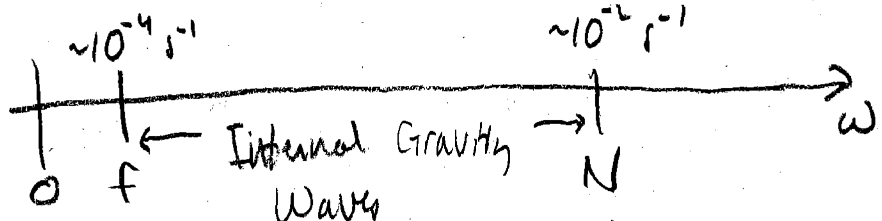
4.5 IGW Properties

recall the dispersion relation

$$\omega^2 = \frac{f^2 m^2 + N^2 K_H^2}{K^2} \qquad K_H^2 = k^2 + l^2 \qquad K^2 = k^2 + l^2 + m^2$$

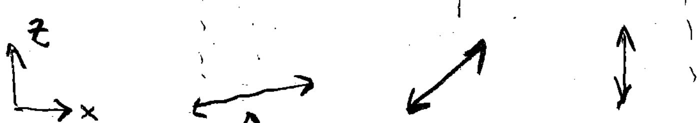
for wave solutions we require real ω, k, l, and m, which limits the possible frequencies



We saw this before for Poincaré Waves

this is new! It is because we can't have parcels oscillate vertically any faster than this

Parcel motion



also moving out of the page ~ like an inertial circle the velocity solutions can be written as

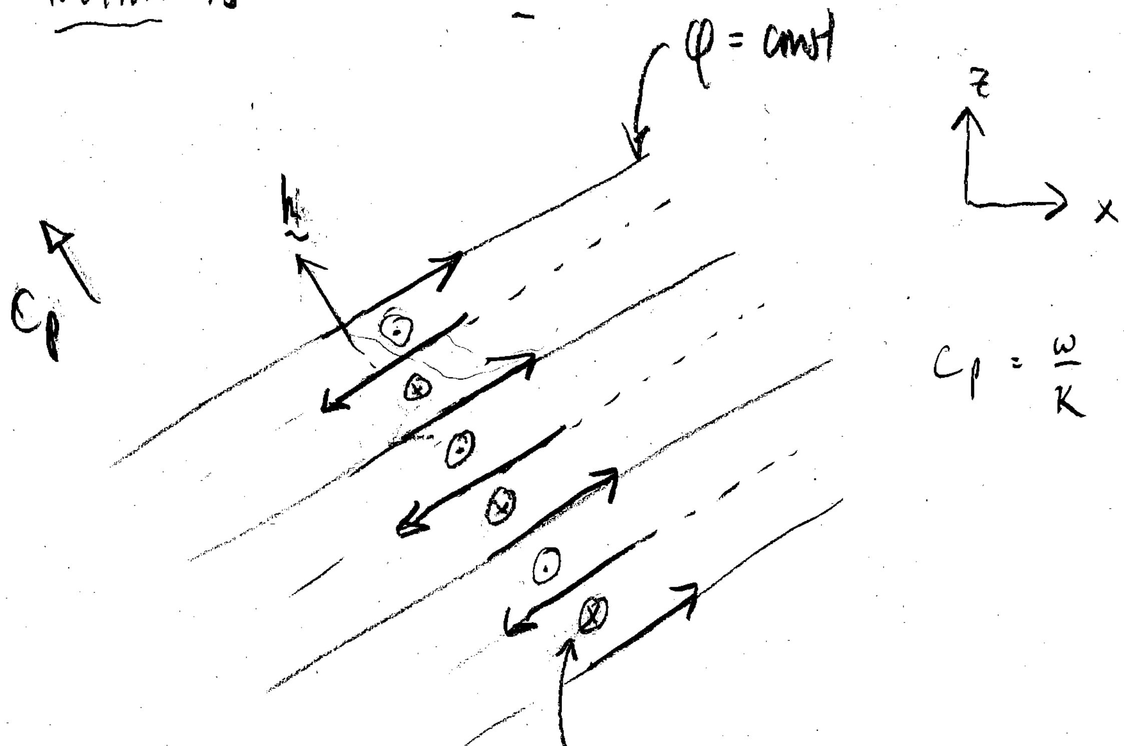
$$u = \mathrm{Re}\{u_0 \exp i\varphi\}$$
$$v = \mathrm{Re}\{v_0 \exp i\varphi\}$$
$$w = \mathrm{Re}\{w_0 \exp i\varphi\}$$

where u_0, v_0, and w_0 are <u>complex</u> constants and phase $\varphi = kx + ly + mz - \omega t$

putting these into $\nabla \cdot \underset{\sim}{u} = 0 \Rightarrow i(ku + lv + mw) = 0$

and $\therefore$ $\underset{\sim}{k} \cdot \underset{\sim}{u} = 0$ where $\underset{\sim}{k} = (k, l, m)$

so the <u>motion</u> is $\perp$ to $\underset{\sim}{k}$



$$c_p = \frac{\omega}{K} \frac{\underset{\sim}{k}}{|\underset{\sim}{k}|}$$

a v-velocity develops because of Coriolis

Exploring other fields u, v, w, ρ', p'

Easy place to start is using $\boxed{\rho}$ $\quad \rho'_t + w\bar{\rho}_z = 0$

and assume $\rho' = \mathrm{Re}\{R \exp i\varphi\}$, $\varphi = kx + ly + mz - \omega t$

