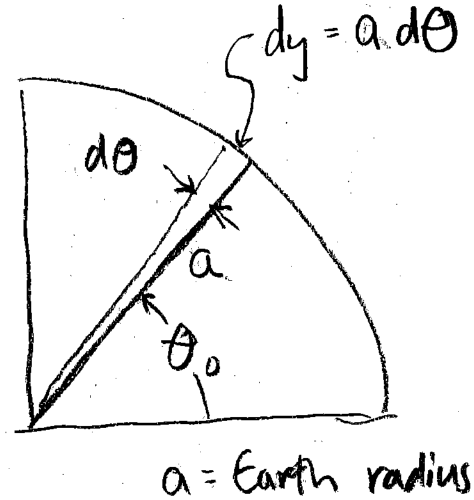
2.6

The β-plane, Rossby Number, Geostrophic Balance, Thermal Wind

The first approximation of <u>sphericity</u> away from the f-plane comes from expanding f as a Taylor series about θ_0:

$$\Rightarrow f = f\Big|_{\theta_0} + \frac{df}{dy}\Big|_{\theta_0} y + \ldots$$



where $y=0$ at θ_0 and

$$\frac{df}{dy}\Big|_{\theta_0} = \frac{df}{d\theta}\frac{d\theta}{dy}\Big|_{\theta_0} = 2\Omega\cos\theta_0 \frac{1}{a}$$

so we write $\boxed{f \cong f_0 + \beta y}$ — we call it the "β-plane" when used in $\boxed{\underset{\sim}{x} \text{ mom}}$

where $\beta \equiv \dfrac{2\Omega\cos\theta_0}{a}$

Used for "Rossby Wave" derivation (later) for $\Omega = 7.292 \times 10^{-5}$ s^{-1}

$a = 6.37 \times 10^{6}$ m

$\theta_0 = 45°$

$$\Rightarrow \quad f_0 = 2\Omega \sin\theta_0 = 1.0 \times 10^{-4} \; s^{-1}$$

$$\beta = \frac{2\Omega \cos\theta_0}{a} = 1.6 \times 10^{-11} \; s^{-1} \, m^{-1}$$

Consider flow on the f-plane ...

(I will write f, but this really means f_0)

$\boxed{x, y \text{ mom}}$ $\quad \dfrac{D \underset{\sim}{u}_H}{Dt} + f \hat{k} \times \underset{\sim}{u} = -\dfrac{1}{\rho_0} \nabla_H p$ $\qquad \underset{\sim}{u}_H = (u, v)$

$\nabla_H = \left(\dfrac{\partial}{\partial x}, \dfrac{\partial}{\partial y}\right)$

$$\text{scaling} \Rightarrow \quad \underset{①}{\frac{U^2}{L}} \qquad \underset{②}{fU} \qquad \sim \qquad \underset{③}{\frac{[p']}{L}}$$

In working out the hydrostatic approximation, we had $f = 0$, so ① had to balance ③

BUT, for planetary flows the actual balance is much different!!

Consider

$$\frac{①}{②} = \frac{U^2}{L}\,\frac{1}{fU} = \frac{U}{fL} \equiv R_o \quad \text{the "Rossby Number"}$$

at mid latitudes $f \sim 10^{-4}\ s^{-1}$, so

<u>atm</u>: $\left.\begin{array}{l} U = 10\ m\ s^{-1} \\ L = 1000\ km = 10^{6}\ m \end{array}\right\} R_o = 0.1$

<u>ocn</u>: $\left.\begin{array}{l} U\ \ 0.5\ m\ s^{-1} \\ L\ \ 50\ km = 5\times 10^{4}\ m \end{array}\right\} R_o = 0.1$ again

