Review of the governing equations

$\boxed{\rho}$ $\rho = \rho(S, T, p)$ or $\rho(T, p, q)$ equation of state

$\boxed{\text{mass}}$ $\underbrace{\frac{1}{\rho}\frac{D\rho}{Dt}}_{\text{fractional change in density following a fluid parcel}} + \underbrace{\nabla \cdot \underset{\sim}{u}}_{\text{on the rhs: } = -\nabla\cdot\underset{\sim}{u} = -\text{divergence} = \text{convergence}} = 0$ continuity or mass conservation

Note $\frac{Dc}{Dt} \equiv \frac{\partial c}{\partial t} + \underset{\sim}{u} \cdot \nabla c = c_t + u c_x + v c_y + w c_z$

for arbitrary scalar c

called material derivative or Lagrangian derivative

= "rate of change of c following a fluid parcel"

can also write continuity as

$\boxed{\text{Mass}}$ $\rho_t + \nabla \cdot (\rho \underset{\sim}{u}) = 0$

$\rho_t \equiv \frac{\partial \rho}{\partial t}$

$\boxed{\underset{\sim}{x} \text{ mom}}$ $$\frac{D\underset{\sim}{u}}{Dt} = -\frac{1}{\rho}\nabla p - g\hat{k}$$ (neglecting viscosity)

momentum conservation ~ Navier-Stokes equations

actually 3 equations

$$\frac{Du}{Dt} = -\frac{1}{\rho}p_x, \quad \frac{Dv}{Dt} = -\frac{1}{\rho}p_y, \quad \frac{Dw}{Dt} = \underbrace{-\frac{1}{\rho}p_z - g}_{\text{hydrostatic}}$$

Note $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$

$\hat{k}$ = vertical unit vector

☆ so we have 5 equations in 5 unknowns u, v, w, p, ρ

(need more to keep track of T or S or q)

For typical atm. + ocn. scales we make several additions + approximations

1. Rotating frame of reference → Coriolis force + a slight redefinition of gravity

2. Length scale ≫ Height scale → Hydrostatic flow

3. Small density changes → Boussinesq approx.
   → incompressible flow $\nabla \cdot \underset{\sim}{u} = 0$

4. Average over turbulent time scales → Reynolds averaging
   → Eddy viscosity + Diffusivity