$\boxed{4.3}$ $\boxed{\text{Mechanical Energy}}$

We develop equations for the Kinetic Energy (KE) and Potential Energy (PE) of the macroscopic flow.
(not concerned with conversion to "internal" energy of molecular motion, because we assume fluid is Boussinesq $\Rightarrow$ ~incompressible)

units • Energy = Force × Distance $\left[J = N\,m = \frac{kg\,m^2}{s^2} = \text{Joules} \right.$

• Power = $\frac{d}{dt}$ Energy $\left[W = J\,s^{-1} = \frac{kg\,m^2}{s^3} = \text{Watts} \right]$

For a <u>fluid</u>

$$KE_V = \frac{KE}{\text{unit volume}} = \frac{1}{2}\rho_0\, \underset{\sim}{u} \cdot \underset{\sim}{u}$$ = work done to accelerate to speed $|\underset{\sim}{u}|$

$$PE_V = \frac{PE}{\text{unit vol.}} = \rho g z$$ = work done against gravity

For the linear SWE's we derive energy equations by forming

① $H\rho u \boxed{x\ \text{mom}} : H\rho u(u_t - fv = -g\eta_x)$

② $H\rho v \boxed{y\ \text{mom}} : H\rho v(v_t + fu = -g\eta_y)$

③ $\rho g \eta \boxed{\text{mass}} : \rho g \eta\left[\eta_t + H(u_x + v_y) = 0\right]$

(ρ = const)

$$① + ② \Rightarrow \left[\tfrac{1}{2} H\rho(u^2+v^2)\right]_t = -\rho g H(u\eta_x + v\eta_y) \quad (*)$$

$$③ \Rightarrow \left[\tfrac{1}{2}\rho g \eta^2\right]_t = -\rho g H \eta(u_x + v_y)$$

so ① + ② + ③ gives

$$\underbrace{\left[\tfrac{1}{2} H\rho(u^2+v^2) + \tfrac{1}{2}\rho g\eta^2\right]_t}_{\text{rate of change of}} = \underbrace{-\rho g H\left[(u\eta)_x + (v\eta)_y\right]}_{\text{convergence of "pressure work"}}$$

$$\frac{KE}{\text{unit horiz. area}} + \frac{APE}{\text{unit horiz. area}} \equiv KE_A + APE_A$$

"available potential energy"

- Note that $PE_A = \int_{-H}^{\eta} PE_v \, dz = \int_{-H}^{\eta} \rho g z \, dz = \tfrac{1}{2}\rho g(\eta^2 - H^2)$

  at rest $\eta = 0$, so define $PE_{A0} = -\tfrac{1}{2}\rho g H^2$, and subtracting this we are left with the "Available Potential Energy" — that which can be converted to KE. $APE_A \equiv PE_A - PE_{A0} = \tfrac{1}{2}\rho g \eta^2$ (for 1 layer SWE flow)

- Note: Coriolis vanished from (*). It does no work because it is $\perp$ to the velocity!

- pressure work is a force (pressure ~ $\rho g \eta$) times velocity — it is how waves move energy around without moving parcels (much)