IGW Properties

Recall the dispersion relation

\[ \omega^2 = f^2 m^2 + N^2 k \frac{k^2}{k^2} \]

\[ k^2 = k_x^2 + k_y^2 + m^2 \]

For wave solutions we require real \( w, h, b, \) and \( m, \) which limits the possible frequencies

\( \sim 10^{-9} \text{ s}^{-1} \)  \( \sim 10^{-4} \text{ s}^{-1} \)

\[ \text{Internal Gravity Waves} \]

\( f \)

\( N \)

This is new! It is because we cannot have parcels oscillate vertically any faster than this.

We saw this before for Poincaré waves.

\[ \text{Parcel motion} \]

\[ \text{also moving out of the page ~ like an inertial circle} \]
the velocity solutions can be written as

\[ u = \text{Re} \left\{ u_0 \exp i \varphi \right\} \]
\[ v = \text{Re} \left\{ v_0 \exp i \varphi \right\} \]
\[ w = \text{Re} \left\{ w_0 \exp i \varphi \right\} \]

where \( u_0, v_0, \) and \( w_0 \) are complex constants and phase \( \varphi = kx + ly + mz - \omega t \)

putting these into \( \nabla \cdot \mathbf{u} = 0 \Rightarrow i( ku + lv + mw) = 0 \)

and : \( k \cdot u = 0 \)

so the motion is \perp \) to \( \mathbf{k} \)

\( \varphi = \text{const} \)

\( C_p = \frac{\omega}{k} \frac{k}{|k|} \)

a \( v \)-velocity develops because of Coriolis
Exploring other fields \( u, v, w, e', p' \)

Easy place to start is using \( \bar{e}' \) \( e' \), and assume 
\[ p' = \text{Re} \left\{ K \exp(i\varphi) \right\}, \quad \varphi = kx + ly + mz - \omega t \]

\[ -i\omega R = -\omega_0 \bar{p}_z \quad (+) \]

\[ R = -i\omega_0 \bar{p}_z \quad \text{(note} \quad \frac{1}{i} = -i) \]

So if we assume \( \omega_0 = \omega \) (real) \( \Rightarrow \omega = \omega \cos \varphi \)

and 
\[ p' = \frac{\omega \bar{p}_z}{\omega} \text{Re} \left\{ (-i) \cos \varphi + (-i) i \sin \varphi \right\} \]

\[ p' = \frac{\omega \bar{p}_z}{\omega} \sin \varphi \quad \text{(or} \quad b = -\frac{p'e'}{\omega} = \frac{\omega N}{\omega} \sin \varphi) \]

sign of \( p' \)

Note:

\( \bar{p}_z \) is negative

\( m, k \) positive as drawn

\( \varphi = \frac{3\pi}{4} \)
Physical interpretation: $\varphi'$ reflects the sign of $w$ that a fluid parcel experienced recently (look at greater $\varphi$)

Notes on the math:

- Working with $\exp i \varphi$ allows us to absorb changes of sine $\leftrightarrow$ cosine into the complex coefficients (like $u_0$, $R$, $u_0$, etc.)

- Procedure (e.g. in (++) is to stay with complex numbers when working on coefficients, then take real part at the end.

This works (for linear systems) because e.g.

$$\frac{d}{dt} \varphi' = \frac{d}{dt} \text{Re}\{R \exp i \varphi\} = \text{Re}\{\frac{d}{dt} [R \exp i \varphi]\}$$