The Rossby Adjustment Problem, Rossby Radius

... linearized SWE on an f-plane summary

High frequency: non-rotating waves

\[ \omega = \mathbf{f} \quad \text{inertial oscillation} \]

\[ \omega = 0 \quad \text{steady geostrophic flow} \]

Since the equations are linear we may superimpose (add) solutions.

Recall the expression from

\[ \left[ \mathbf{y}_{\text{max}} - \mathbf{y}_{\text{min}} \right] x \]

\[ \Rightarrow \quad (\zeta - \frac{\mathbf{f}}{H} \eta)_t = 0 \quad \Rightarrow \quad \zeta - \frac{\mathbf{f}}{H} \eta = \left( \zeta - \frac{\mathbf{f}}{H} \eta \right)_{t=0} \]

\[ \text{a linearized form of conservation of "potential vorticity"} \]

Squash it and \( \zeta < 0 \)

"anti-cyclonic"

Initial fluid column

\( \zeta = 0 \)

Stretch it and \( \zeta > 0 \)

"cyclonic"
(2) allows us to solve the "Rossby adjustment problem" with \[ \eta = -\eta_0 \text{sgn}(x) \]

\[ z=0 \quad \eta \]

\[ z=-H \]

X=0

What happens when we start with some \( \eta \) disturbance, and \( u=v=0 \)?

The i.c. is clearly not in geostrophic balance, so it will evolve:

Surface disturbances radiate away as Poincaré waves

Fluid accelerates to the right

\[ \Delta x \rightarrow \text{recall } \Delta u = -f \Delta x \]

Complicated!

However, the eventual steady state must:

1. be in geostrophic balance \[ \mathbf{V} = \mathbf{V}_g = \frac{f}{\mathbf{h}} \eta \mathbf{x} \] (\( \eta_0 = 0 \))

2. satisfy (2) \[ \zeta = \frac{f}{\mathbf{h}} \eta = \left( \zeta - \frac{f}{\mathbf{h}} \eta \right) \bigg|_{t=0} \]

Potential vorticity is conserved
The math is easy:

\[ \xi - \frac{f}{H} \eta = \frac{f}{H} \eta_0 \text{sgn}(x) \quad (***) \]

and

\[ \xi_1 = V_x - \xi_0 \eta = \frac{\partial}{\partial x} \left( \frac{f}{H} \eta \right) = \frac{f}{H} \eta_{xx} \]

thus (****) is

\[ \eta_{xx} - \frac{f^2}{gH} \eta = \frac{f^2}{gH} \eta_0 \text{sgn}(x) \]

where

\[ A = \sqrt{\frac{gH}{f}} \]

the "Rossby radius of deformation" \( c \)?

\[ \rightarrow \text{how far a non rotating wave gets in time } 1/f \]

Solution to (****) eq. for \( x > 0 \)

(I) Homogeneous solution \( \eta_{xx} - \frac{1}{a^2} \eta = 0 \) = guess \( \eta = A e^{x/a} + B e^{-x/a} \)

(II) Particular solution \( \eta = \text{const} = C \) will work

\[ \text{sum must satisfy boundary conditions:} \]

\[ \eta(0) = 0, \quad \eta(x \to \infty) = -\eta_0 \]

\[ \Rightarrow \quad \eta_{xx} + \eta_{xx} = \eta = C + B e^{-x/a} = -\eta_0 + \eta_0 e^{-x/a} \]
Full solution is
\[ Y = \begin{cases} -1 + e^{-x/a}, & x > 0 \\ 1 - e^{x/a}, & x < 0 \end{cases} \]

\[ U = 0 \]
\[ U = \frac{\gamma_0}{\alpha} \exp \left( -\frac{|x|}{a} \right) \]

- Original "step" of \( Y \) only slumped a distance "\( a \)" before geostrophic balance was attained
  \( \Rightarrow \) no further changes (except by friction)

- Only a little of the original (infinite) potential energy of the \( Y \) field was extracted
  \( (\frac{1}{3} \Rightarrow \text{KE}, \quad \frac{2}{3} \text{ goes to radiated Poincaré waves}) \)