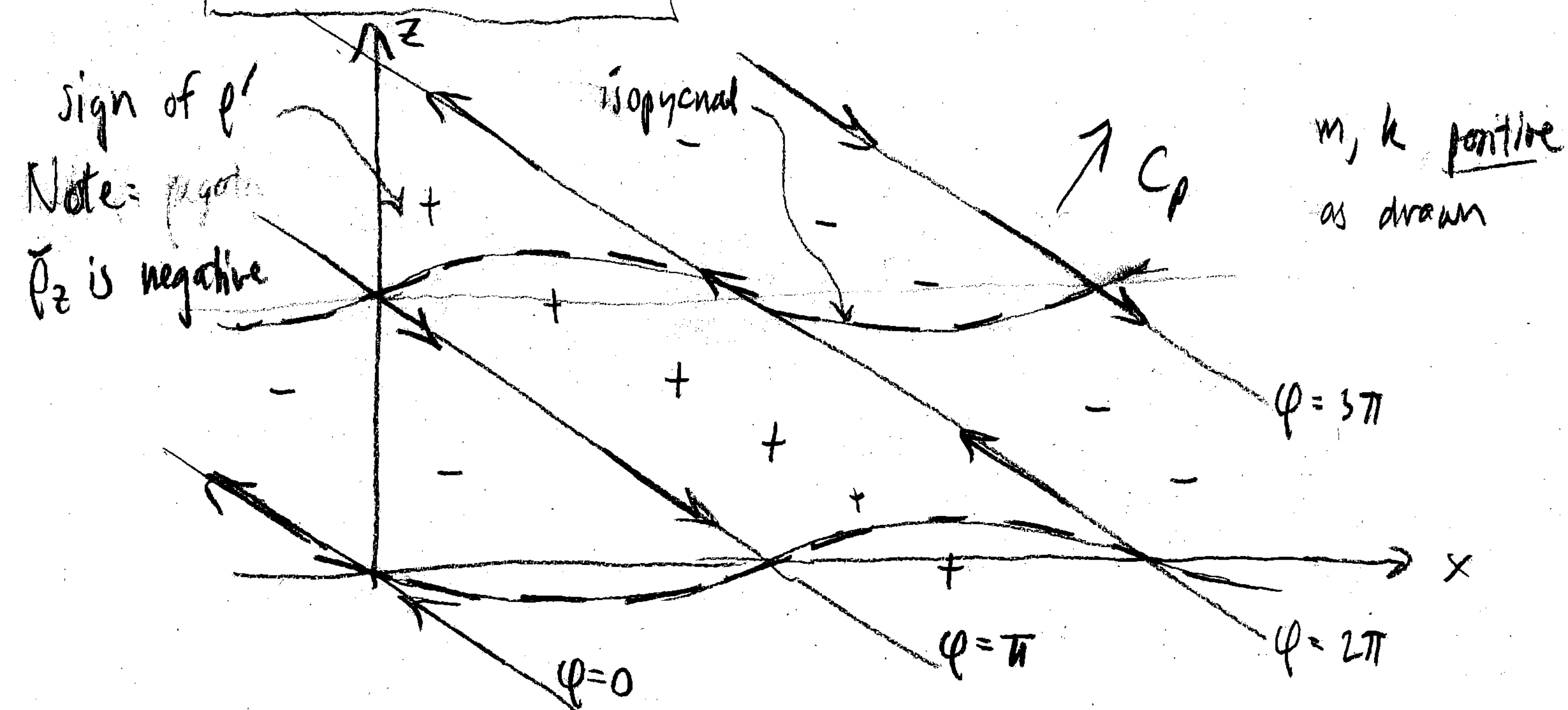
$$\Rightarrow \quad -i\omega R = -w_0\,\bar{\rho}_z \quad (\dagger\dagger)$$

$$\Rightarrow \quad R = \frac{-i\,w_0\,\bar{\rho}_z}{\omega} \qquad \left(\text{note } \frac{1}{i} = -i\right)$$

so if we assume $w_0 = W$ (real) $\Rightarrow$ $\boxed{w = W\cos\varphi}$

$$\text{and} \quad \rho' = \frac{W\bar{\rho}_z}{\omega}\,\mathrm{Re}\left\{(-i)\cos\varphi + (-i)\,i\sin\varphi\right\}$$

$$\Rightarrow \quad \boxed{\rho' = \frac{W\bar{\rho}_z}{\omega}\sin\varphi} \qquad \left(\text{or } b = -\frac{g\rho'}{\rho_0} = \frac{WN^2}{\omega}\sin\varphi\right)$$



Physical interpretation: ρ' reflects the sign of w that a fluid parcel experienced recently (look at greater φ)

Notes on the math:

- working with $\exp i\varphi$ allows us to absorb changes of sine $\leftrightarrow$ cosine into the complex coefficients (like w_0, R, u_0, etc.)
- Procedure (e.g. in (++)) is to stay with complex numbers when working out coefficients, then take real part at the end.

This works (for linear systems) because e.g.

$$\frac{\partial}{\partial t}\rho' = \frac{\partial}{\partial t} \mathrm{Re}\left\{R \exp i\varphi\right\} = \mathrm{Re}\left\{\frac{\partial}{\partial t}\left[R \exp i\varphi\right]\right\}$$