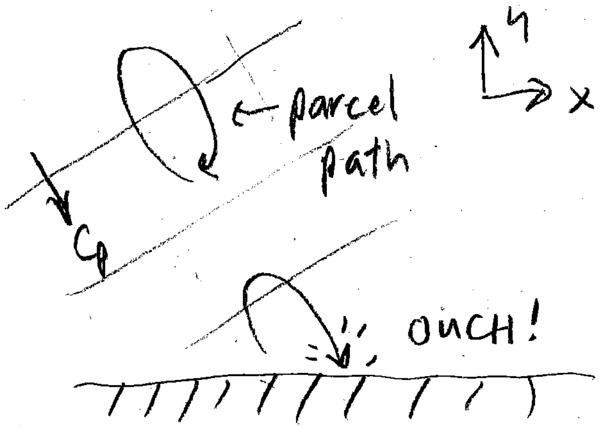
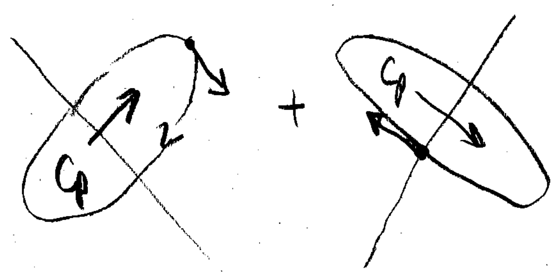
Kelvin Waves

Q: how do Poincaré Waves work in the presence of a lateral boundary?



could fix with a superposition of two waves



Or: look for a solution with $v = 0$ everywhere

$\boxed{x \text{ mom}}$ $$u_t - f\cancelto{0}{v} = -g\eta_x$$

$\boxed{y \text{ mom}}$ $$\cancelto{0}{v_t} + fu = -g\eta_y$$

$\boxed{\text{mass}}$ $$\eta_t + Hu_x + H\cancelto{0}{v_y} = 0$$

these combine to give the usual non-rotating wave equation

$$\eta_{tt} - gH\eta_{xx} = 0$$

which allow solutions of the form

$$\eta = \eta_0 G(y) \cos(kx - \omega t)$$

and $$u = \frac{\eta_0}{H} c_0 G(y) \cos(kx - \omega t)$$

$c_0 = \sqrt{gH}$

$c_p \rightarrow$ k positive

$c_p \leftarrow$ k negative

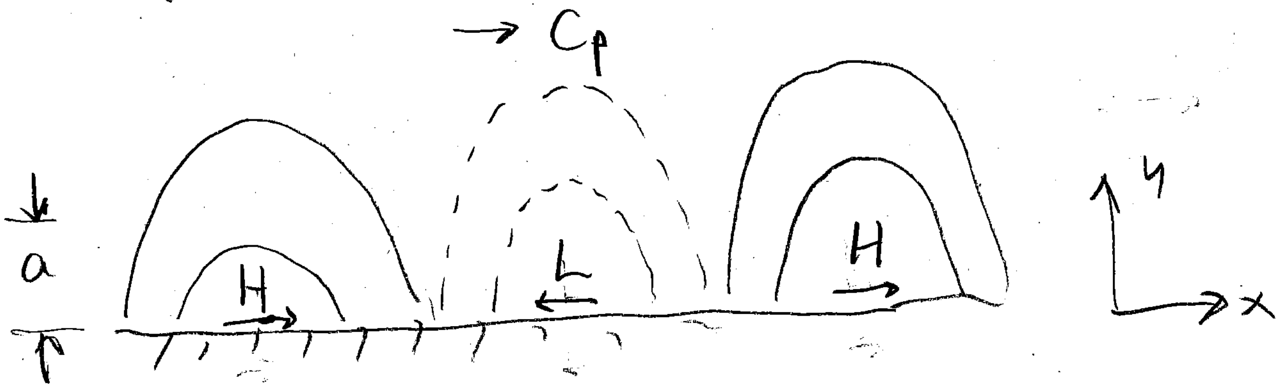
Then use $\boxed{y \text{ mom}}$ to find an equation for $G(y)$ ②

$$(\star)\ \boxed{fu = -g\eta_y} \Rightarrow f \frac{\cancel{\eta_0}}{H} c_0 G = -g \cancel{\eta_0} \frac{dG}{dy}$$

$$\text{or } \frac{dG}{dy} + \frac{f\sqrt{gH}}{gH} G = 0 \quad \text{or} \quad \boxed{\frac{dG}{dy} + \frac{1}{a} G = 0} \quad \left(a = \frac{\sqrt{gH}}{f}\right)$$

$\therefore G = e^{-y/a}$ (any scaling constant absorbed into η_0)

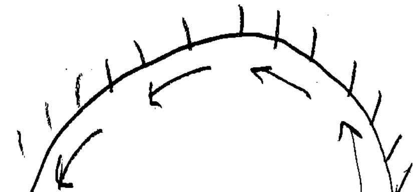
So the $\eta(x,y,t)$ field looks like



Kelvin waves:

