3.4

The Rossby Adjustment Problem, Rossby Radius

... linearized SWE on an f-plane summary

parcel motion

high frequency : non-rotating waves

$\omega = f$: inertial oscillations

$\omega = 0$: steady geostrophic flow

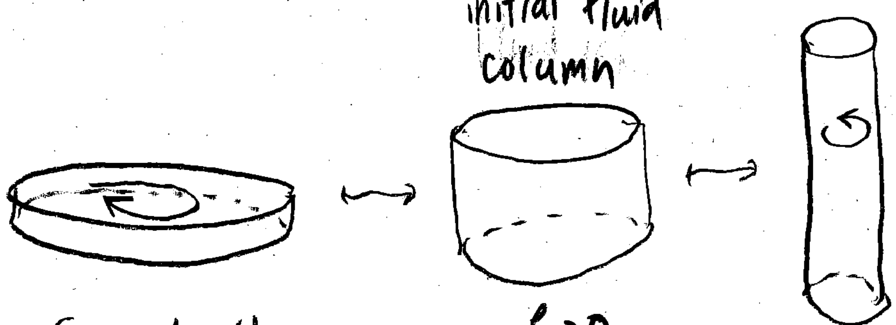
Since the equations are linear we may superimpose (add) solutions

- Recall the expression from $\boxed{y\ \text{mom}}_x - \boxed{x\ \text{mom}}_y$

$$\Rightarrow \left(\zeta - \frac{f}{H}\eta\right)_t = 0 \Rightarrow \boxed{\zeta - \frac{f}{H}\eta = \left(\zeta - \frac{f}{H}\eta\right)\Big|_{t=0}} \quad (*)$$

a linearized form of conservation of "potential vorticity"

initial fluid column



Squash it and $\zeta < 0$

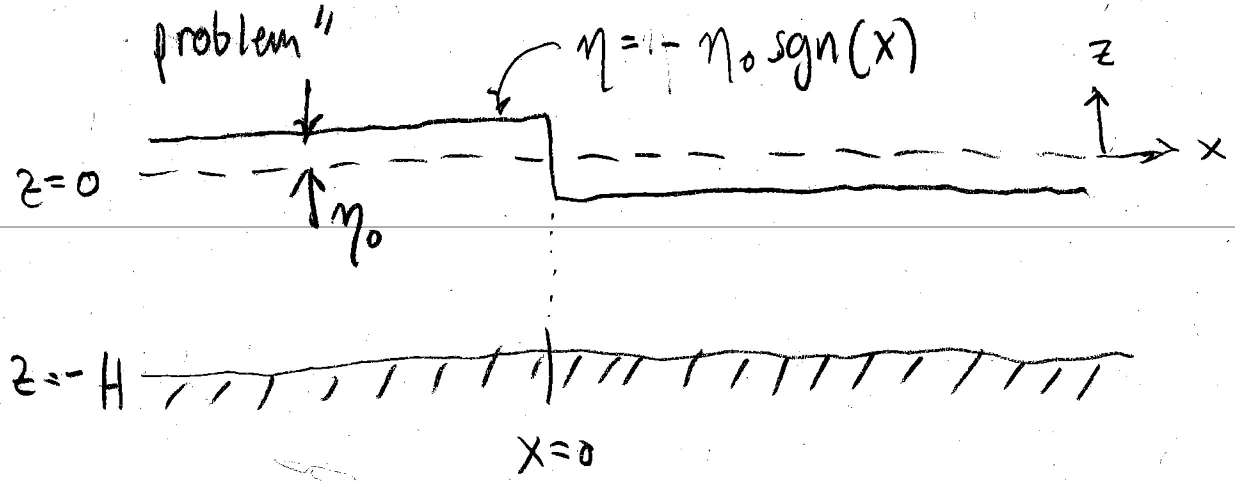
$\zeta = 0$

Stretch it and $\zeta > 0$

"anti-cyclonic"

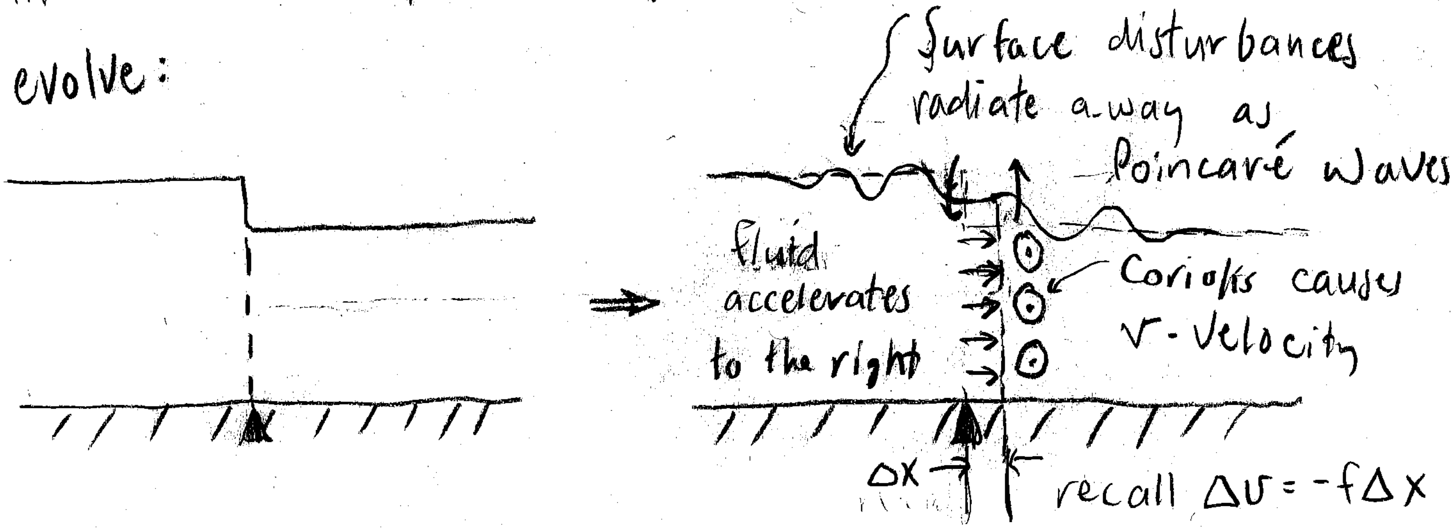
"cyclonic"

