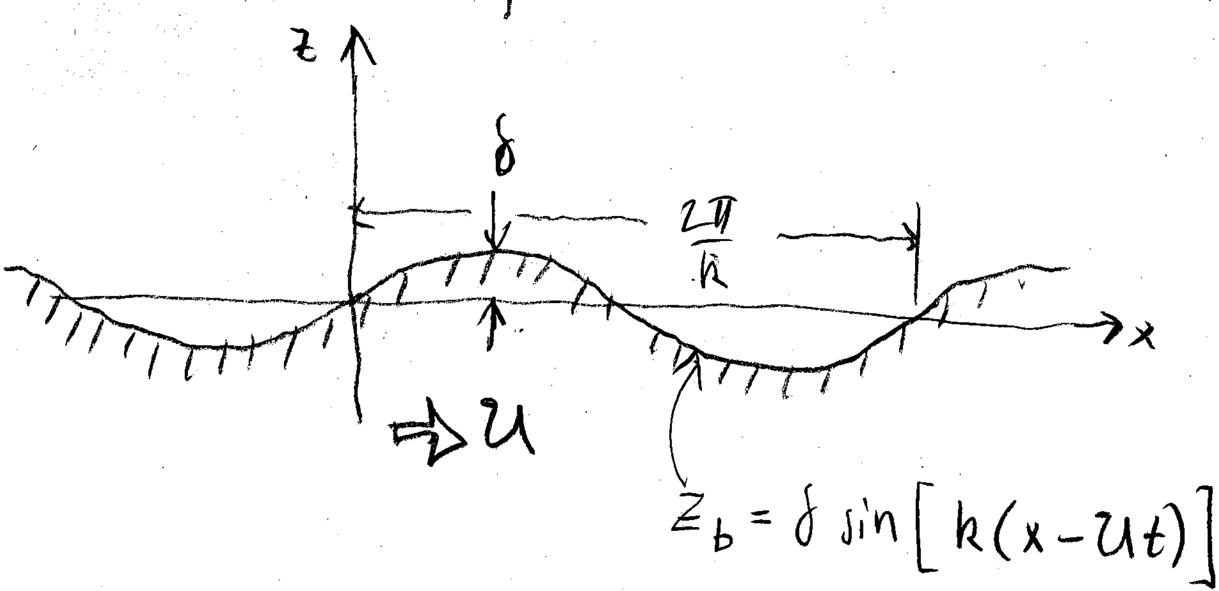


We could derive a solution with $\underset{\sim}{u} = (-\mathcal{U} + u', v', w')$

then $\frac{D}{Dt} \cong -\mathcal{U}\frac{\partial}{\partial x}$ instead of $\frac{\partial}{\partial t}$

<u>Or</u> make a Galilean transformation into f.o.r moving at velocity $-\mathcal{U}$ (to the left, with speed $\mathcal{U}$)

→ Now fluid is motionless except for waves, and the ground is moving



this gives a b.c. for the wave problem

$$w(z=z_b) \simeq w(z=0) = \frac{\partial z_b}{\partial t} = -\delta k U \cos\left[k(x-Ut)\right]$$

The general solution is $w = Re\{ w_0 \exp i(kx + mz - \omega t)\}$

$\Rightarrow \quad w = -\delta k U \cos(kx + mz - \omega t)$

with: • k set by topography

• $\omega = Uk$ set by frequency of parcel encounter with hills

Still Need to know m ...