- non-dispersive
- $C_p = |\underline{C}_g| = \sqrt{gH}$
- $\boxed{y \text{ mom}}$ is geostrophic $(\star)$ and solutions that decay away from the coast require that C_p is only toward positive x

$\Rightarrow$ K-waves always travel cyclonically around ocean basins



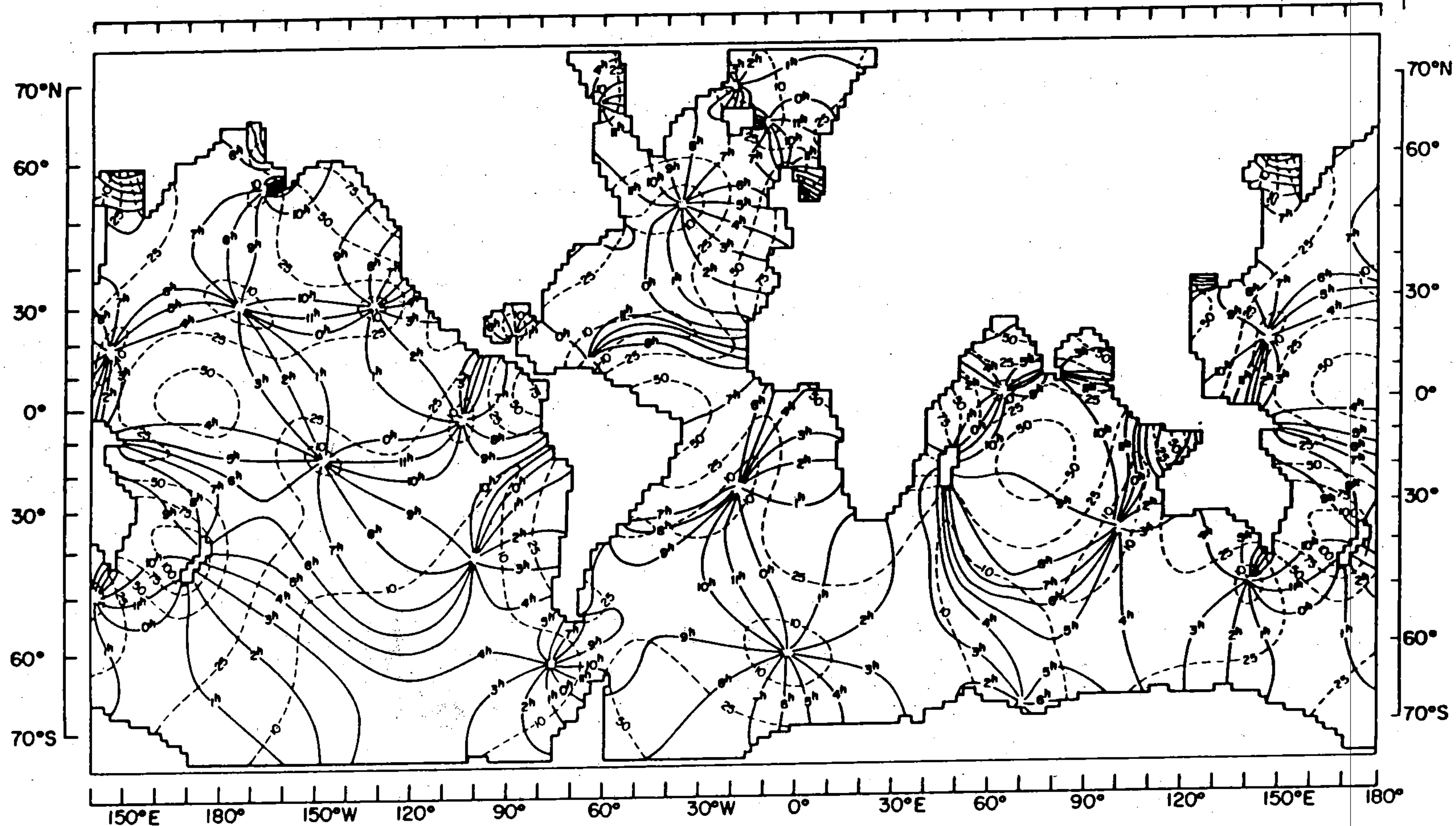
N. Hemisphere

- they have no low frequency limit
  (unlike Poincaré waves that have $\omega > f$)

- Tides can, in part, be described as
  Kelvin waves.

Gill (1982) Atmosphere-Ocean Dynamics

p.392



(a)

Fig. 10.9. (a) The M_2 tide as computed from a numerical model by Accad and Pekeris (1978, Fig. 8); the phase is shown by solid lines marked in Greenwich hours, and the range is shown by dashed lines, in centimeters. (b) The 12.5-hr free mode of oscillation as computed by Platzman *et al.* (1981). Phase contours are denoted by solid lines and co-range lines by dashed lines.