1.3

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Properties of Air + Water

Equation of State

Potential Temperature + Density

Buoyancy Frequency w/ Compressibility

Air	Seawater
N_2 78.1%, O_2 21.0%, ...	H_2O, NaCl, ...
$\rho = \rho(T, p, q)$ ← Eqn. of state →	$\rho = \rho(S, T, p)$
specific humidity $\propto \dfrac{\text{mass of water vapor}}{\text{mass of moist air}}$	salinity $\propto \dfrac{\text{mass of salt}}{\text{mass of seawater}}$
$\rho \to 0$ kg m^{-3} top of atm.	$\rho \sim 1000$ kg m^{-3} freshwater
$\rho \sim \frac{1.2}{e}$ kg m^{-3} at 7-8 km (scale height)	$\rho \sim 1025$ kg m^{-3} ocn. surface
$\rho \sim 1.2$ kg m^{-3} sea level	$\rho \sim 1055$ kg m^{-3} abyss (largely due to compressibility)
varies by $\mathcal{O}(1)$	varies by only ~ 5%

More about seawater $\rho = \rho(S, T, p)$

- most important density variation is due to temperature, although salinity is important in coastal & polar regions

<u>TYPICAL VALUES</u> - at $T = 10°C$, $S = 35$: units are "practical salinity" = psu $\cong \frac{\text{kg salt}}{\text{kg seawater}} \times 1000$

and $p = 0$

$\Rightarrow \rho \cong 1027 \text{ kg m}^{-3}$

with rates of change :

$$\alpha \equiv -\frac{1}{\rho}\frac{\partial \rho}{\partial T}\Big|_p \cong 1168 \times 10^{-7} \text{ K}^{-1} \quad , \quad \frac{\partial \rho}{\partial S} = 0.781 \ \frac{\text{kg}}{\text{m}^{-3}} \ (\text{per psu})$$

(these vary with T, S, p)

so: increasing salinity by 1 psu $\Rightarrow \Delta\rho \cong 0.78 \text{ kg m}^{-3}$

increasing temperature by $1°C$ $\Rightarrow \Delta\rho \cong -0.12 \text{ kg m}^{-3}$

Speed of sound: $c_s = \sqrt{\frac{\partial p}{\partial \rho}} = 1500 \text{ m s}^{-1}$

Note: Often the dynamically-important density changes are just a few kg m^{-3} !

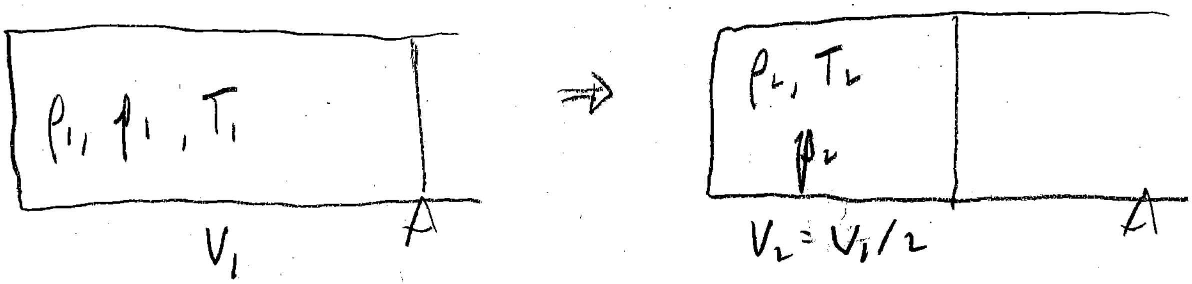
More about air - neglecting moisture $\rho = \rho(T, p, \cancel{q})$

Dry air ~ an "ideal gas" $\Rightarrow$ $\boxed{p = \rho R T}$ (*)

°K (°K − 273.15 = °C)

R = gas constant = 287 J kg^{-1} K^{-1}

What if we compress air in a cylinder?



Clearly $\rho_2 = 2 \times \rho_1$, But what about p & T?

expect both to increase, but how much?

→ Need another equation to use with (*)

From thermodynamics:

If the change is (i) "adiabatic" (no heat added)

and (ii) "reversible" (e.g. no viscous dissipation)

then the change is "isentropic" and

$$\boxed{\frac{p}{\rho^\gamma} = \text{const.}} \quad (**)$$

where $\gamma = 1.4$ for dry air

"gamma"

so for isentropic changes of an ideal gas
from p to p_r (r = "reference")

$$(**) \Rightarrow \boxed{\frac{p}{\rho^\gamma} = \frac{p_r}{\rho_r^\gamma}}$$

then, using $p = \rho R T$ it is easy to show:

$$\left\{ \frac{T_r}{T} = \left(\frac{p_r}{p}\right)^{\frac{\gamma-1}{\gamma}} \right. \quad \text{and} \quad \frac{\rho_r}{\rho} = \left(\frac{p_r}{p}\right)^{\frac{1}{\gamma}}$$

use °K!