(*) allows us to solve the "Rossby adjustment problem"



What happens when we start with some η disturbance, and $u = v = 0$?

- The i.c. is clearly <u>not</u> in geostrophic balance, so it will evolve:



COMPLICATED!

However, the eventual <u>steady</u> <u>state</u> must:

1. be in geostrophic balance $v = v_g = \frac{g}{f} \eta_x$ $(u_g = 0)$
2. Satisfy (*) $\zeta - \frac{f}{H}\eta = \left(\zeta - \frac{f}{H}\eta\right)\Big|_{t=0}$

potential vorticity is conserved

The math is easy: (3)

$$\zeta - \frac{f}{H}\eta = \frac{f}{H}\eta_0 \,\mathrm{sgn}(x) \qquad (**)$$

and $\zeta = v_x - \cancel{u_y} = \frac{\partial}{\partial x}\left(\frac{g}{f}\eta_x\right) = \frac{g}{f}\eta_{xx}$

thus $(**)$ is $\eta_{xx} - \frac{f^2}{gH}\eta = \frac{f^2}{gH}\eta_0 \,\mathrm{sgn}(x)$

or $\eta_{xx} - \frac{1}{a^2}\eta = \frac{1}{a^2}\eta_0 \,\mathrm{sgn}(x) \qquad (+)$

where $\boxed{a \equiv \frac{\sqrt{gH}}{f}}$ the "Rossby radius of deformation" ¿?

→ how far a non rotating wave gets in time $1/f$

Solution to (+) eq. for $x \geq 0$

(I) Homogeneous solution $\eta_{xx} - \frac{1}{a^2}\eta = 0 \Rightarrow$ guess $\eta = \cancel{A e^{x/a}} + B e^{-x/a}$

(II) Particular solution $\eta = \text{const} = C$ will work | expect bounded solution as $x \to \infty$

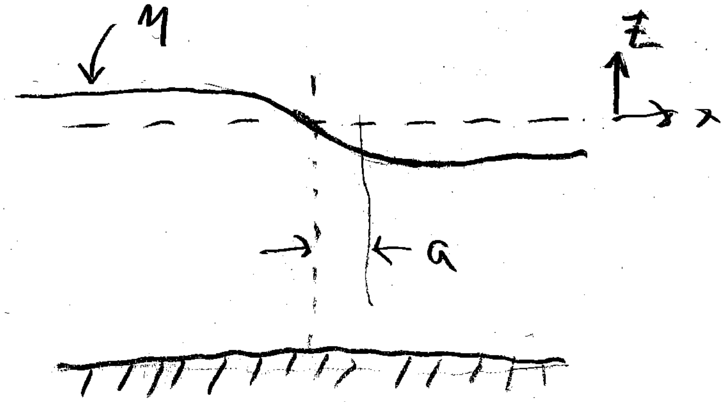
sum must satisfy boundary conditions: (2)

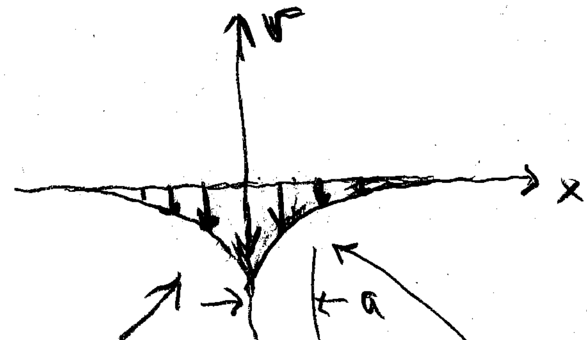
$$\eta(0) = 0, \quad \eta(x\to\infty) = -\eta_0$$

$$\Rightarrow \eta_I + \eta_{II} = \eta = C + Be^{-x/a} = -\eta_0 + \eta_0 e^{-x/a}$$

Full solution is

$$\frac{\eta}{\eta_0} = \begin{cases} -1 + e^{-x/a}, & x > 0 \\ 1 - e^{x/a}, & x < 0 \end{cases} \qquad \Bigg\| \qquad u = 0, \quad v = \frac{-g\eta_0}{fa} \exp\left(\frac{-|x|}{a}\right)$$

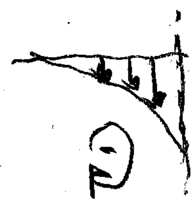




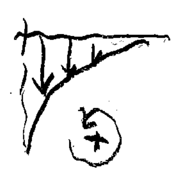
• Original "step" of η only slumped a distance "a" before geostrophic balance was attained

⇒ no further changes (except by friction)

for x < 0
fluid columns squashed
⇒ q becomes negative



but for x > 0 fluid columns are stretched
⇒ q positive



• Only a little of the original (infinite) potential energy of the η field was extracted
($\frac{1}{3}$ → KE, $\frac{2}{3}$ goes to radiated Poincaré waves)