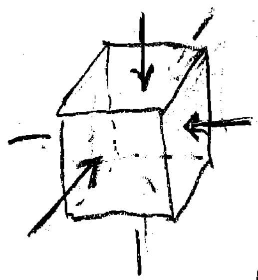
1.2

1/19/2012

Pressure, Hydrostatic Pressure, Buoyancy Frequency

pressure is fundamental

- a force per unit area: the direction of the force is defined by the direction of the normal to the area, so for a fluid parcel this is typically compressive



and p is a scalar

- units: Pa ("Pascal") $= \frac{N}{m^2} = \frac{kg\, m}{s^2}\frac{1}{m^2}$

- "bar" $= 10^5$ Pa ~ atm. pressure at sea level

  1 bar = 1000 mb ("millibars")

- equal to the added pressure at 10 m depth in the ocean

- in static equilibrium with gravity:

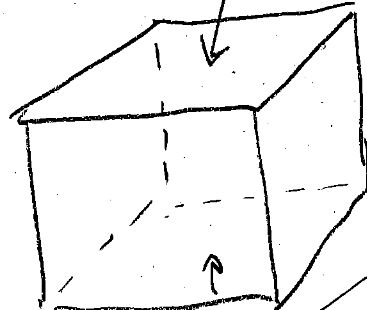
$$\boxed{p = \frac{\text{the weight of fluid overhead}}{\text{unit area}}}$$

- consider a "fluid parcel" of size $dx \cdot dy \cdot dz$

z, y, x

- extra weight on lower surface

$p(z)$ known



what is $p(z-dz)$?

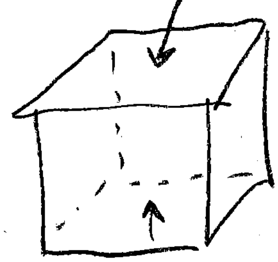
$= \text{mass} \times \text{gravity}$

$$= \rho \cdot \text{volume} \cdot g = \rho \cdot dx \cdot dy \cdot dz \cdot g$$

"rho" = density = $\dfrac{\text{mass}}{\text{unit volume}}$ $[\text{kg m}^{-3}]$

$$\therefore \text{extra pressure} = \frac{\text{force}}{\text{area}} = \frac{\rho \cdot \cancel{dx} \cdot \cancel{dy} \cdot dz \cdot g}{\cancel{dx} \cdot \cancel{dy}} = \rho g \cdot dz$$

so we have



$p = p_0$

$p = p_0 + \rho g\, dz$

$$\Rightarrow \frac{\partial p}{\partial z} = \lim_{dz \to 0} \left[\frac{p_0 - (p_0 - \rho g\, dz)}{dz} \right] = -\rho g$$

use a partial derivative because $p = p(x, y, z, t)$

So we have

$$\boxed{\frac{\partial p}{\partial z} = -\rho g}$$

Hydrostatic Balance (a simplified form of the z-momentum eqn.)

Note: ρ may vary in space, but since we are taking the limit $dz \to 0$ it is exactly the local value