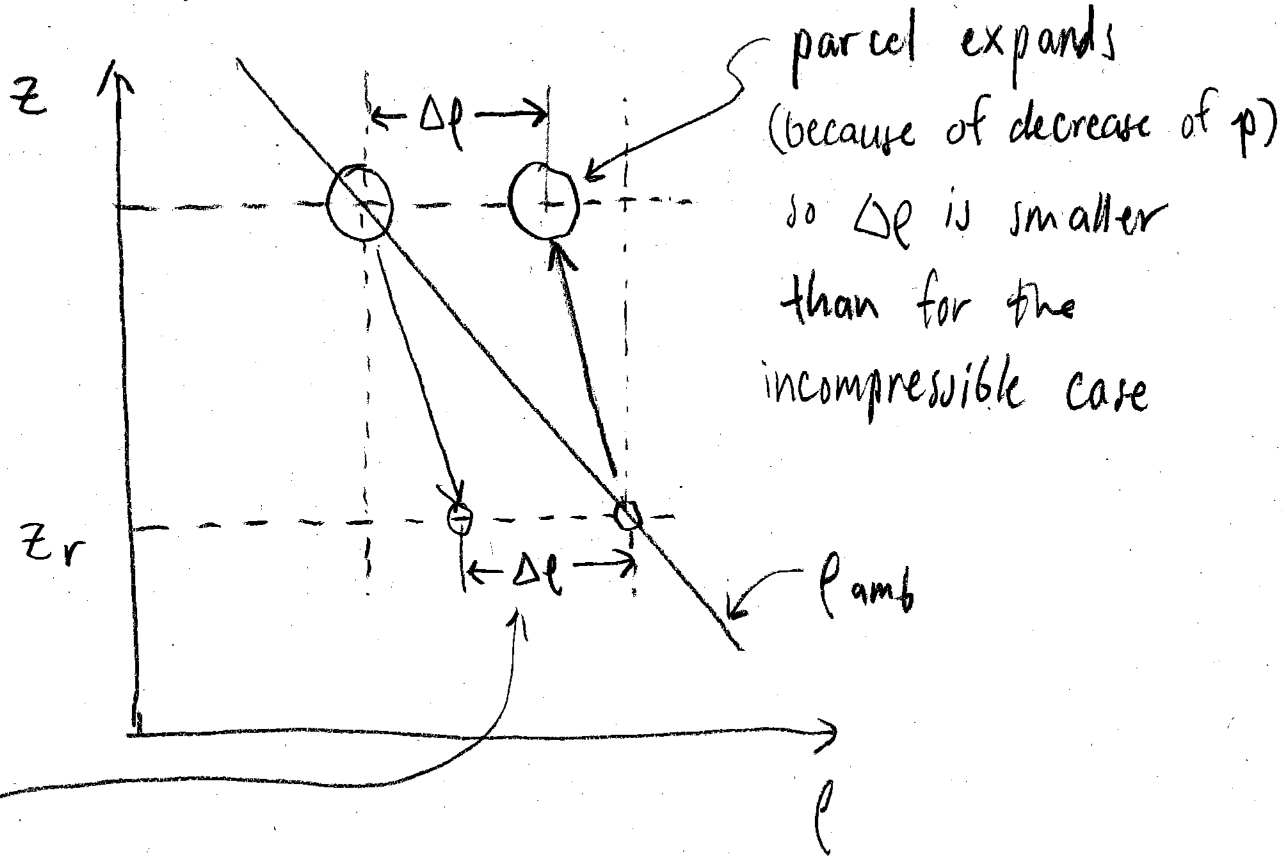
and the usual notation is

$$\left.\begin{aligned} T_r &\equiv \Theta = \text{potential temperature} \\ \rho_r &\equiv \rho_{pot} = \text{potential density} \end{aligned}\right\}$$

These quantities are used for seawater too!

These are the temperature & density after an isentropic change of pressure to p_r

Next, re-consider the Buoyancy Frequency, but for a compressible fluid



$\Delta\rho$ is most easily approximated by considering isentropic displacement of ambient fluid to p_r

and thus $\Delta\rho = -\Delta z \dfrac{\partial \rho_{pot}}{\partial z}$

(note $\rho_{pot} = \rho_{amb}$ at z_r, where $p = p_r$)

Hence $N = \sqrt{-\dfrac{g}{\rho_r}\dfrac{\partial \rho_{pot}}{\partial z}}$ a better estimate of the Buoyancy Frequency

(most accurate near z_r)

Typical values of N

Lower stratosphere	$1.7 \times 10^{-2}\ s^{-1}$	
Troposphere	$10^{-2}\ s^{-1}$	$\frac{2\pi}{N} = 10$ minutes
Upper Ocean	$10^{-2}\ s^{-1}$	
Deep Ocean	$10^{-3}\ s^{-1}$	

¿Why does N increase upwards in both cases?
{prompted interesting discussion! save time for it}

Q: Why use potential density?

A: A fluid with $\rho_{pot} = $ const. is <u>not</u> <u>stratified</u> even though it has $\frac{\partial \rho_{amb}}{\partial z} < 0$