For simplicity, assume flow is in regime (2) Hydrostatic, Non-rotating

$$\omega^2 = \frac{\cancel{f^2 m^2} + N^2k^2}{m^2 + \cancel{k^2}} \Rightarrow \omega = \pm \frac{Nk}{m} \quad (i)$$

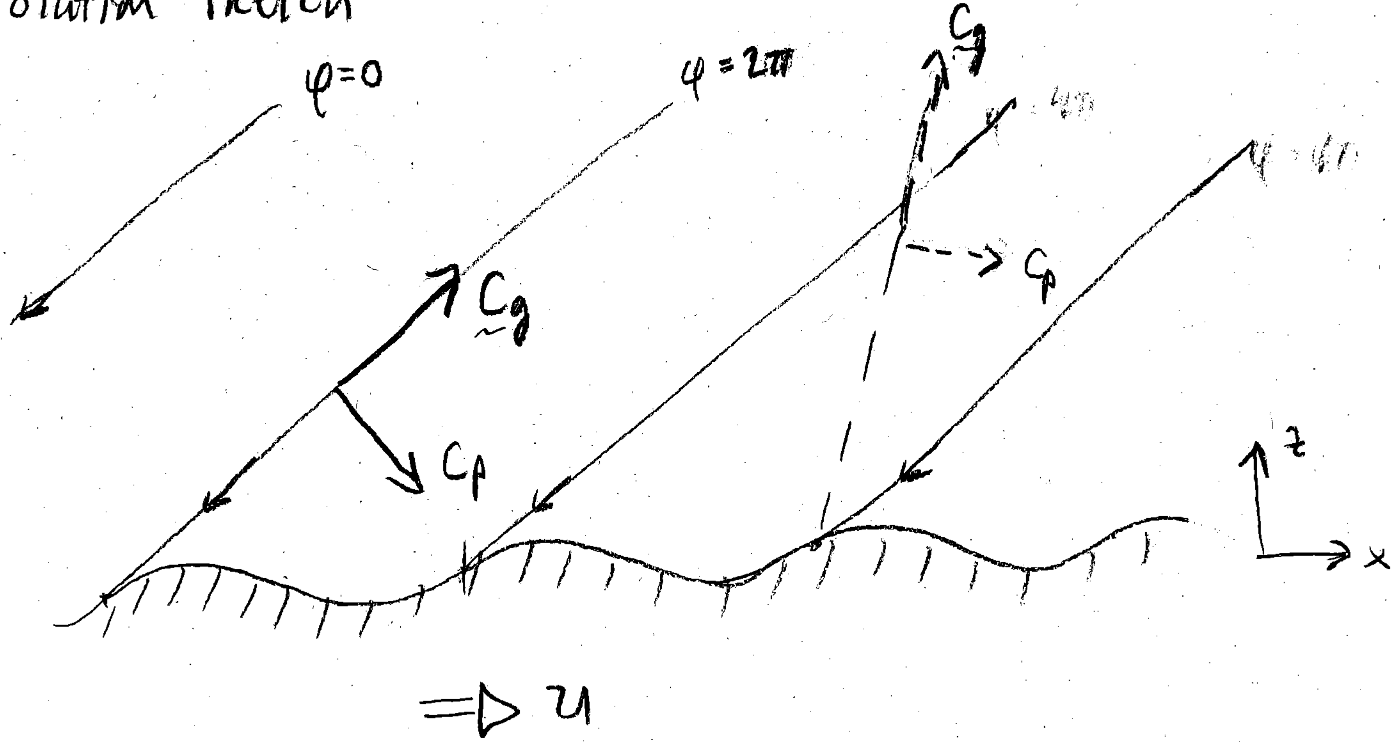
$\Rightarrow \quad C_g^z = \frac{\partial \omega}{\partial m} = \mp \frac{Nk}{m^2}$ (ii), physically we require

upward energy propagation (forced from below)

So, assuming k positive, we take positive root in (ii)

+ hence negative root in (i) $\Rightarrow \quad m = -\frac{Nk}{\omega} = -\frac{N}{U}$ (negative)

Solution sketch



$$\alpha^2 = \frac{\omega^2 - \cancel{f^2}}{N^2 - \cancel{\omega^2}} = \frac{\omega^2}{N^2} \Rightarrow \alpha = \pm \frac{\omega}{N} = \frac{uk}{N}$$

take pos. root

—— $\frac{uk}{N}$ smaller - - - - - $\frac{uk}{N}$ bigger

- pattern for real mountain waves is more complex