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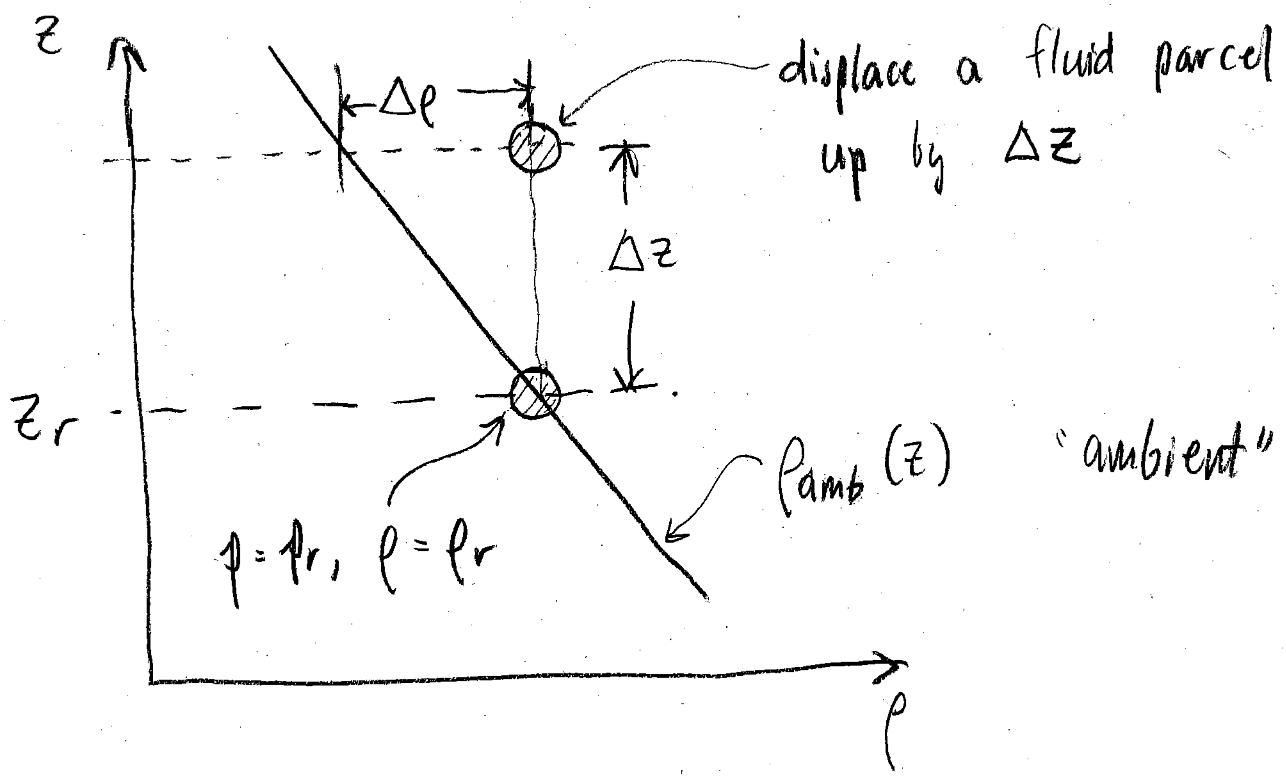
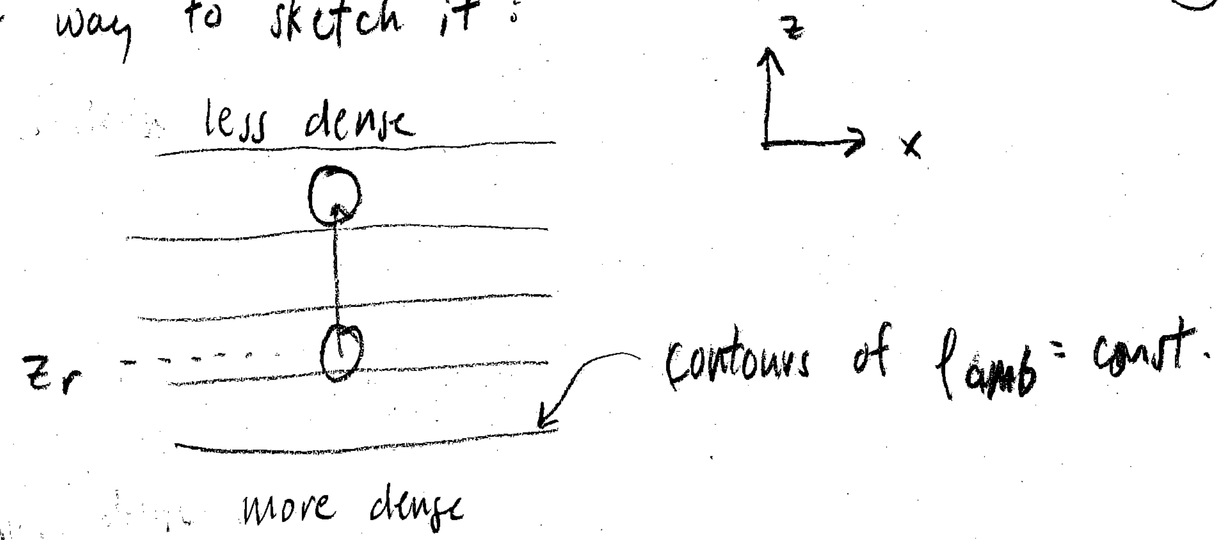
Buoyancy Frequency: consider a stratified, incompressible fluid



$$\Delta\rho = \rho_r - \rho_{amb}$$

Another way to sketch it:



The parcel density is now <u>greater</u> than that of the surrounding fluid, by an amount

$$\Delta \rho = -\Delta z \frac{\partial \rho_{amb}}{\partial z}$$

An approximate <u>force balance</u> for the parcel is:

$$\underbrace{\rho_r \frac{\partial^2 \Delta z}{\partial t^2}}_{\frac{\text{mass}\,\cdot\,\text{acceleration}}{\text{unit vol.}}} = \underbrace{-\frac{\partial p}{\partial z}}_{\frac{\text{force}}{\text{unit vol.}}\text{ due to pressure gradient of ambient fluid}} \underbrace{-\rho_r g}_{\frac{\text{weight of parcel}}{\text{unit vol.}}}$$

$$-\frac{\partial p}{\partial z} = g \rho_{amb} \quad (\text{at } z_r + \Delta z)$$

$$\Rightarrow \rho_r \frac{\partial^2 \Delta z}{\partial t^2} = g(\rho_{amb} - \rho_r) = -g \Delta \rho = g \frac{\partial \rho_{amb}}{\partial z} \Delta z$$

So we have a 2^{nd} order ODE for $\Delta z(t)$

$$\frac{\partial^2 \Delta z}{\partial t^2} = \frac{g}{\rho_r} \frac{\partial \rho_{amb}}{\partial z} \Delta z$$

guess a solution of the form $\Delta z = A \cos \omega t$

$$\Rightarrow \omega = \sqrt{\frac{-g}{\rho_r} \frac{\partial \rho_{amb}}{\partial z}} \equiv N$$ the "Buoyancy Frequency"

or Brunt-Väisälä Frequency

Note: this turns out to be the highest frequency possible for internal waves