∴ the horizontal momentum balance is mainly

acceleration + Coriolis = pressure gradient

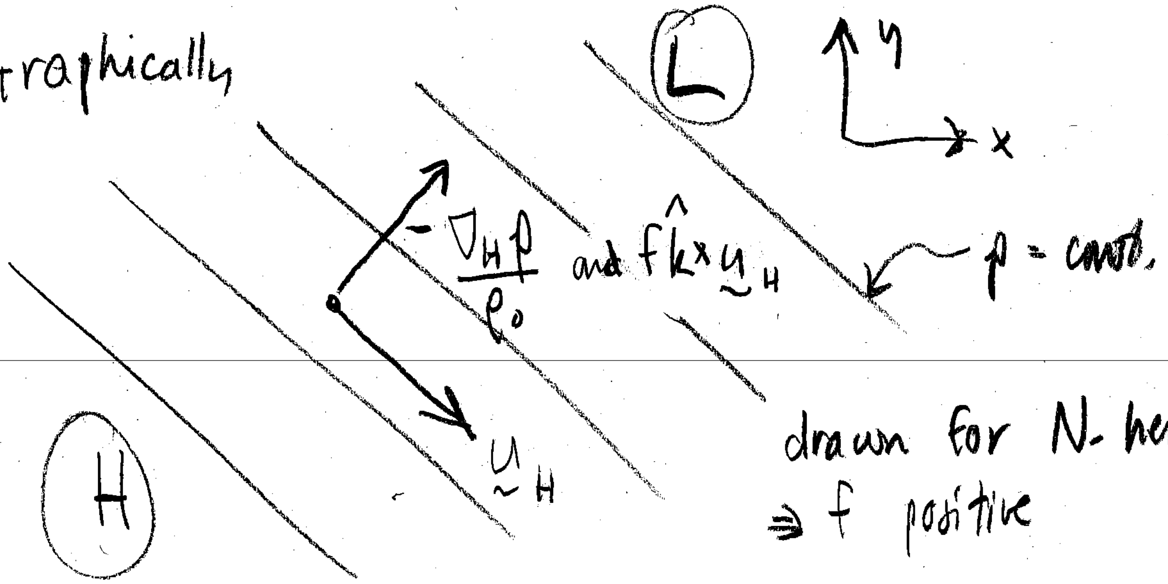
small

or $\boxed{f\hat{k}\times \underline{u}_H = -\frac{1}{\rho_0}\nabla_H p \quad \text{"geostrophic balance"}}$

i.e. $\boxed{x \text{ mom}}\quad -fv = -\frac{1}{\rho_0}p_x$

$\boxed{y \text{ mom}}\quad fu = -\frac{1}{\rho_0}p_y$

Graphically



drawn for N- hemisphere
$\Rightarrow f$ positive

$\underset{\sim}{u}_H$ is along "isobars" (lines of constant pressure)

"push it and it moves <u>to the right</u>" (N. hemisphere)

Another consequence of geostrophy relates to <u>vertical</u> <u>shear</u>

consider $\boxed{\hat{x}\text{ mom}}$ $\quad -fv = -\frac{1}{\rho_0} p_x$

$\boxed{z\text{ mom}}$ $\quad p_z = -\rho g$

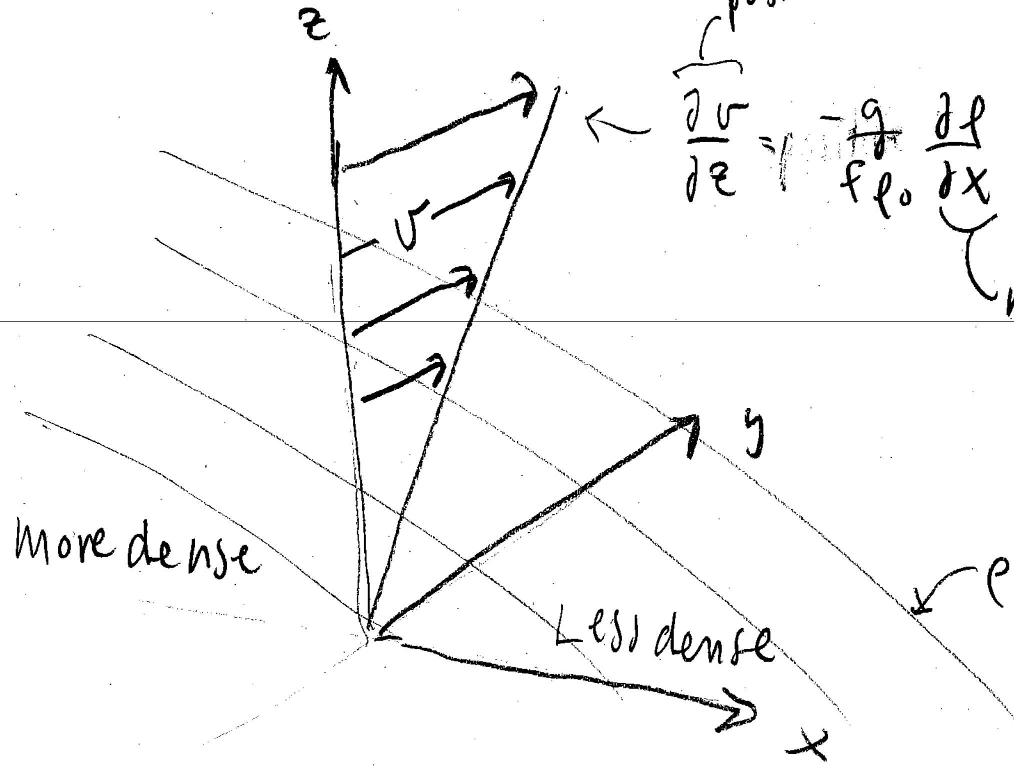
$$\frac{\partial}{\partial z}\boxed{x\text{ mom}} \Rightarrow \boxed{-f\frac{\partial v}{\partial z} = \frac{-1}{\rho_0} p_{xz} = \frac{g\rho_x}{\rho_0}}$$

similarly from $\boxed{y\text{-mom}}$: $\boxed{f u_z = \frac{g\rho_y}{\rho_0}}$

called the "Thermal Wind"

(recall $\rho \sim \frac{1}{T}$ in the atm.)

So vertical shear requires a horizontal density gradient



$$\underbrace{\frac{\partial v}{\partial z}}_{\text{positive}} = \frac{-g}{f\rho_0}\underbrace{\frac{\partial \rho}{\partial x}}_{\text{negative}}$$